Research Journal of Applied Sciences, Engineering and Technology 7(8): 1516-1528, 2014
DOI:10.19026/rjaset.7.428
ISSN: 2040-7459; e-ISSN: 2040-7467

Submitted: May 08, 2013 Accepted: June 10, 2013 Published: February 27, 2014

Research Article

Thermal Stresses in an Anisotropic Thin Plate Subjected to Moving Line Heat Sources

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Abstract: The aim of this present study is to investigate thermal stresses inside a thin anisotropic mild steal plate during moving line heat source. The parabolic heat conduction model is used for the prediction of the temperature history. The temperature distributions are determined numerically using finite difference method. Thermal stresses are computed numerically. It is found that the thermal conductivity ratio affect in both temperature and thermal stresses distributions, in additional to the speed and heat source intensity.

Keywords: Anisotropic, moving line heat source, parabolic heat conduction model, thermal stresses

INTRODUCTION

Heat conduction problems in anisotropic material have numerous important applications in various branches of science; that is the thermal conductivity varies with direction. Crystals, wood, sedimentary rocks and many others are example of anisotropic materials. Heat conduction problems involving a moving heat source have several applications, such as metal cutting, flame of laser hardening of metals, welding and others. The early published in this literature on this topic by Al-Huniti *et al*. (2004) investigated the variation of thermal and residual stresses inside a thin mild steel plate during welding process. It was found that welding speed and heat source intensity are the main factors that affect the residual stress formation in the plate. Al-Nimr and Naji (2000) described the thermal behavior of anisotropic material using hyperbolic heat conduction model, which assumed different phase lags between each component of the heat flux vector and the summation of temperature gradient in all direction of the orthogonal coordinate system. Zibdeh and Al-Farran (1995) presented a steady-state solution for the thermal stresses of a homogeneous, orthotropic hollow cylinder subjected to asymmetric temperature distribution at the outer surface and heat convection into a medium at zero reference temperature at the inner surface. The results show that the orientation of fibers of each layer affects the distribution of the stresses. Hou and Komanduri (2000) presented general solution for temperature rise at any point due to stationary/moving plane heat source of different shapes and heat intensity distributions (uniform, parabolic, rectangular and normal) using the Jaeger's classical heat source method. The solutions for the stationary heat source were obtained from the moving source solution by simply equating velocity of sliding to zero. The result shows that the temperature rise as well as its distribution around the heat source depends on several factors, including the heat intensity and its distribution, the shape and size of the heat source, the thermal properties and the velocity of sliding. Al-Huniti *et al*. (2001) investigated the dynamic thermal and elastic behavior of a rod due to a moving heat source. The hyperbolic heat conduction model was used to predict the temperature history. Also they presented the effect of different parameters such as moving source speed and the convection heat transfer. Laplace transformer and Riemann-sum approximations were used to determine the temperature, displacement and stresses distribution within the rod. Francis (2002) investigated the simulations of the welding process with moving heat sources for butt and tee joints using finite element analyses. From the transient heat transfer equation he obtained the thermal analysis, followed by a separate mechanical analysis based on the thermal history. Also presented the residual stresses for both butt and tee joints. The results shows for the butt joint that the maximum residual longitudinal normal stress was within 3.6% of published data and for a fully transient analysis this maximum stress was within 13% of the published result. And also shows for the tee, the maximum residual stresses were found to be 90-100% of the room-temperature yield strength. Araya (2004)

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presented a numerical simulation of the temperature field and the removed material resulting from the impingement of a moving laser beam on a ceramic surface. Finite volume was used to predict the temperature field including phase changes generated during the process. Ozisisik (1993) derives the energy equation in anisotropic material with its boundary condition. The same reference offers different analytical technique to solve energy equation. Also, this reference classifies different types of anisotropic materials.

In the present study, the temperature and thermal stresses distribution are investigated through an anisotropic thin plate subjected to moving line heat source. The temperature is determined numerically from the heat equation solution using finite deferent method definition implicit scheme. The calculation of thermal stresses is also based on numerical method that was used by Al-Huniti *et al*. (2004).

MATERIALS AND METHODS

Analysis:

Formulation of heat equation: Figure 1 shows a schematic diagram of the physical domain for a square thin anisotropic metal plate with moving line source. The plate has a length *L* and thickness *w*. The line moving heat source has constant speed *v* in y-direction.

Assumptions: The assumption can be summarized as follows:

- Plate is thin ($\frac{w}{L} \ll 1$).
- The heat source is a line heat source.
- The speed of moving heat source is constant.
- The thermal properties of material under consideration are constant with temperature. Without this assumption, the heat conduction equation nonlinear (Al-Huniti *et al*. 2004).
- Convection and radiation losses are neglected.

The line heat source is moving at a constant speed *V* along *y* direction and is releasing its energy continuously while moving. Hence, the form of the heat source is given by (Ozisisik, 1993):

$$g(t,x,y) = g_0\delta(y - vt)\delta(x)$$

where, $g(t, x, y)$ is the volumetric source W/m^3 and g_0 is the line source W/m.

A cartesian coordinate system $(x\ y)$ is chosen fixed to the work piece ($w/L<<1$), so can be neglected the z-axis. The transient heat conduction equation with heat generation in Cartesian coordinates for a thin solid can be expressed as follow (Al-Huniti *et al*., 2004):

$$\rho C\frac{\partial T}{\partial t} = k_{11}\frac{\partial^2 T}{\partial x^2} + k_{22}\frac{\partial^2 T}{\partial y^2} + 2k_{12}\frac{\partial^2 T}{\partial xy} - \frac{hu}{w}(T - T_\infty) - \frac{h1}{w}(T - T_\infty) + g_0\delta(x)\delta(y - vt) \quad (1)$$

where,

- ρ = The mass density (kg/m^3)
- C = The specific heat (J/kg.k)
- k = The thermal conductivity (W/m.k)
- T = Temperature at any location (K)
- $h1$ = Lower convection heat transfer coefficient (W/m^2.K)
- hu = The upper convection heat transfer coefficient (W/m^2.K)
- T_∞ = The ambient temperature (K)
- v = The velocity of moving heat source (m/s)

Let $h1 = hu = h$ then the equation becomes:

$$\rho C\frac{\partial T}{\partial t} = k_{11}\frac{\partial^2 T}{\partial x^2} + k_{22}\frac{\partial^2 T}{\partial y^2} + 2k_{12}\frac{\partial^2 T}{\partial xy} - 2\frac{h}{w}(T - T_\infty) + g_0\delta(x)\delta(y - vt) \quad (2)$$

Initial and boundary condition: Initially, the plate temperature is assumed to be uniform and equal to

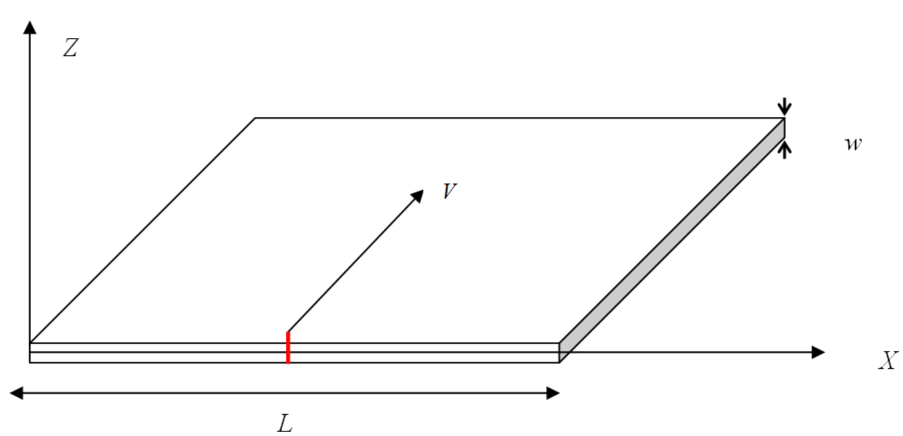

Fig. 1: Thin anisotropic plate with moving line heat source

ambient temperature. Hence, the initial and boundary condition are:

$$\begin{aligned} T(x,y,0) &= T_\infty \\ T(0,y,t) &= T_\infty \\ T(L,y,t) &= T_\infty \\ T(x,0,t) &= T_\infty \\ T(x,L,t) &= T_\infty \end{aligned} \tag{3}$$

The heat conduction equation for anisotropic equation with moving line heat source (2) will be transformed into dimensionless parameter form. For this, the following dimensionless parameters are introduced:

$$\theta = \frac{T - T_\infty}{T_\infty} \quad \varsigma = \frac{x}{L} \quad \eta = \frac{y}{L} \quad \tau = \frac{t}{t_0}$$
$$\gamma = \frac{L^2}{wk_{11}}(2h) \; G_0 = \frac{g_0}{T_\infty k_{11}} \quad V = \frac{vt_0}{L} \quad t_0 = \frac{L^2 \rho C}{k_{11}}$$
$$\varepsilon_{22} = \frac{k_{22}}{k_{11}} \quad {}_{12} = \frac{k_{12}}{k_{11}}$$

Substituting these parameters in Eq. (2) gives:

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial \varsigma^2} + \varepsilon_{22}\frac{\partial^2 \theta}{\partial \eta^2} + 2\varepsilon_{12}\frac{\partial^2 \theta}{\partial \varsigma \eta} - \gamma\theta + G_0 \delta(\varsigma)\delta(\eta - V\tau) \tag{4}$$

The dimensionless initial and boundary conditions are:

$$\begin{aligned} \theta(\varsigma,\eta,0) &= 0 \\ \theta(0,\eta,\tau) &= 0 \\ \theta(L,\eta,\tau) &= 0 \\ \theta(\varsigma,0,\tau) &= 0 \\ \theta(\varsigma,L,\tau) &= 0 \end{aligned} \tag{5}$$

The Following a procedure to solve the governing equations by implicit method, which transformed to algebraic equations as shown below:

$$\begin{aligned} \frac{\theta_{i,j}^{n+1} - \theta_{i,j}^{n}}{\Delta\tau} &= \frac{\theta_{i+1,j}^{n+1} - 2\theta_{i,j}^{n+1} + \theta_{i-1,j}^{n+1}}{\Delta\varsigma^2} \\ &+ \varepsilon_{22}\frac{\theta_{i,j+1}^{n+1} - 2\theta_{i,j}^{n+1} - \theta_{i,j-1}^{n+1}}{\Delta\eta^2} \\ &+ 2\varepsilon_{12}\frac{\theta_{i+1,j+1}^{n+1} - \theta_{i+1,j-1}^{n+1} - \theta_{i-1,j+1}^{n+1} + \theta_{i-1,j-1}^{n+1}}{4\Delta\varsigma\eta} \\ &- \gamma\,\theta_{i,j}^{n+1} + G_0\delta(\varsigma)\delta(\eta - V\tau) \end{aligned} \tag{6}$$

Let $\Delta\varsigma = \Delta\eta$, arranging the above Eq. (6) gives the final solutions for parabolic heat equation:

$$\begin{aligned} &(2\mathrm{A} + 2\mathrm{A}\varepsilon_{22} + \gamma\Delta\tau + 1)\theta_{i,j}^{n+1} - \mathrm{A}(\theta_{i+1,j}^{n+1} + \theta_{i-1,j}^{n+1}) \\ &- \mathrm{A}\varepsilon_{22}(\theta_{i,j+1}^{n+1} + \theta_{i,j-1}^{n+1}) \\ &- \mathrm{A}\frac{\varepsilon_{12}}{2}(\theta_{i+1,j+1}^{n+1} - \theta_{i+1,j-1}^{n+1} - \theta_{i-1,j+1}^{n+1} + \theta_{i-1,j-1}^{n+1}) = \\ &\theta_{i,j}^{n} + G_0\delta(\varsigma)\delta(\eta - V\tau)\Delta\tau \end{aligned} \tag{7}$$

where, $A = \dfrac{\Delta\tau}{\Delta\varsigma^2}$

Thermal stresses: The thermal stresses present in many branches of engineering, considered an important factors affect on the life of material, so that become a very significant in application involving large temperature difference and they important to determine the life of material, when the temperature rise in a homogeneous body, different element of body tend to expand by different amount, an amount proportional to the local temperature raise.

After determine the dimensionless temperature distributions in the plate, it can be found the thermal stresses distribution. Each point have strain, it come from tow affect, the first effect is due to uniform expansion and is proportional to the temperature raise, the other come from the stresses resulting from external load, but the material is assumed homogeneous with no external loads. The stresses variation in the *y* direction is much less than those in x direction (Fig. 1) (Al-Huniti *et al*., 2004). The thermal stresses due to temperature raise can be determine from the following equation (Al-Huniti *et al*., 2004):

$$\sigma_{th} = E\alpha\Delta T \tag{8}$$

Which σ_{th} value of thermal stresses limited at specific temperature, E the young's modulus of elasticity and α coefficient of thermal expansion? These proprieties dependant on the temperature and very considerably at elevated temperature, so that to determine the thermal stresses we must evaluate these parameters (E, α) the Fig. 2 and 3 shown the approximate variation of these parameters with the temperature for mild steal (Al-Huniti *et al*., 2004).

Solution: After applying finite difference we have a system of linear algebraic Eq. (7). To solve algebraic equation we should create grids (steps) for the time and space (ς, η), let us divided the length of the plate in to equal space $\Delta\varsigma = \Delta\eta$ = 0.1 and for dimensionless time $\Delta\tau = 1.38888888\ 89\mathrm{e}-3$, then linear algebraic Eq. (7) can be written as matrix equation:

$$\Psi\theta = b$$

So that, in each time step we have a matrix to be solved. The matrix inverse method is used to determine

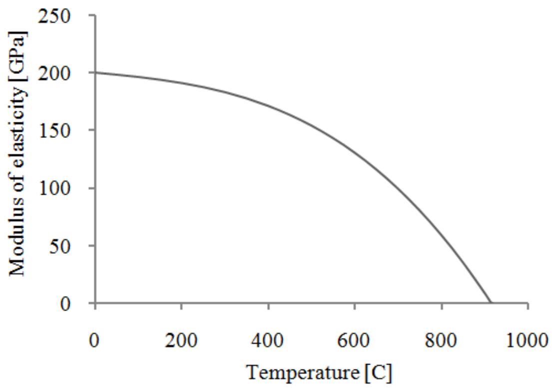

Fig. 2: Modulus of elasticity for mild steel with temperature

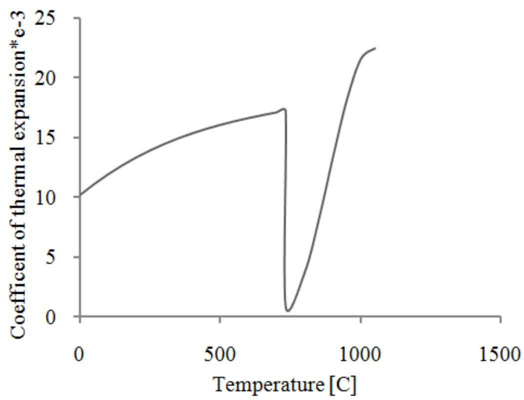

Fig. 3: Coefficient of liner expansion for mild steel with temperature

the dimensionless temperature distribution in each time step. Hence, when the matrix inverse is known ψ^{-1}, the solution θ is simply the product of the matrix inverse ψ^{-1} and the right hand side vector *b* as shown in equation below:

$$\theta = \Psi^{-1} b$$

Solving the thermal stresses Eq. (8) need the property of parameter in each temperature value, Fig. 2 and 3 will be used to update the property at each temperature values.

RESULTS AND DISCUSSION

Equations (7) represented the dimensionless solution of the parabolic heat equations of moving line; afford the transient dimensionless temperature variation with space and time. The property of material will be used (mild steal) (Al-Huniti *et al*., 2004) are given in Table 1.

The ratio of thermal conductivity (ε_{22}, ε_{12}) will be change during the solution to understand the effect of anisotropic on the temperature and thermal stresses distribution at moving line heat source.

Figure 4 shows the transient thermal behavior of the plate of specific point: (ς = 0.5, η = 0.7) at different ratio of thermal conductivity (ε_{22} ε_{12}), during moving line heat source. It is clear from figure when the ratio of thermal conductivity increases the peak value of temperature distribution decrease and cooling process occur slowly these mean the conductivity within the thin plate increase. In the other hand decrease the ratio

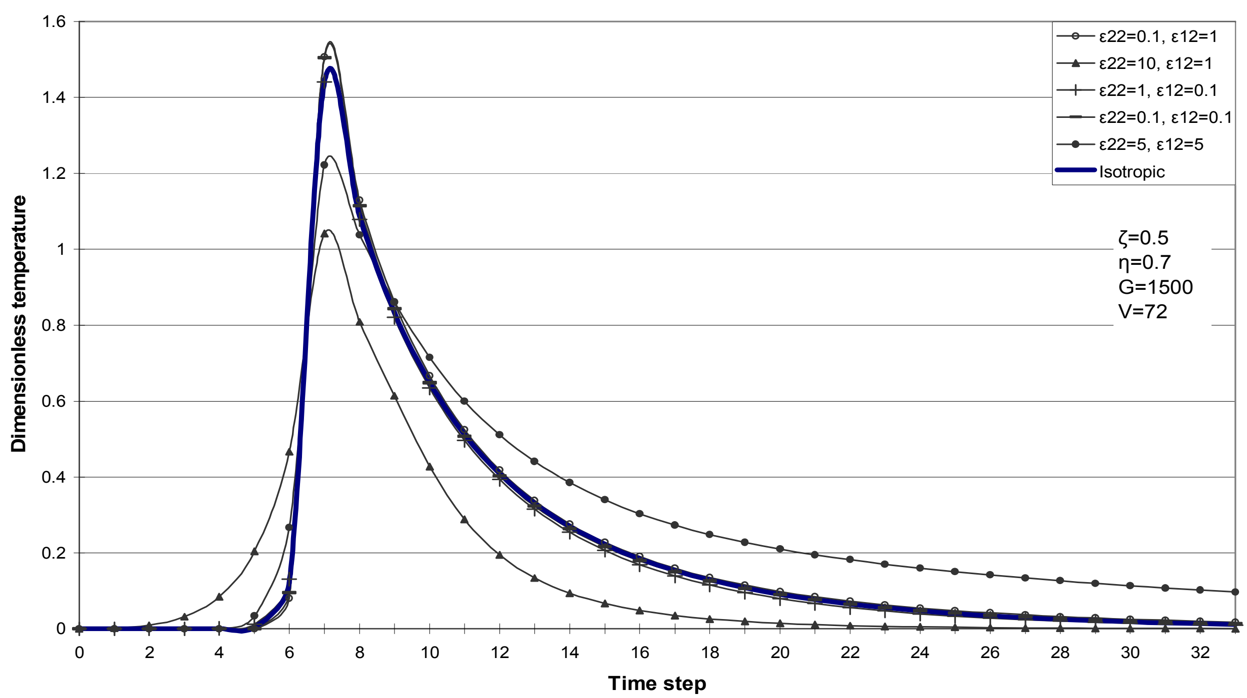

Fig. 4: Temperature distributions of anisotropic (different cases)

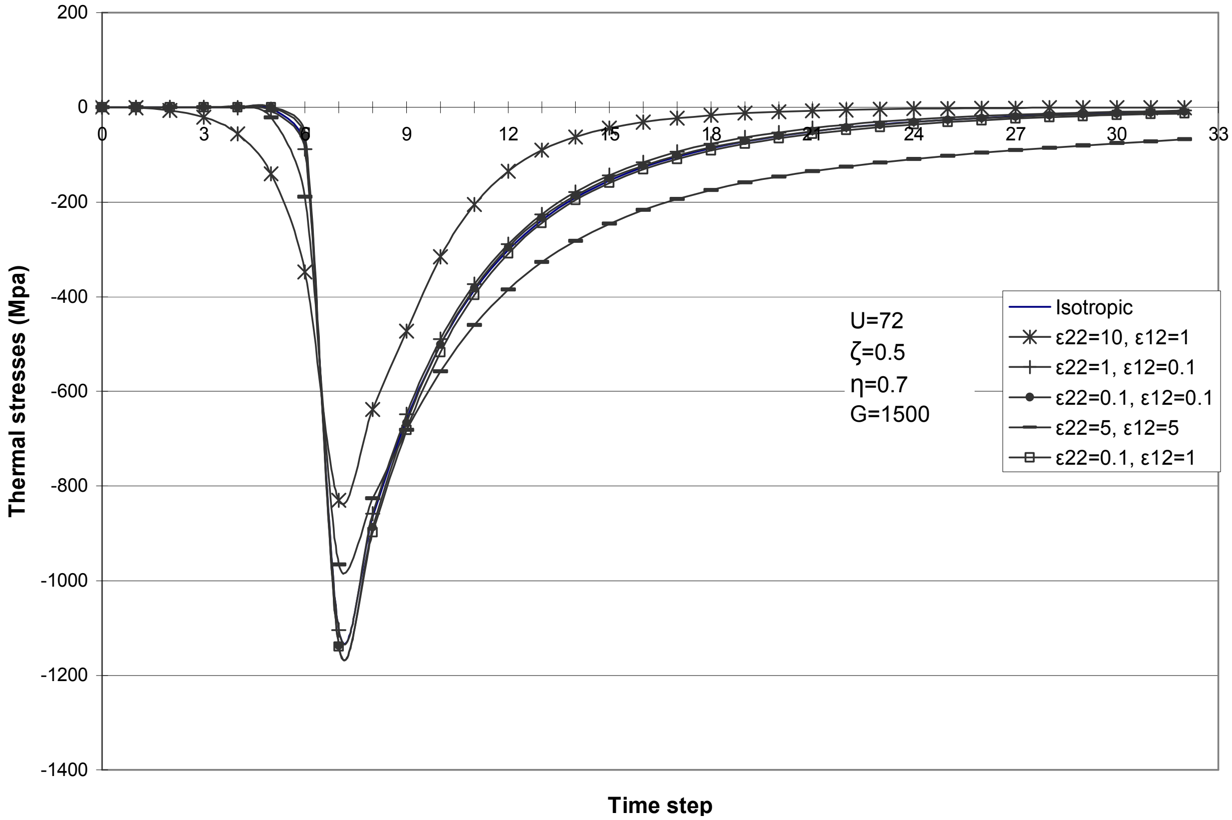

Fig. 5: Thermal stresses distribution of anisotropic (different case)

Table 1: Properties of mild steal

Mass density (ρ)	7800 kg/m^3
Specific heat (C)	450 J/kg °C
Thermal conductivity (K_{11})	65 W/m °k
Plate length (L)	1 m
Plate thickness (w)	0.005 m
Velocity of moving heat source	0.001333 m/s
Plate initial and ambient temperature (T_∞)	300 K
Convection heat transfer coefficient (from steal to air) (h)	15 W/m^{2}K
Heat source intensity (G)	1500

of thermal conductivity (ε_{22} ε_{12}) let the cooling process within the plate occurred rapidly and the peak value of temperature distribution increase. Where the Fig. 5 shows the transient thermal stresses of the same point, it is clear from figure when the ratio of thermal conductivity increase the peak value of thermal stresses distributions decrease due to the temperatures behavior as shown in Fig. 4. Where the peak of temperature distribution decrease, where the plate become more able to passes the heat through the plate.

Figure 6 to 8 shows the transient thermal behavior of the plate along the longitudinal line along ς = 0.5 at different ratio of thermal conductivity (ε_{22} ε_{12}), during moving line heat source. It is a clear from Fig. 7 the temperature distribution became more uniform along longitudinal line when the ratio of thermal conductivity decrease while in other hand when the ratio of thermal conductivity increase the temperature distribution near the edges less than other's there are due increase the ratio of conductivity, this mean the conductivity for the plate increase and became more abler to passing the temperature through the plate. The sketches in Fig. 9 to 11 illustrate the distribution of dimensionless temperature for these cases at specific time step = 7. Where a Fig. 12 to 14 shows the transient thermal stresses distributions for this longitudinal line. It is a clear from Figure 13 the peak value of thermal stresses increase along longitudinal line when the ratio of thermal conductivity decrease, Where the thermal stresses increase by increasing temperature, but after moving the heat source the area cool down then the stresses decrease with time.

Figure 15 shows the transient thermal behavior of specific point: (ς = 0.5, η = 0.7) at different values of heat source input, namely for G = 500, 1000, 1500 and 2100. It is illustrated the heat intensity increase, higher temperature distributions will be given, due to the more heat reached the plate, this situation for all cases for anisotropic. Where Fig. 16 show the transient thermal stresses of this point at different values of heat sources.

It is illustrated the thermal stresses intensity increase, due higher temperature distributions that given due to the more heat reached the plate. This situation for all cases for anisotropic.

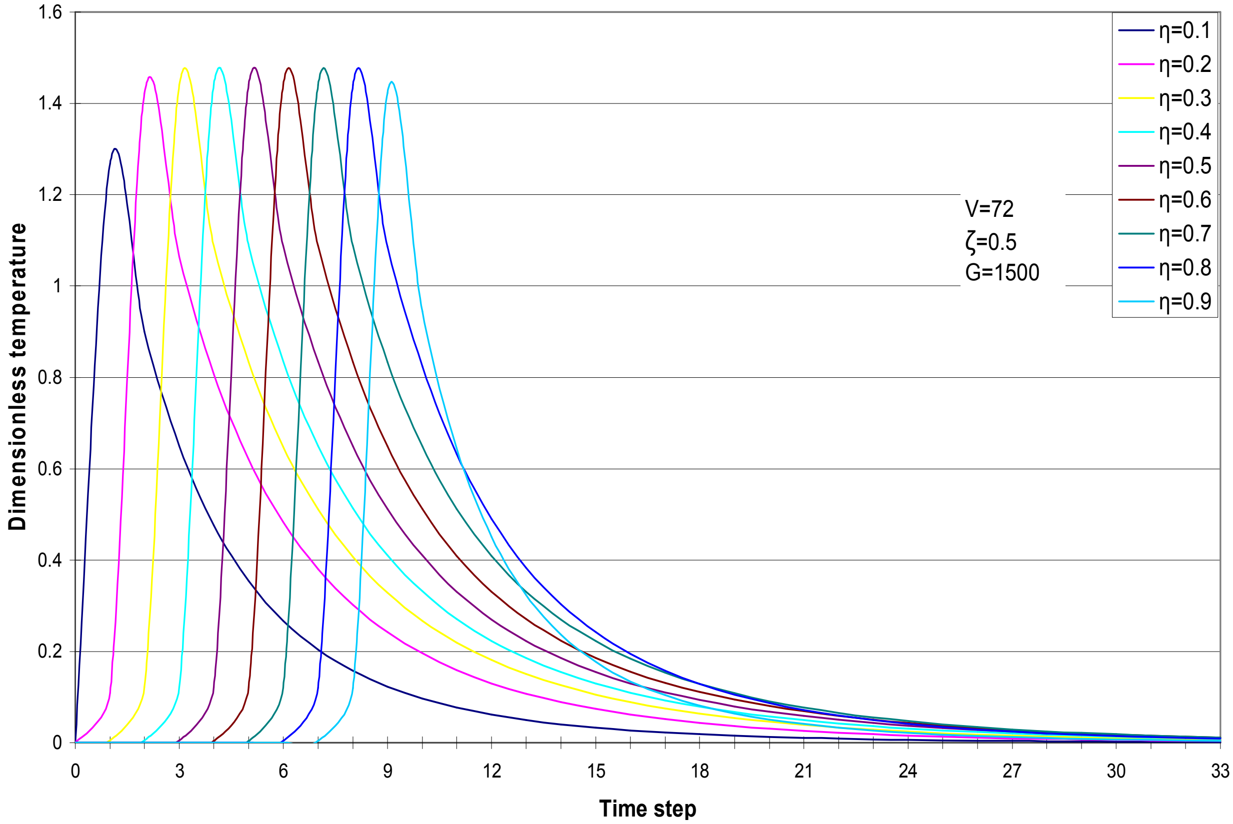

Fig. 6: Temperature distributions along direction for isotropic

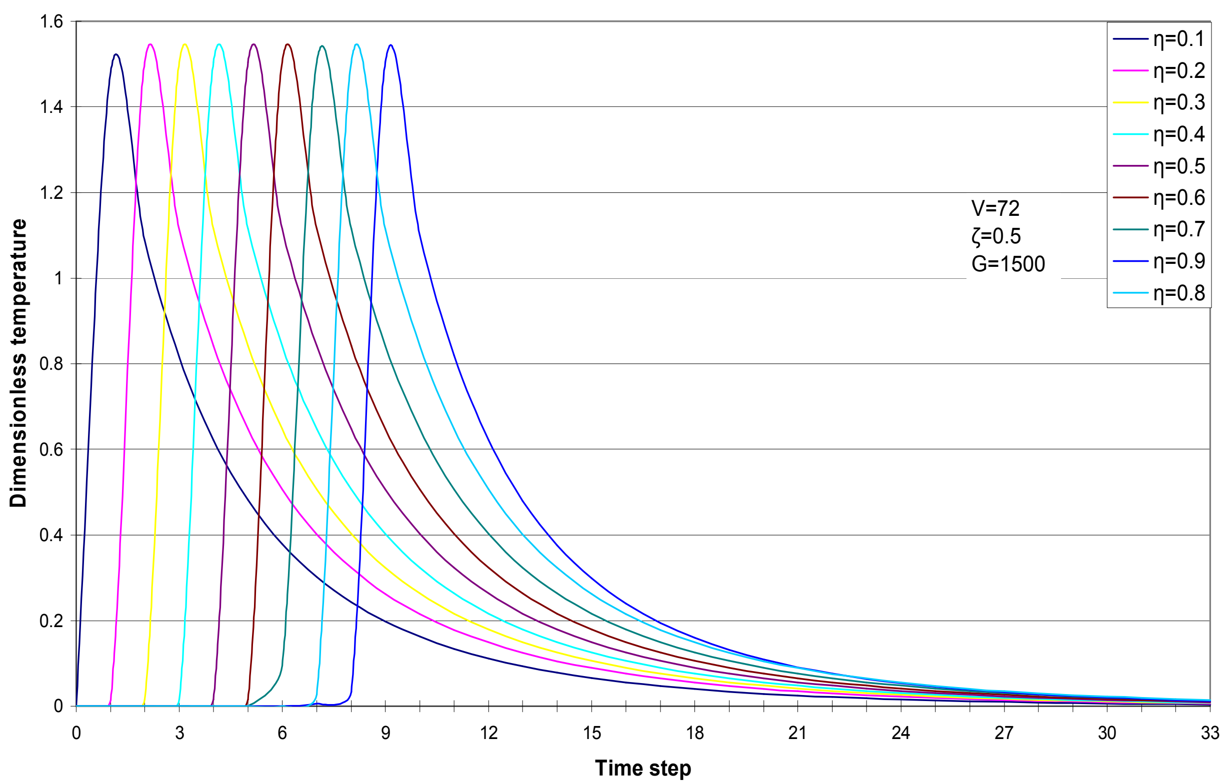

Fig. 7: Temperature distribution along direction with time for anisotropic ($\varepsilon_{22} = 0.1$, $\varepsilon_{12} = 0.1$)

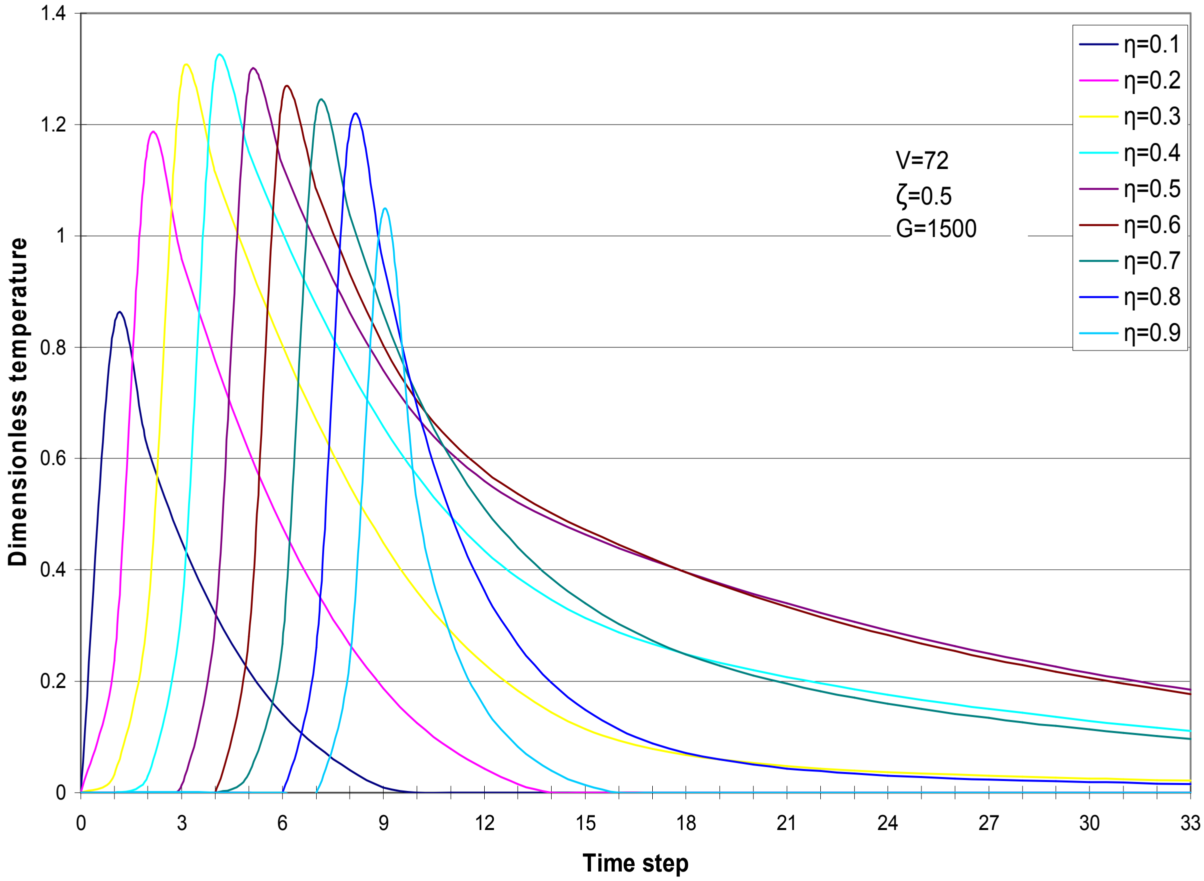

Fig. 8: Temperature distribution along direction with time for anisotropic ($\varepsilon_{22} = 5$, $\varepsilon_{12} = 5$)

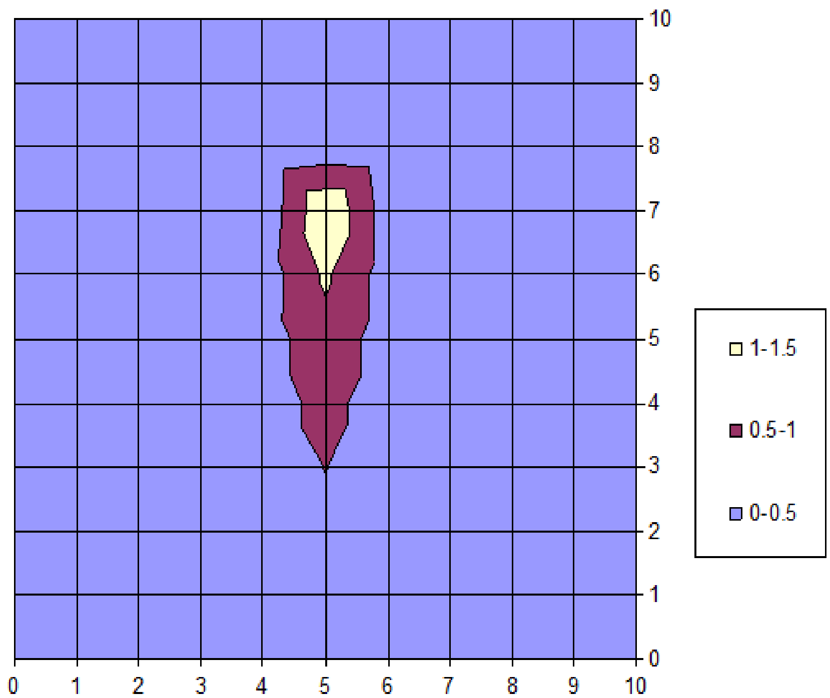

Fig. 9: Sketch for temperature distributions for isotropic

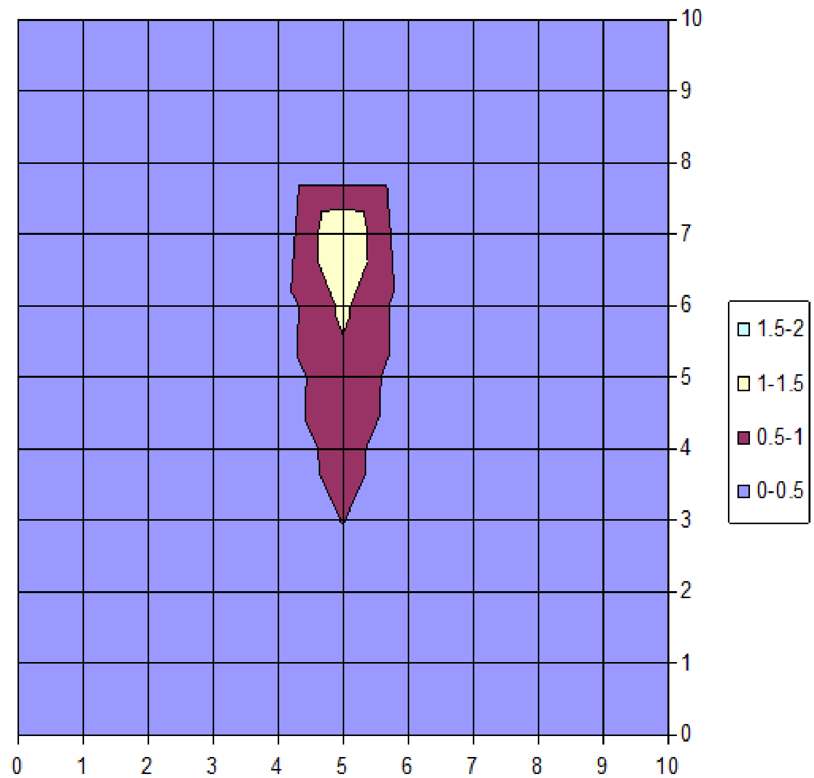

Fig. 10: Sketch for temperature distributions for anisotropic ($\varepsilon_{22} = 0.1$, $\varepsilon_{12} = 0.1$)

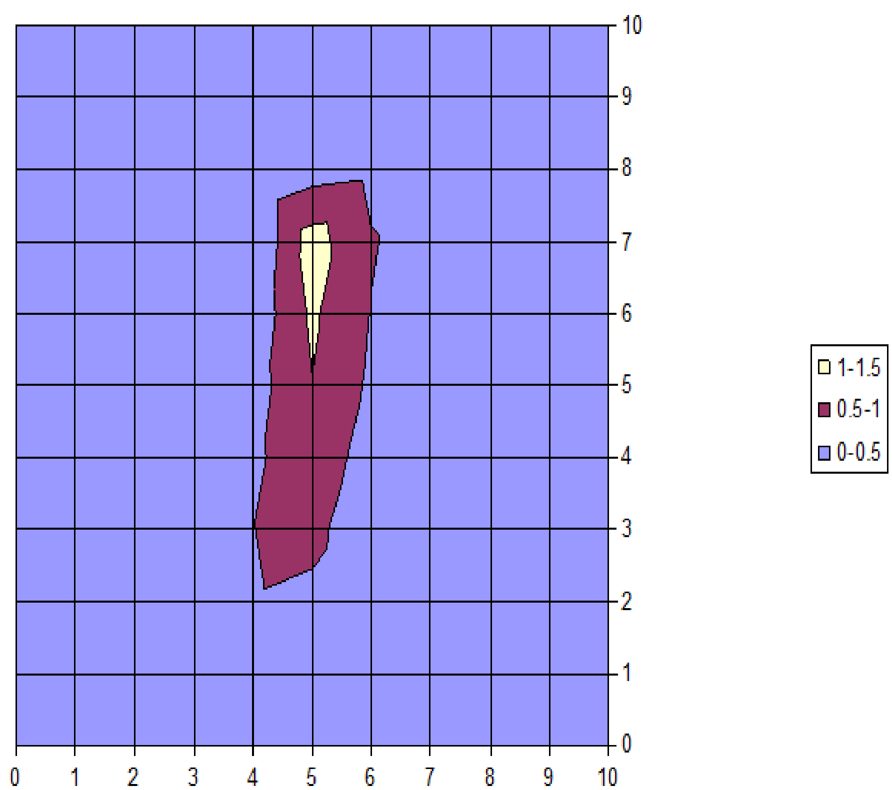

Fig. 11: Sketch for temperature distributions for anisotropic ($\varepsilon_{22} = 5$, $\varepsilon_{12} = 5$)

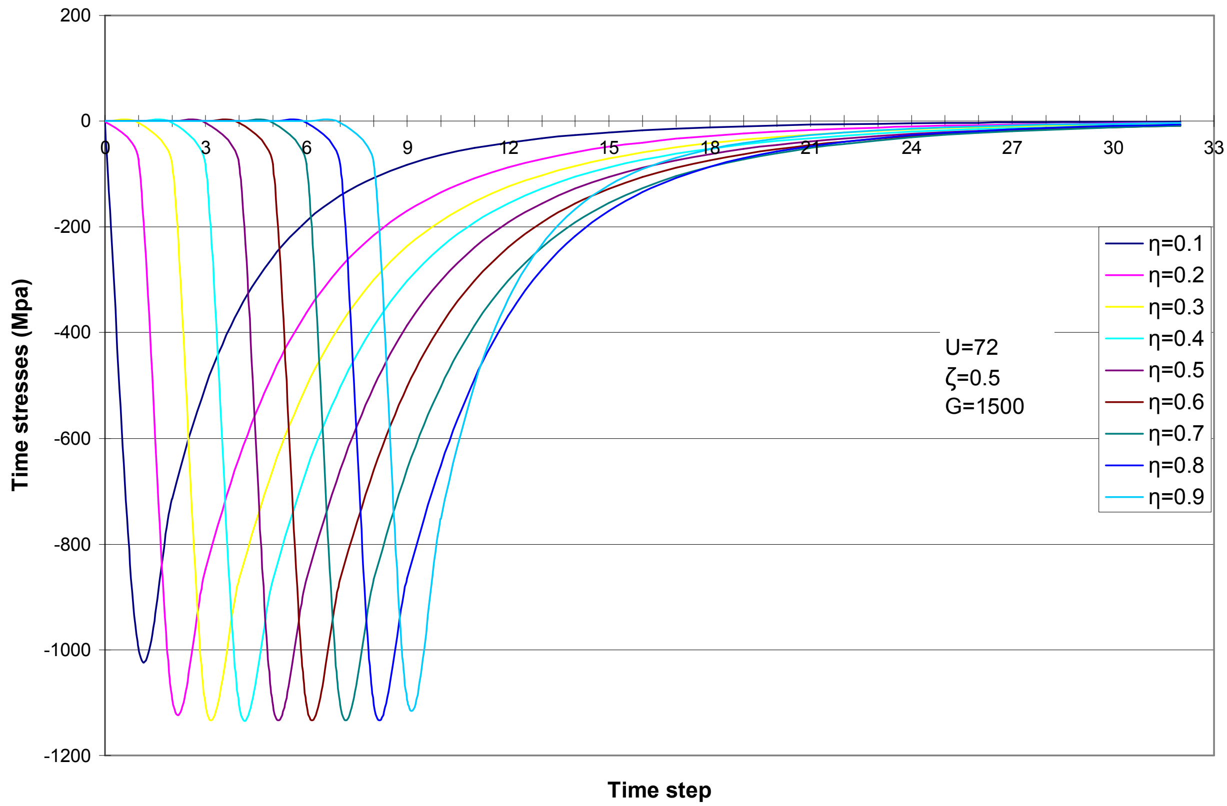

Fig. 12: Thermal stresses a long moving line direction for isotropic

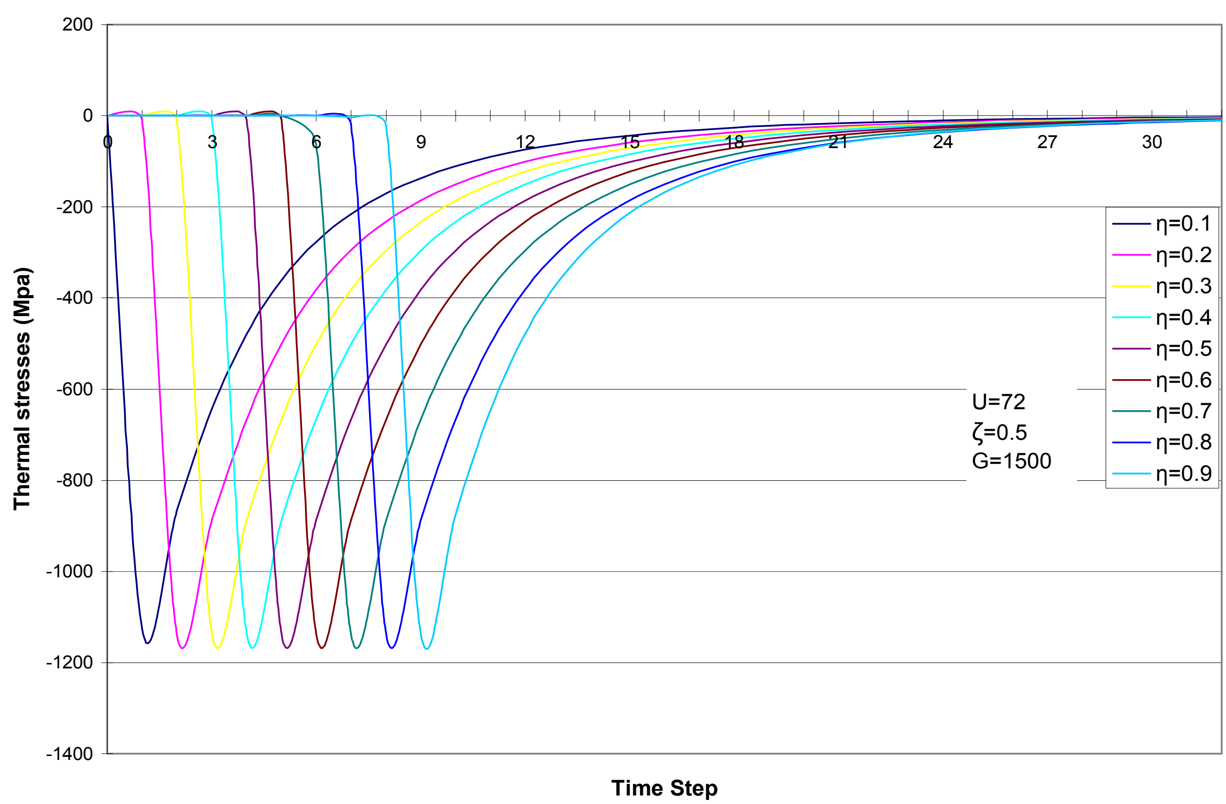

Fig. 13: Thermal stresses ($\varepsilon_{22} = 0.1$, $\varepsilon_{12} = 0.1$) along direction of moving line heat source

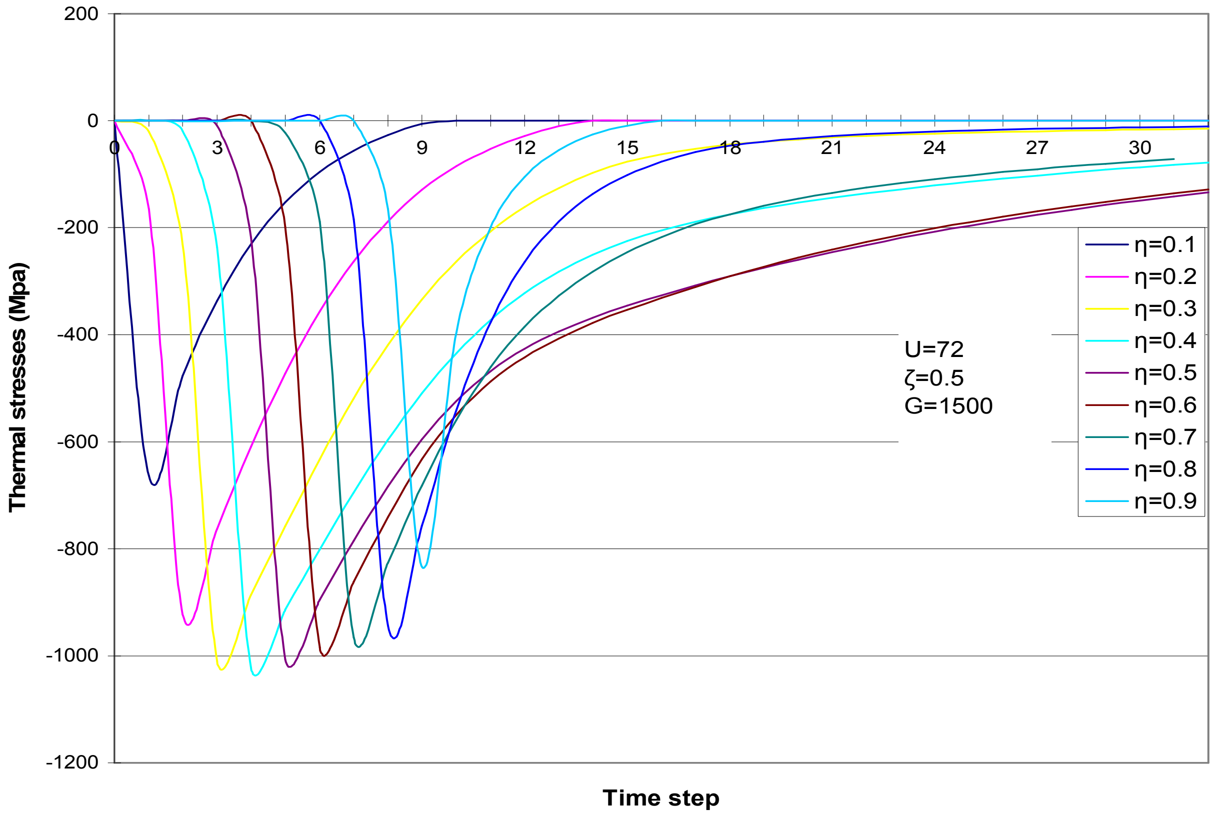

Fig. 14: Thermal stresses ($\varepsilon_{22} = 5$, $\varepsilon_{12} = 5$) along direction of moving line heat source

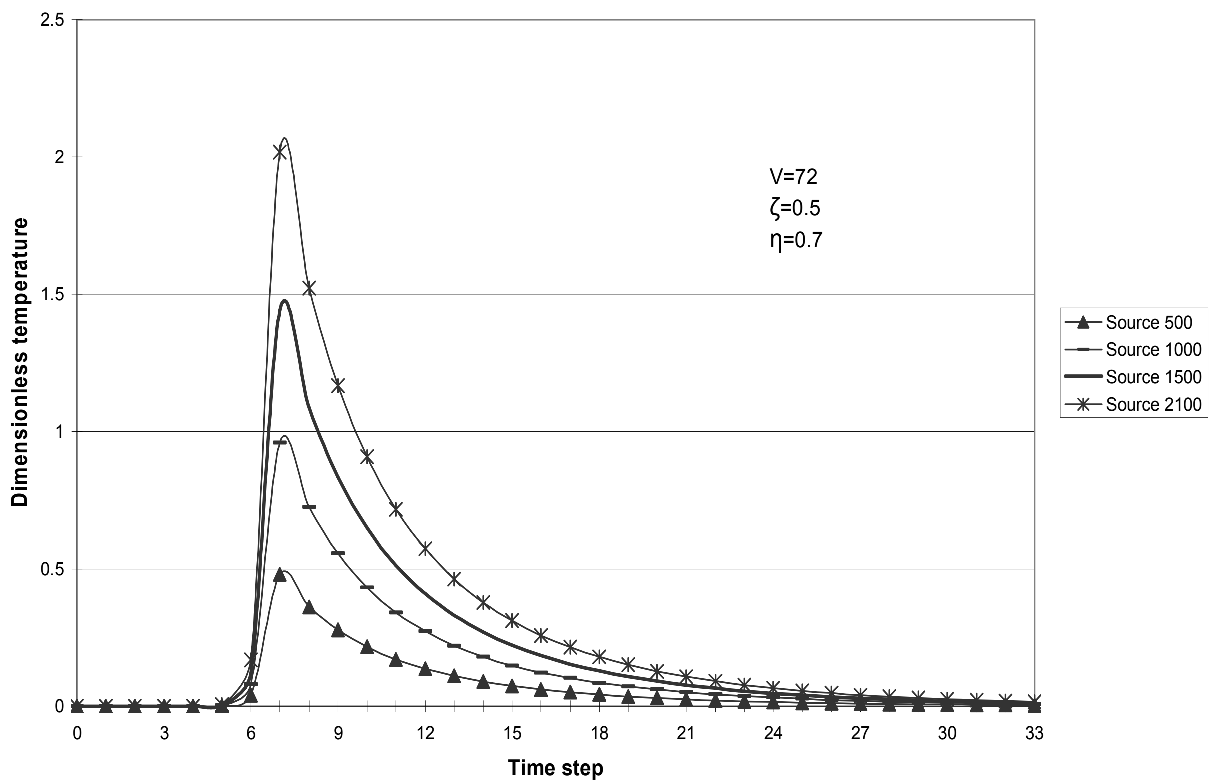

Fig. 15: Temperature distributions for isotropic with different heat source

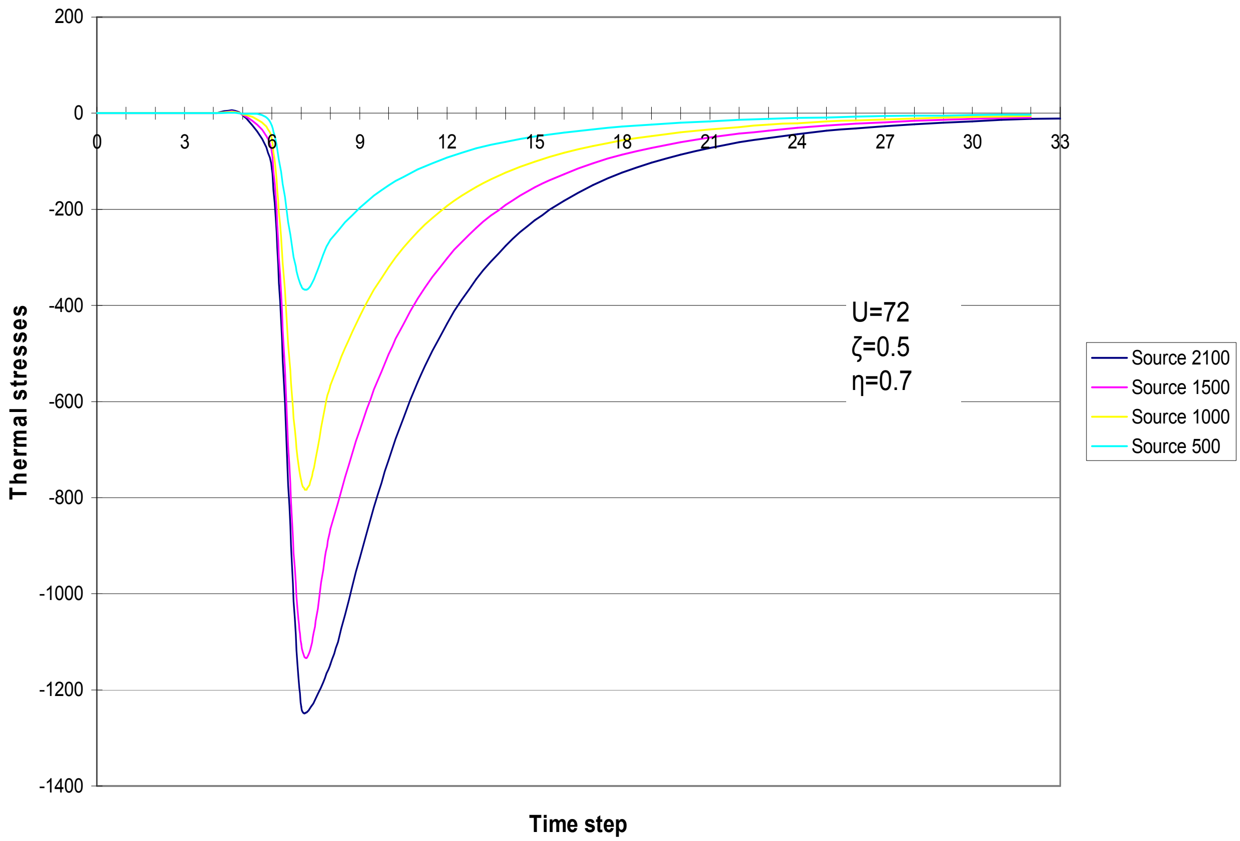

Fig. 16: Thermal stresses distributions for isotropic with different heat source value

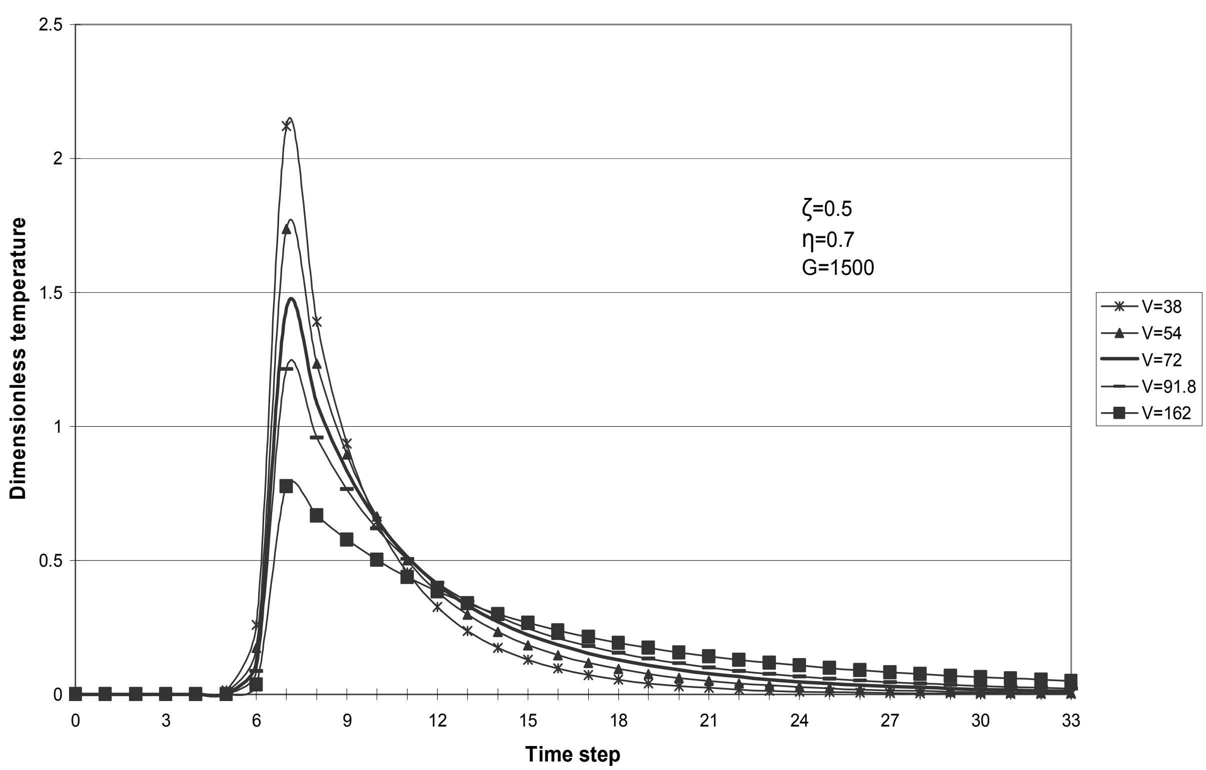

Fig. 17: Temperature distribution for isotropic with different velocity

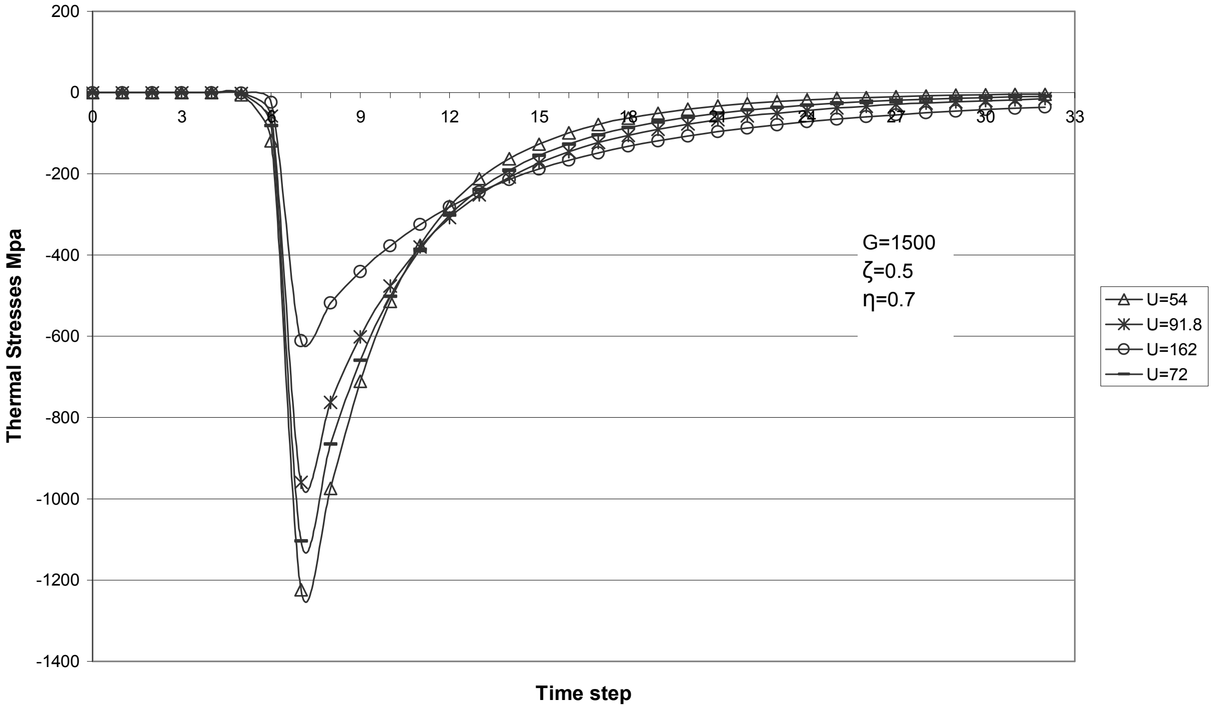

Fig. 18: Thermal stresses for isotropic with different velocity

Figure 17 shows the effect of changing the moving heat source speed on thermal cycle. Five moving heat source dimensionless speed are taken into consideration: V = 36, 54, 72, 91.8 and 160. From figure illustrate that is the moving heat source speed decrease, higher temperature value will be reached inside the plate, this mean more amount of power delivered to the plate per unit time. This situation for all cases for anisotropic. Where Fig. 18 show the effect of changing the moving heat source speed on the thermal stresses. From figure illustrate that when moving heat source decrease, higher thermal stresses value will be reached inside the plate, due to increase the temperature this mean more amount of power delivered to the plate per unit time. This situation for all cases for anisotropic.

CONCLUSION

The thermal stresses in thin anisotropic plate involving a moving line heat source are presented. The parabolic heat conduction model is used to evaluate the thermal behavior of thin anisotropic plate.

The governing equation is derived and solved using finite difference method by implicit scheme.

The effect of dimensionless ratio of thermal conductivity and moving heat source value or speed were studied. The peak value of temperature and thermal stresses distribution in the plate is found decrease rapidly with increase the ratio of thermal conductivity.

The thermal stresses in the plate are found follow the behavior of temperature.

The temperature and thermal stresses of the plate are found to increase at small moving heat source speed.

This due to fact that decrease the speed means the source release more amount of energy.

Decrease of the value of moving heat source are found to decrease the temperature and thermal stresses. This is due to the fact decrease the value means the mount of energy reached to the plate will be reduce.

NOMENCLATURE

L : Plate side length, m
C : Specific heat, J/m^3 k
g, g_0 : Heat source per unite length
E : Young's modulus, N/m^2
w : Plate thickness, m
k : Thermal conductivity, W/m k
ε_{12}, ε_{22} : Ratio of thermal conductivity in xy and y direction
q : Heat flux vector, W/m^2
T_∞ : Plate initial and ambient temperature, K
T : Temperature, k
t : Times
V : Moving line heat source speed
G : Dimensional heat source
h_l, h_u : Lower and upper surface heat convection coefficients

Greek symbols:

α : Coefficient of thermal expansion
τ : Dimensionless time
ς : Dimensionless coordinate transverse directions

η : Dimensionless coordinate in longitudinal direction
Θ : Dimensionless temperature
σ_{th} : Thermal Stresses
ρ : Mass Density, kg/m^3
δ : Unit step function

ACKNOWLEDGMENT

The Authors extend their appreciation to the Deanship of Scientific Research at King Saud University for funding the work through the research group project No. RGP-VPP-036.

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