

## Research Article

### Analysis and Optimization of Static Characteristics in Overall Operating Conditions of Electromagnetic Actuator

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**Abstract:** This analysis focused on looking for a method to optimize electromagnetic actuators in overall operating conditions. With disadvantages of slow convergence and poor accuracy, available optimization methods are difficult to achieve optimization results of electromagnetic actuators in overall operating conditions. In order to solve the problem, an optimization method suitable for overall operating conditions is proposed based on the electromagnetic difference model. The method combines advantages of the analytical method and finite element method. It uses numerical method to establish static characteristic sensitivity matrix of optimization system and parameter optimization results are obtained by optimizing difference model of target characteristics and current characteristics. In this study, a diesel speed electromagnetic actuator as an example, quantitative analysis of system parameters effect on static characteristics in overall operating conditions is done. Using the optimization method, in overall operating conditions, average electromagnetic force and standard deviation electromagnetic force of the electromagnetic actuator are optimized. The simulation results verify the optimization method is simple, effective and applicable.

**Keywords:** Electromagnetic actuator, optimization, quantitative analysis, sensitivity matrix, static characteristics

## INTRODUCTION

Due to a simple structure, good reliability and easy control, electro-magnetic actuators are widely used to drive fuel pump racks in speed position loop control system of diesel engines. With the progress of science, technology and social needs, characteristics of electromagnetic actuators such as precision, life, heating, energy consumption and linearity, static and dynamic characteristics and so on, are also demanded higher than before. Therefore, optimization design of electromagnetic actuators must be done to meet those demands.

Performance optimization methods of electromagnetic actuators can be divided into the Finite Element Method (FEM) and the analytical method. With high precision, FEM is particularly suitable to optimize the static and dynamic characteristics at a operating condition, but it has disadvantages such as a long calculation time and without the ability to optimize static and dynamic characteristics in overall operating conditions. At the same time, by the functional limitations of modeling software, its optimization algorithms cannot meet the multi-level needs of users. The analytical method is suitable to optimize static and dynamic multi-objective characteristics. Its calculation is fast, but due to the complexity of the system model

and the difficulty of setting boundary parameters (such as magnetic reluctance and magnetic leakage), the model is not accurate and optimization results have a large error. Therefore, a lot of researches are done to improve the optimization method. Regression and fitting methods take advantage of Design of Experiment (DOE) to calculate dynamic and static characteristics of the system in overall operating condition planes and in independent and interaction conditions of system parameters and then, with the regression method or neural network method, the relationship model between the system parameters and characteristics is established and the system parameters are optimized with various optimization algorithms (Yucel and Michielssen, 2012; Yun-Dong *et al.*, 2006). The method is simple, but a large amount of data is needed to improve accuracy of the model and to reflect the relativity of the system parameters and internal rules of the system. In magnetic reluctance network method, those parts of magnetic flux which are of evenly distribution and relatively regular shape in magnetic field are taken as an independent unit and the unit's magnetic reluctance is calculated. Moreover, all units are connected with magnetic network to form system models (Nakamura and Ichinokura, 2008; Fukuoka *et al.*, 2012). Before the establishment of magnetic reluctance network, it is necessary to make a qualitative analysis on the magnetic

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field distribution. If the magnetic field distribution is too complex to make a qualitative analysis, the application of this method will be limited. Magnetic reluctance network method is simple, convenient and accurate, but its modeling process is complex. Space mapping is to create a mapping relation between magnetic circuit model and finite element model and use the magnetic circuit mode to do an optimization (Encica *et al.*, 2007; Laurentiu *et al.*, 2008). The key to this method is to establish the mapping relationship of the magnetic circuit and the finite element model, but this relationship, especially in the case of the saturated magnetic field, is difficult to obtain accurate. Multidisciplinary optimization can get an optimal solution for making full use of cooperative effects, which are generated on mutual interaction of multidiscipline (Oriol *et al.*, 2007; Li *et al.*, 2012). Multidisciplinary optimization can shorten design cycles by parallel computing and design, but computational complexity caused by multiple iterations of the various sub-disciplines computing model, information exchange complexity caused by the organization and management of the information exchange between the various disciplines and other issues are difficult to handle.

Sensitivity analysis of the geometrical structural parameters can express the change relationship between the performance of the system output and structural parameters and it is a powerful tool for analysis and design of the system features. An automatic differentiation method is proposed to calculate sensitivity (Enciu *et al.*, 2011). The average minimum objective function of system parameters and performance is established to optimize geometrical parameters of electro-magnetic actuator mode. Dynamic networks and static finite element model are used to simulate system dynamic characteristics and nonlinear load characteristics and a sensitivity analysis of the system characteristics is done through the establishment of the electromagnetic force table and an experimental design method (Neubert *et al.*, 2011). An electromagnetic lumped parameter model is established and sensitivity analysis is done (Yu *et al.*, 2010). A fast predictable calculation method of dynamic characteristics after a change of the electromagnetic system size parameters is proposed (Zhai *et al.*, 2010). The method that combines rapidity of magnetic circuit method and accuracy of finite element method can solve the contradiction of efficiency and accuracy of calculating the electromagnetic system problem. Those literatures make useful explorations of the sensitivity analysis of system performance analysis and optimization.

In this study, a new optimization method is proposed to solve the problems of electromagnetic actuators performance optimization, which combined the advantages of numerical and analytical method. In the method, a numerical model of the electromagnetic

actuators is established with finite element software and sensitivity of system parameters relative to system outputs is obtained. Then, through establishing a difference model of the target and measurement electromagnetic force and taking advantage of the sensitivity, the model parameters are optimized. The method avoids establishing complicated analytical models of electromagnetic actuators. A test proves that the method is effective and feasible.

## STATIC CHARACTERISTICS OF ELECTRO-MAGNETIC ACTUATOR

The structure of the electro-magnetic actuator for diesel engine speed governing is shown in Fig. 1. The static characteristic of the electromagnetic actuator is a relationship between electromagnetic force, current and stroke. When structure and material of the electromagnetic are fixed on, its electromagnetic force is a function of current and stroke. The function can be expressed as:

$$F = f(I, x) \quad (1)$$

According to the electromagnetic actuator structure, the model of the electromagnetic system is created with finite element software ANSOFT. By simulation, its static characteristic curves are shown in Fig. 2. According to Fig. 2, in working stroke (4-16 mm), the static characteristic curves of the electromagnetic actuator are relatively level and smooth as a proportional solenoid.

Figure 3 is the curve of average electromagnetic force and standard deviation. The relation between electromagnetic force and current is approximately linear. Electromagnetic force's maximum standard deviation is less than 25 N. The larger current is, the larger standard deviation and relative standard deviation of electromagnetic force are. When current is 2A, relative standard deviation of electromagnetic force is  $\Delta\delta = 0.014588$ ;  $I = 4A$ ,  $\Delta\delta = 0.029105$ ;  $I = 6A$ ,  $\Delta\delta = 0.066778$ .

## QUANTITATIVE ANALYSIS OF THE STATIC CHARACTERISTICS

The static characteristics of the electromagnetic actuator are determined by its geometrical structural parameters without considering its material. Therefore, the parameters of electro-magnetic actuator in Table 1 are selected to do quantitative analysis of the static characteristics.

**The effect of side air gap:** Figure 4 reflects the effect of side air gap on the static characteristic of the electromagnetic actuator. According to Fig. 4, in full

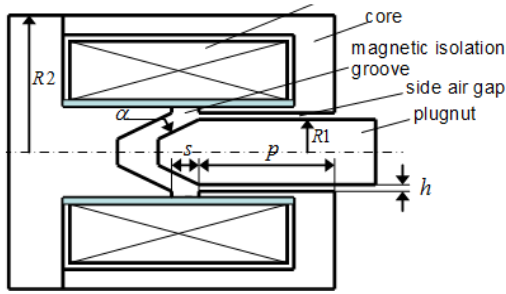


Fig. 1: Model of the electromagnetic actuator

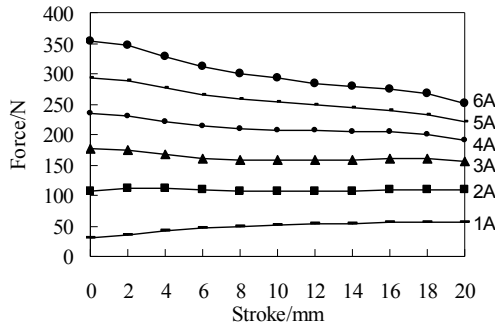


Fig. 2: Static characteristic

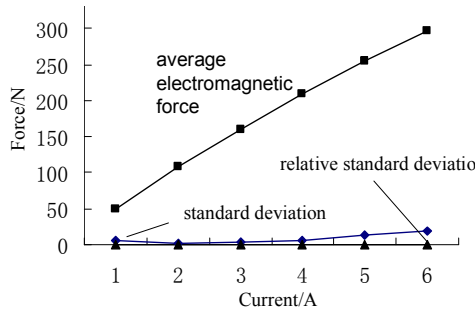


Fig. 3: Average electromagnetic force and standard deviation

Table 1: Main parameters

Paramters	Initial value	Change 1	Change 2
Side air gap $h$	0.3 mm	-0.1 mm	+0.1 mm
Plugnut radius $R1$	19.5 mm	-1 mm	+1 mm
Core radius $R2$	49 mm	-1 mm	+1 mm
plugnut taper $\alpha$	3:26.5	4:26.5	2:26.5
Magnetic isolation groove size $S$	19 mm	-1 mm	+1 mm
Magnetic isolation groove position $p$	0 mm	-1 mm	+1 mm

stroke and working current range (2-6A), the bigger side air gap is, the smaller electromagnetic force is. Figure 5 reflects the effects of side air gap relative to average electromagnetic force and standard deviation in working stroke (4-16 mm). According to Fig. 4, the effects are approximately consistent.

**The effects of plugnut radius:** Figure 6 reflects the effect of plugnut radius on the static characteristic of

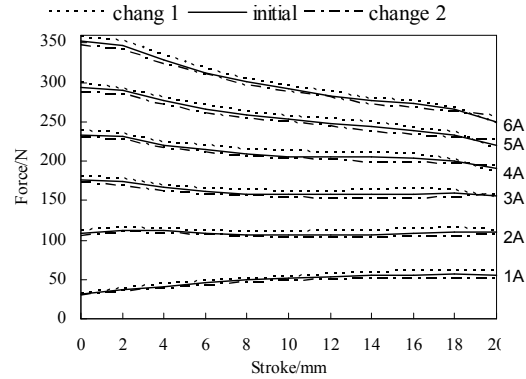


Fig. 4: Static characteristic

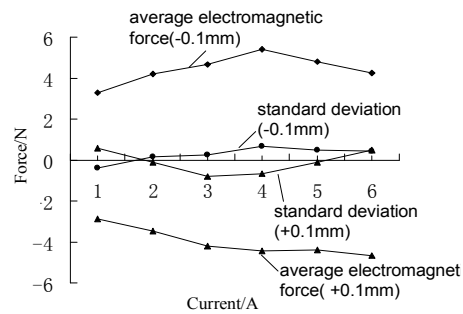


Fig. 5: Change of average electromagnetic force and standard deviation

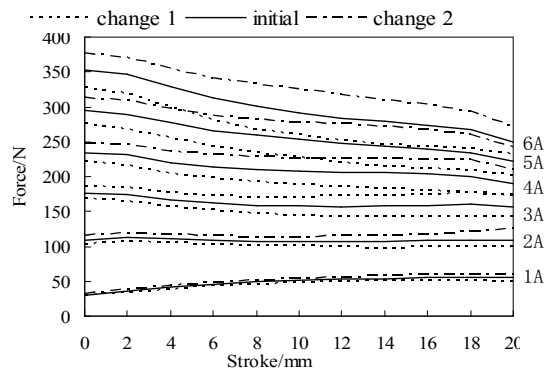


Fig. 6: Static characteristic

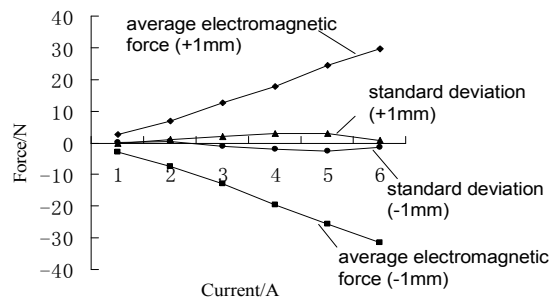


Fig. 7: Change of average electromagnetic force and standard deviation

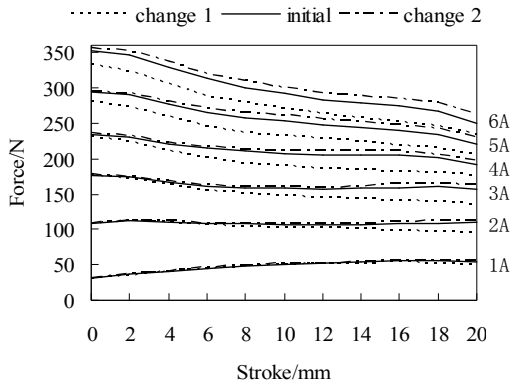


Fig. 8: Static characteristic

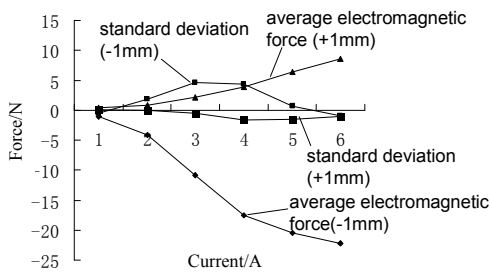


Fig. 9: Change of average electromagnetic force and standard deviation

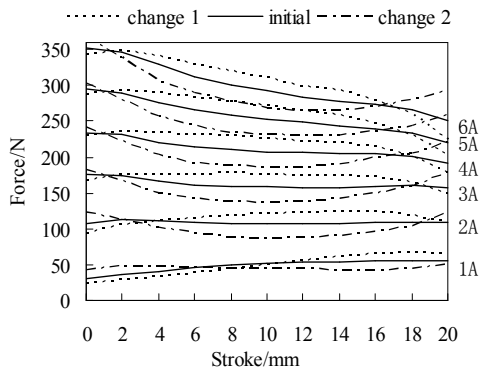


Fig. 10: Static characteristic

the electromagnetic actuator. According to Fig. 6, in full stroke and working current range, the bigger plugnut radius is, the larger electromagnetic force is.

Figure 7 reflects the effects of plugnut radius relative to average electro-magnetic force and standard deviation in working stroke (4-16 mm). According to Fig. 7, the effects are approximately consistent.

**The effect of core radius:** Figure 8 reflects the effect of core radius on the static characteristics of the electromagnetic actuator. According to Fig. 8, in full stroke and working current range, the bigger core radius is, the larger electromagnetic force is. Figure 9 reflects the effects of core radius relative to average

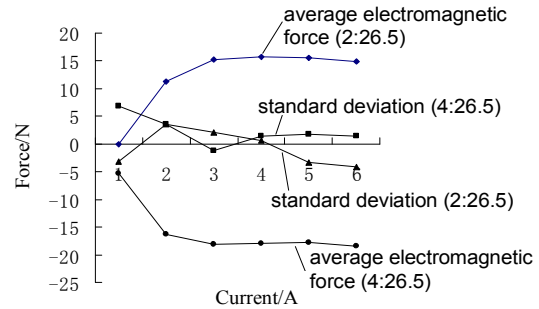


Fig. 11: Change of average electromagnetic force and standard deviation

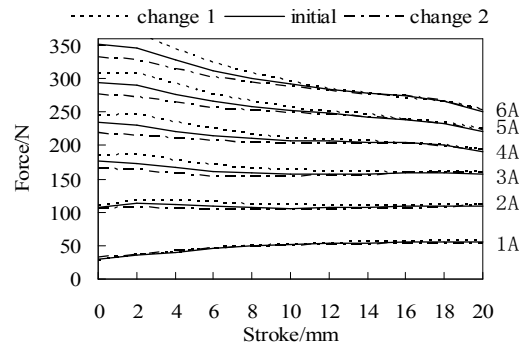


Fig. 12: Static characteristic

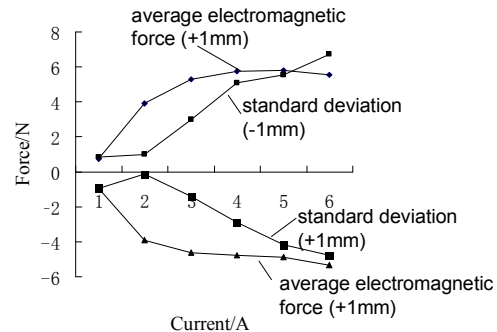


Fig. 13: Change of average electromagnetic force and standard deviation

electromagnetic force and standard deviation in working stroke (4-16 mm). According to Fig. 9, the effects are not approximately consistent. The effect of core radius increase is larger than core radius decrease. It is mainly due to magnetic flux area difference caused by the radius increase or decrease and the decrease of magnetic flux area may lead to magnetic saturation.

**The effects of plugnut taper:** Figure 10 reflects the effect of plugnut taper on the static characteristics of the electromagnetic actuator. According to Fig. 10, in full stroke and working current range, the relation between plugnut taper and electromagnetic force is complicated, but in working stroke (4-16 mm), the

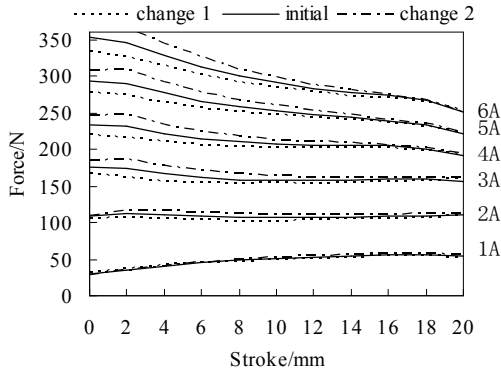


Fig. 14: Static characteristic

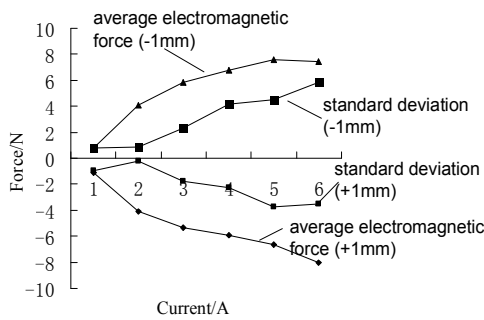


Fig. 15: Change of average electromagnetic force and standard deviation

bigger core radius is, the smaller electromagnetic force is. Figure 11 reflects the effects of plugnut radius relative to average electromagnetic force and standard deviation in working stroke (4-16 mm). According to Fig. 11, the effects are approximately consistent.

**The effect of magnetic isolation groove position:** Figure 12 reflects the effect of magnetic isolation groove position on the static characteristics of the electromagnetic actuator. According to Fig. 12, in stroke (0-16 mm) and working current range, the bigger magnetic isolation groove is, the smaller electromagnetic force is. Note: the positive direction is toward left hand in Fig. 1 to 13 reflects the effects of magnetic isolation groove position relative to average electro-magnetic force and standard deviation in working stroke (4-16 mm). According to Fig. 13, the effects are approximately consistent.

**The effect of magnetic isolation groove size:** Figure 14 reflects the effect of magnetic isolation groove size on the static characteristics of the electromagnetic actuator. According to Fig. 14, in full stroke and working current range, the bigger the magnetic isolation groove is, the larger the electromagnetic force is. Figure 15 reflects the effects of magnetic isolation groove size relative to average electromagnetic force and standard deviation in working stroke (4-16 mm). According to Fig. 15, the effects are approximately consistent.

### OPTIMIZATION MODEL OF STATIC CHARACTERISTICS

In order to optimize the static performance of the electromagnetic actuators in overall operating conditions, it is necessary to establish its optimization model. Assuming that  $\theta = [\theta_1 \theta_2 \theta_3 \dots \theta_{n-1} \theta_n]^T$  denotes mainly geometrical structure parameters of the electromagnetic actuator. The output of the system can be written as follows by Eq. (1):

$$F = f(\theta, I, x) \tag{2}$$

It can also be written as:

$$F = f(\theta, u) \tag{3}$$

where,  $u [I \ x]^T$ . When the input is an operating condition  $u_1$ , the output of the system is:

$$F_1 = f(\theta, u_1) \tag{4}$$

If the expected output of system is  $\hat{F}_1$  at input  $u_1$ , the system geometrical parameters should be  $\hat{\theta}$ :

$$\hat{\theta} = [\hat{\theta}_1 \ \hat{\theta}_2 \ \hat{\theta}_3 \ \dots \ \hat{\theta}_{n-1} \ \hat{\theta}_n]^T$$

$$\hat{F}_1 = f(\hat{\theta}, u_1) \tag{5}$$

Hence,

$$\hat{F}_1 - F_1 = \Delta F = f(\hat{\theta}, u_1) - f(\theta, u_1) \tag{6}$$

Assuming,

$$\Delta\theta = \hat{\theta} - \theta$$

where,

$$\Delta\theta = [\Delta\theta_1 \ \Delta\theta_2 \ \Delta\theta_3 \ \dots \ \Delta\theta_{n-1} \ \Delta\theta_n]^T$$

Equation (6) can be expressed as:

$$\Delta F_1 = f(\theta + \Delta\theta, u_1) - f(\theta, u_1) \tag{7}$$

With Taylor's expansion and removing the high order terms, Eq. (7) can be written as follow:

$$\Delta F_1 = f(\theta + \Delta\theta, u_1) - f(\theta, u_1) \approx \frac{\partial f(\theta, u_1)}{\partial \theta_1} \Delta\theta_1 + \frac{\partial f(\theta, u_1)}{\partial \theta_2} \Delta\theta_2 + \dots + \frac{\partial f(\theta, u_1)}{\partial \theta_n} \Delta\theta_n$$

Table 2: Results of parameter optimization

Parameters	Initial value $\theta$	Results $\Delta\theta$	Constraints [b, ub]
Side air gap $h$	49 mm	+0.7723 mm	-1~1 mm
Plugnut radius $R1$	19.5 mm	+0.5339 mm	-1~1 mm
Core radius $R2$	0.35 mm	-0.0964 mm	-0.1~0.1 mm
Plugnut taper $\alpha$	3:26.5	2.7223:26.5	4:26.5~2:26.5
Magnetic isolation groove size $S$	0 mm	-1.5406 mm	-2~2 mm
Magnetic isolation groove position $p$	19 mm	-0.9921 mm	-2~2 mm

Table 3: Results of characteristic optimization

Optimization target	Optimization results	Optimization residual	Simulation results
-0.3996	-0.5234	0.1238	-1.012440
-0.3756	-0.1577	-0.2179	1.972280
-0.5660	-0.3225	-0.2435	0.927783
-1.2671	-0.6098	-0.6573	-2.421230
-2.0617	-1.3155	-0.7462	-4.202600
-0.8024	-1.9753	1.1729	-1.823000
4.4776	4.9818	-0.5042	6.940800
10.3476	10.8118	-0.4642	11.754000
16.1464	15.9972	0.1492	15.562000
21.7225	20.9516	0.7709	20.747000
25.5762	25.5030	0.0732	25.244000
29.0439	29.5798	-0.5359	27.992000

Namely,

$$\Delta F_1 = \begin{bmatrix} \frac{\partial f(\theta, \mathbf{u}_1)}{\partial \theta_1} & \frac{\partial f(\theta, \mathbf{u}_1)}{\partial \theta_2} & \dots & \frac{\partial f(\theta, \mathbf{u}_n)}{\partial \theta_n} \end{bmatrix} \begin{bmatrix} \Delta \theta_1 \\ \Delta \theta_2 \\ \vdots \\ \Delta \theta_n \end{bmatrix} = \mathbf{J}_1 \Delta \theta \quad (8)$$

After transformation, Eq. (8) can be written as follow:

$$\Delta \theta = \mathbf{J}_1^{-1} \Delta F_1 \quad (9)$$

If requirements  $\Delta F = [\Delta F_1 \ F_2 \ \dots \ \Delta F_n]^T$  and sensitivity matrix  $\mathbf{J} = [\mathbf{J}_1 \ \mathbf{J}_2 \ \dots \ \mathbf{J}_n]^T$  in overall operating conditions can be obtained, system optimization model can be built as follow:

$$\Delta \theta = \mathbf{J}^{-1} \Delta F \quad (10)$$

To solve the Eq. (10), a target function  $\min G = \|\Delta F - \mathbf{J}\Delta\theta\|$  is established and least square method is adopted to solve it. Finally, with the optimization method, the geometrical parameter errors of the electromagnetic actuator can be obtained.

Because the accurate analytical model of the electromagnetic force is difficult to be gotten, it is not easy to construct the error sensitivity matrix  $\mathbf{J}$  directly with partial differential. To solve the problem, difference operator is used to displace partial differential. Central difference is as follow:

$$\frac{\partial f_i(\theta, \mathbf{u}_1)}{\partial \theta_j} \approx \frac{f(\theta_1, \theta_2, \dots, \theta_{j-1}, \theta_j + 1, \theta_{j+1}, \dots, \theta_n, \mathbf{u}_1) - f(\theta_1, \theta_2, \dots, \theta_{j-1}, \theta_j - 1, \theta_{j+1}, \dots, \theta_n, \mathbf{u}_1)}{2}$$

### OPTIMIZATION CALCULATION

The average electromagnetic force and standard deviation in overall operating conditions are optimized as example. In condition of structure dimensions constraints, the target is to increases 10% average electromagnetic force and to reduce 10% standard deviation.

The optimization model:

$$\min_{\Delta \theta} G = \|\Delta F - \mathbf{J}\Delta\theta\|$$

Constraints:  $b \leq \Delta\theta \leq ub$

The sensitive matrix is established as follows. The first six rows of the matrix is sensitive values of the parameters of the electromagnetic force standard deviation and last six rows of the matrix are sensitive values of the parameters of the average electromagnetic force.

After 29 iterations, the optimization of parameters and parameter optimization results are shown in Table 2.

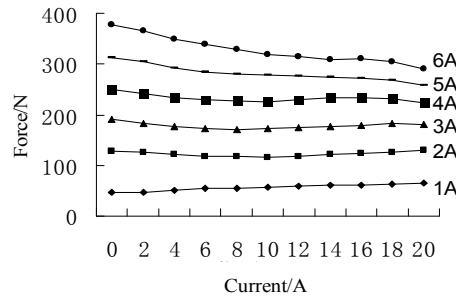


Fig. 16: Static characteristic

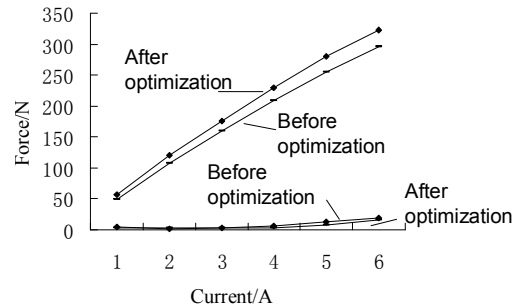


Fig. 17: Average electromagnetic force and standard deviation

Performance optimization results are shown in Table 3. In the column of "Optimization target", the first six rows are the optimization target value of the standard deviation of the electro-magnetic force and last six rows are the optimization target value of the average electromagnetic force. The column of "Simulation results" is the simulation results after parameter optimization. The simulation results show that the optimization target of the average electromagnetic force standard deviation is reached. "Optimization residual" = "Target value" - "Optimization results":

$$J = \begin{bmatrix} -4.8074 & 0.2551 & -0.1483 & -5.0650 & -0.8991 & 0.8702 \\ 1.2778 & -0.3718 & -0.4944 & -0.0185 & -0.5785 & 0.5626 \\ 5.2247 & -1.5933 & -1.2609 & 1.6570 & -2.1805 & 2.0734 \\ 6.8435 & -2.4493 & -2.7921 & -0.3846 & -3.9749 & 3.1850 \\ 2.9838 & -2.7049 & -3.9048 & -2.5539 & -4.8460 & 4.1028 \\ -0.3178 & -1.0480 & -4.7657 & -2.8411 & -5.7458 & 4.6927 \\ -30.847 & 2.7027 & 0.7099 & 3.4234 & -0.8446 & 0.8457 \\ -38.253 & 7.1420 & 2.5304 & 10.1705 & -3.9064 & 2.3209 \\ -44.30 & 12.7089 & 6.4477 & -12.7392 & -4.9552 & 4.0186 \\ -49.1453 & 18.7312 & 10.7328 & -14.6621 & -5.2605 & 5.3817 \\ -45.9941 & 25.1191 & 13.4036 & -16.7390 & -5.3314 & 6.2365 \\ -44.4043 & 30.5909 & 15.3730 & -18.9984 & -5.4310 & 6.5867 \end{bmatrix}$$

Figure 16 is a static characteristic curve of the parameter optimization of the electromagnetic actuator after optimization. Figure 17 shows the standard deviation of the electromagnetic force and the average electromagnetic force.

### CONCLUSION

This study presents an optimization method based on numerical calculation of the sensitivity matrix and a difference model of target characteristics and current characteristics. The method combines the advantages of the analytical method and the finite element method and performance of electromagnetic actuators in overall operating conditions can be optimized. Numerical method is effective to establish the sensitive matrix of the optimization model and it can solve the precise and complex problem when create a distributing parameters model of electromagnetic actuators with analytical method and can also solve the problem of sensitive matrix difficult to be built with analytical method. With the optimization method, electromagnetic force of an electromagnetic actuator in overall operating conditions is optimized. The method is verified effective and feasible. The method is also suitable for the static and dynamic characteristics optimization of other electromagnetic actuators in overall conditions.

### ACKNOWLEDGMENT

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