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Research Article

Analysis of Nonlinear Discrete Time Active Control System with Boring Chatter

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Abstract: In this work we study the design and analysis for nonlinear discrete time active control system with boring charter. It is shown that most analysis result for continuous time nonlinear system can be extended to the discrete time case. In previous studies, a method of nonlinear Model Following Control System (MFCS) was proposed by Okubo (1985). In this study, the method of nonlinear MFCS will be extended to nonlinear discrete time system with boring charter. Nonlinear systems which are dealt in this study have the property of norm constraints $\| f\ (v\ (k))\| \leq \alpha + \beta \| v\ (k)\|^{\gamma}$, where $\alpha \geq 0$, $\beta \geq 0$, $0 \leq \gamma \leq 1$. When $0 \leq \gamma < 1$. It is easy to extend the method to discrete time systems. But in the case $\gamma = 1$ discrete time systems, the proof becomes difficult. In this case, a new criterion is proposed to ensure that internal states are stable. We expect that this method will provide a useful tool in areas related to stability analysis and design for nonlinear discrete time systems as well.

Keywords: Discrete time systems, disturbance, internal states, nonlinear control system

INTRODUCTION

Nonlinear control systems are those control systems where nonlinearity plays a significant role, either in the controlled process or in the controller itself. Nonlinear plants arise naturally in numerous engineering and natural systems, including mechanical and biological systems, aerospace and automotive control, industrial process control and many others. Nonlinear control theory is concerned with the analysis and design of nonlinear control systems. It is closely related to nonlinear systems theory in general, which provides its basic analysis tools (Binder *et al*., 2009). There has been much excitement over the emergence of new mathematical techniques for the analysis and control of nonlinear systems. In addition, great technological advances have bolstered the impact of analytic advances and produced many new problems and applications which are nonlinear in an essential way (Sastry, 1999). Based on genetic algorithms, we propose a practical design method for robust nonlinear controllers of uncertain nonlinear system. We demonstrate its applicability by tackling with the flight control problems of landing in a wind shear (Mori and Torisu, 2000). The fuzzy system is constructed to approximate the nonlinear system dynamics. Based on this fuzzy approximation suitable adaptive control laws and appropriate parameter update algorithms for nonlinear uncertain (or unknown) systems are developed to achieve H_∞ tracking performance (Martin, 2009).

An iterative learning control method is proposed for nonlinear discrete-time systems with well-defined relative degree, which uses the output data from several previous operation cycles to enhance tracking performance (Sun and Wang, 2001). Present a control methodology for a class of discrete time nonlinear systems that depend on a possibly exogenous scheduling variable (Raul and Passino, 2003).

One of the most important problems in control theory is concerned with system stability, especially with systems affected by external disturbances (Zhong *et al*., 2004). Finite-time stability of nonlinear discrete-time systems is studied. Some new analysis results are developed and applied to controller design (Mastellone *et al*., 2004).

In previous studies, a method of nonlinear Model Following Control System (MFCS) was proposed by Okubo (1985), Wang and Okubo (2008) and Akiyama (1998). In this study, the method of nonlinear MFCS will be extended to nonlinear discrete time system. Nonlinear systems which are dealt in this study have the property of norm constraints:

$$\|f(v(k))\| \leq \alpha + \beta \|v(k)\|^{\gamma}$$

where $\alpha \geq 0$, $\beta \geq 0$, $0 \leq \gamma \leq 1$. When $0 \leq \gamma < 1$. It is easy to extend the method to discrete time systems. But in the case $\gamma = 1$ discrete time systems, the proof becomes difficult. In this case, a new criterion is proposed to ensure that internal states are stable. We expect that this method will provide a useful tool in areas related to stability analysis and design for nonlinear discrete time systems as well.

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The expression of problems: A controlled object is described in (1), (2) and the accordant model is given in (3) and (4):

$$x(k+1) = Ax(k) + Bu(k) + B_f f(k, x(k)) + d(k) \tag{1}$$

$$y(k) = Cx(k) + d_0(k) \tag{2}$$

$$x_m(k+1) = A_m x_m(k) + B_m r_m(k) \tag{3}$$

$$y_m(k) = C_m x_m(k) \tag{4}$$

where, $x(k) \in R^n, u(k) \in R^l, f(k, x(k)) \in R^{l_f}$, $y(k) \in R^l, d(k) \in R^n, d_0(k) \in R^l, x_m(k) \in R^{n_m}$, $r_m(k) \in R^{l_m}, y_m(k) \in R^l$. The available state is output $y(k)$. assuming $f(v(k)) = f(k, x(k))$, $v(k) = C_f x(k)$ and the nonlinear function f(v(k)) is available and satisfies the following constraint:

$$\|f(v(k))\| \le \alpha + \beta \|v(k)\|^{\gamma} \tag{5}$$

where, $\alpha \ge 0$, $\beta \ge 0$, $0 \le \gamma < 1$. ||.|| is Euclidean norm.

Assume that $[C\ A\ B]$ is controllable and observable. Zeros of $C\ [zI - A]^{-1}B$ are stable. Disturbance $d(k)$ and $d_0(k)$ are bounded and satisfy (6):

$$D_k(z)d(k) = 0, D_k(z)d_0(k) = 0 \tag{6}$$

$D_k(z)$ is a scalar characteristic polynomial of disturbances. Output error is given as below:

$$e(k) = y(k) - y_m(k) \tag{7}$$

The aim of boring control system design is to obtain a control law which makes output error zero and keeps internal states bounded.

Design of nonlinear discrete time system with boring charter: We consider shift operator z and the follows are established:

$$C[zI - A]^{-1}B = N(z)/D(z)$$
$$C[zI - A]^{-1}B_f = N_f(z)/D(z)$$
$$C_m[zI - A_m]^{-1}B_m = N_m(z)/D_m(z)$$

where, $D(z) = |zI - A|, D_m(z) = |zI - A_m|$. Then the representations of input-output equation are described as followings:

$$D(z)y(k) = N(z)u(k) + N_f(z)f(v(k)) + w(k)$$

$$D_m(z)y_m(k) = N_m(z)r_m(k)$$
$$w(k) = C adj[zI - A]d(k) + D(z)d_0(k)$$

where, $\partial_{ri}(N(z)) = \sigma_i$, $\partial_{ri}(N_f(z)) < \sigma_{fi}$, $\partial_{ri}(N_m(z)) = \sigma_{mi}$ and $\Gamma_r(N(z)) = N_r$ ($\Gamma_r(\cdot)$ is the coefficient matrix of the element with maximum of row degree), as well as $N_r \ne 0$. Since the disturbances satisfy (6), then:

$$D_k(z)w(k) = 0$$

The first step of design, a monic and stable polynomial $T(z)$ which has the degree of $\rho(\rho \ge n_d + 2n - n_m - 1 - \sigma_i)$ is chosen. Then, the $R(z)$, $S(z)$ can be derived from:

$$T(z)D_m(z) = D_k(z)D(z)R(z) + S(z) \tag{8}$$

where the degree of each polynomial is:

$$\partial T(z) = \rho,\ \partial D_m(z) = n_m, \partial D_k(z) = n_d$$
$$\partial R(z) = \rho + n_m - n_d - n,\ \partial D(z) = n$$
$$\partial S(z) \le n_d + n - 1$$

The output error $e(k)$ is represented as below:

$$T(z)D_m(z)e(k) = D_k(z)R(z)N(z)u(k) + D_k(z)R(z)N_f(z)f(v(k)) + S(z)y(k) - T(z)N_m(z)r_m(k) \tag{9}$$

Let the right hand side of above equation equal zero, that $u(k)$ will be described as follows:

$$u(k) = -N_r^{-1}Q^{-1}(z)[D_k(z)R(z)N(z) - Q(z)N_r]u(k) - N_r^{-1}Q^{-1}(z)D_k(z)R(z)N_f(z)f(v(k)) - N_r^{-1}Q^{-1}(z)S(z)y(k) + N_r^{-1}Q^{-1}(z)T(z)N_m(z)r_m(k) \tag{10}$$

where, $Q(z) = diag[z^{\delta_i}], \delta_i = \rho + n_m - n + \sigma_i$.

Then the state space expression of u (k) is shown as follows:

$$u(k) = -H_1\xi_1(k) - J_2y(k) - J_3f(v(k)) - H_3\xi_3(k) + J_4r_m(k) + H_4\xi_4(k) \tag{11}$$

Here,

$$Q^{-1}(z)[D_k(z)R(z)N(z)-Q(z)N_r] = N_r H_1(zI-F_1)^{-1}G_1$$

$$Q^{-1}(z)S(z) = N_r(z)[H_2(z)(zI-F_2)^{-1}G_2+J_2]$$

$$Q^{-1}(z)D_k(z)R(z)N_f(z) = N_r(z)[H_3(z)(zI-F_3)^{-1}G_3+J_3]$$

$$Q^{-1}(z)T(z)N_m(z) = N_r(z)[H_4(z)(zI-F_4)^{-1}G_4+J_4]$$

The followings must be satisfied:

$$\xi_1(k+1) = F_1\xi_1(k)+G_1u(k) \quad (12)$$

$$\xi_2(k+1) = F_2\xi_2(k)+G_2y(k) \quad (13)$$

$$\xi_3(k+1) = F_3\xi_3(k)+G_3f(v(k)) \quad (14)$$

$$\xi_4(k+1) = F_4\xi_4(k)+G_4r_m(k) \quad (15)$$

where, $|zI - F_i| = |Q(z)|$, (i = 1, 2, 3, 4). $u(k)$ of (10) is obtained from $e(k) = 0$. Model following control system can be realized if system internal states are bounded.

Boundness of internal states: In this study, $\gamma = 1$. System inputs are reference input signal $r_m(k)$ and disturbances $d(k)$, $d_0(k)$ which are all assumed to be bounded. The bounded can be easily proved if there is no nonlinear part $f(v(k))$. But if $f(v(k))$ exists, the bounded has relation with it.

First, the overall system can represent by state space in below:

$$z(k+1) = \begin{bmatrix} A-BJ_2C & -BH_1 & -BH_2 & -BH_3 \\ -G_1J_2C & F_1-G_1H_1 & -G_1H_2 & -G_1H_3 \\ G_2C & 0 & F_2 & 0 \\ 0 & 0 & 0 & F_3 \end{bmatrix} z(k) + \begin{bmatrix} BH_4 \\ G_1H_4 \\ 0 \\ 0 \end{bmatrix}\xi_4(k) + \begin{bmatrix} B_f-BJ_3 \\ -G_1J_3 \\ 0 \\ G_3 \end{bmatrix} f(v(k)) + \begin{bmatrix} BJ_4 \\ G_1J_4 \\ 0 \\ 0 \end{bmatrix} r_m(k) + \begin{bmatrix} d(k)-BJ_2d_0(k) \\ -G_1J_2d_0(k) \\ G_2d_0(k) \\ 0 \end{bmatrix} \quad (16)$$

$$y(k) = Cx(k)+d_0(k) \quad (17)$$

where, $z^T(k) = (x^T(k), \xi_1^T(k), \xi_2^T(k), \xi_3^T(k))$. $\xi_4(k)$ is bounded. Here, necessary part about boundness is considered. System can be simplified as follows:

$$z(k+1) = A_s z(k) + B_s f(v(k)) + d_s(k) \quad (18)$$

$$v(k) = C_s z(k) \quad (19)$$

The contents of A_s, B_s, C_s, $d_s(k)$ are shown clearly in (16) and (17). In order to obtain desired conclusion, it is sufficient to prove that $z(k)$ is bounded.

Then, we prove that A_s is stable. A_s and its characteristic polynomial is calculated as follows:

$$A_s = \begin{bmatrix} A-BJ_2C & -BH_1 & -BH_2 & -BH_3 \\ -G_1J_2C & F_1-G_1H_1 & -G_1H_2 & -G_1H_3 \\ G_2C & 0 & F_2 & 0 \\ 0 & 0 & 0 & F_3 \end{bmatrix} \quad (20)$$

$$|zI - A_s| = |Q(z)|^2 V_s(z) T^l(z) D_m^l(z) \quad (21)$$

where, $V_s(z)$ is the zeros polynomial of $C[zI - A]^{-1} B = W(z)^{-1} U(z)$, that is $V_s(z) = |U(z)|/|N_r|$. As $|Q(z)|$, $V_s(z)$, $T(z)$, $D_m(z)$ are all stable polynomials. Therefore, A_s is a stable system matrix.

In this case, $v(k)$ and $f(v(k))$ are scalars. To use regular transformation $z(k) = T\bar{z}(k)$, (18) and (19) can be transformed to Kalman canonical form:

$$\bar{z}(k+1) = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} & \bar{A}_{13} & \bar{A}_{14} \\ 0 & \bar{A}_{22} & 0 & \bar{A}_{24} \\ 0 & 0 & \bar{A}_{33} & \bar{A}_{34} \\ 0 & 0 & 0 & \bar{A}_{44} \end{bmatrix} \bar{z}(k) + \begin{bmatrix} \bar{B}_1 \\ \bar{B}_2 \\ 0 \\ 0 \end{bmatrix} f(v(k)) + \begin{bmatrix} \bar{d}_{s1}(k) \\ \bar{d}_{s2}(k) \\ \bar{d}_{s3}(k) \\ \bar{d}_{s4}(k) \end{bmatrix} \quad (22)$$

$$v(k) = \begin{bmatrix} 0 & \bar{C}_2 & 0 & \bar{C}_4 \end{bmatrix} \bar{z}(k) \quad (23)$$

where, $\bar{z}^T(k) = [\bar{z}_1^T(k), \bar{z}_2^T(k), \bar{z}_3^T(k), \bar{z}_4^T(k)]$.

Since A_s is stable matrix, then $\bar{A}_{ii}(i = 1,2,3,4)$ also are stable. Obviously, $\bar{z}_3(k), \bar{z}_4(k)$ are bounded. $\bar{z}_2(k)$ can be rewritten as follows:

$$\bar{z}_2(k+1) = \bar{A}_{22}\bar{z}_2(k) + \bar{B}_2 f(v(k)) + \bar{d}_2(k) \tag{24}$$

$$v(k) = \bar{C}_2\bar{z}_2(k) + \bar{d}_v(k) \tag{25}$$

where, $\bar{d}_2(k), \bar{d}_v(k)$ are bounded. Subsequently, the transfer function from $f(v(k))$ to $v(k)$ can be calculated as:

$$H(z) = \bar{C}_2[zI - \bar{A}_{22}]^{-1}B_2 \tag{26}$$

It can be also calculated in terms of original system parameter:

$$H(z) = C_f[zI - A]^{-1}B_f - C_f[zI - A]^{-1}B \cdot [C[zI - A]^{-1}B]C[zI - A]^{-1}B_f \tag{27}$$

Thus, $v(k)$ can be rewritten as follows:

$$v(k) = H(z)f(v(k)) + d_v(k) \tag{28}$$

where $d_v(k)$ is bounded. Let $\hat{v}(k) = v(k) - d_v(k)$, we can get $\hat{f}(\hat{v}(k)) = f(v(k)) = f(\hat{v}(k)) + d_v(k)$:

$$H(z) = \frac{\hat{v}(k)}{\hat{f}(\hat{v}(k))} = \frac{g_{n'}z^{n'-1} + g_{n'-1}z^{n'-2} + \cdots + g_2 z + g_1}{z^{n'} + h_{n'}z^{n'-1} + \cdots + h_2 z + h_1} \tag{29}$$

The state space observable canonical form of (29) can be written as following:

$$\hat{z}(k+1) = \begin{bmatrix} 0 & \cdots & 0 & -h_1 \\ 1 & \ddots & 0 & -h_2 \\ & \ddots & 0 & \vdots \\ 0 & & 1 & -h_{n'} \end{bmatrix}\hat{z}(k) + \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_{n'} \end{bmatrix}\hat{f}(\hat{v}(k))$$

$$= \hat{A}\hat{z}(k) + g\,\hat{f}(\hat{v}(k)) \tag{30}$$

$$\hat{v}(k) = [0,\cdots,0,1]\hat{z}(k) = \hat{C}\hat{z}(k) = \hat{z}_{n'}(k) \tag{31}$$

Consider system (30), (31), assume that $\hat{f}(\hat{v}(k))$ satisfies the constraint as below:

$$|\hat{f}(\hat{v}(k))| \le \beta|(\hat{v}(k))| + M, \beta \ge 0, M > 0 \tag{32}$$

If the $S \le \sum_{i=1}^{n'}(\beta|g_i| + |h_i|) \le \mu < 1$, then $\hat{z}(k)$ is bounded.

Lemma: Consider system (30) and (31); assume that $\hat{f}(\hat{v}(k))$ satisfies the constraint as below (32).

If the any one following conditions holds, then $\hat{z}(k)$ is bounded:

Define: $D = def[d_{ij}]_{n' \times n'}$

$$d_{ij} = \begin{cases} |\hat{a}_{ij}|, 1 \le i, j \le n' - 1 \\ |\hat{a}_{in'}| + \beta|g_i|, i = 1,2,\cdots,n' \end{cases} \tag{33}$$

$$E = I - D \tag{34}$$

The matrix E is an M-matrix.

$\rho(D) < 1$, where $\rho(D)$ is spectrum radius of D.

Proof: Define $[\cdot]_{abs}$ is a operator which take absolute of elements of vector or matrix. Construct Lyapnov function as following:

$$V(k) = \sum_{i=1}^{n'} r_i|\hat{z}_i(k)| = r^T[\hat{z}(k)]_{abs} \tag{35}$$

where $\gamma_i > 0$ (I = 1, 2, …., n). The difference of $V(k)$ along the trajectories of the system (30) and (31) is given by:

$$\begin{aligned} \Delta V(k) &= r^T[\hat{z}(k+1)]_{abs} - V(k) \\ &= r^T\{[\hat{A}\hat{z}(k)]_{abs} + [g\hat{f}(\hat{v}(k))]_{abs} - [\hat{z}(k)]_{abs}\} \\ &\le r^T\{[\hat{A}\hat{z}(k)]_{abs} + \beta[g\hat{z}_{n'}(k)]_{abs} \\ &\quad + M[g]_{abs} - [\hat{z}(k)]_{abs}\} \\ &\le r^T(D-I)[\hat{z}(k)]_{abs} + M_r \\ &= -r^T E[\hat{z}(k)]_{abs} + M_r \end{aligned} \tag{36}$$

where, M and M_r are positive constants. Because E is M-matrix, there are vectors $r^T = [r_1, r_2, \cdots, r_{n'}]$ and $q^T = [q_1, q_2, \cdots, q_{n'}], q_i > 0, (i = 1,2,\cdots,n')$ and satisfy:

$$r^T E = q^T \tag{37}$$

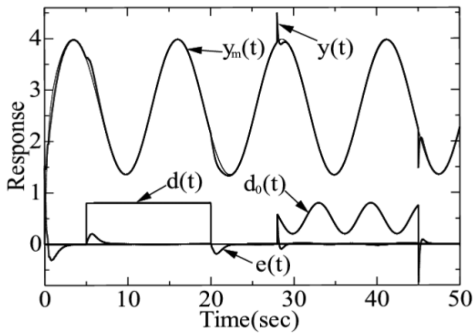

Fig. 1: Responses of the discrete-time system

Hence, following conclusion can be obtained:

$$\Delta V(k) \leq -q^T[\hat{z}(k)]_{abs} + M_r$$
$$\leq -\mu_m V(k) + M_r \quad (38)$$

where, $0<\mu_m<1$, $M_r>0$. It is similar to (36) to prove that V (k) is bounded. Hence, $\hat{z}(k)$ is bounded.

From E = I - D, λ (E) = I - λ (D), λ (E) >0 Base on above lemma, the theorem can be described as following.

Theorem: Consider the system:

$$z(k+1) = Az(k) + Bf(v(k)) + d(k) \quad (39)$$

$$v(k) = Cz(k) + d_0(k) \quad (40)$$

where, $z(k) \in R^n$, $v(k) \in R$, $f(v(k)) \in R$, $d(k) \in R^n$, $d_0(k) \in R^2$. A is a stable system matrix and disturbance of $d(k)$ and $d_0(k)$ are bounded. The nonlinear function $f(v(k))$ satisfies the following constraint:

$$\|f(v(k))\| \leq \alpha + \beta \|v(k)\|^2$$

where, $\alpha \geq 0, \beta \geq 0$.

If the any one following conditions holds, then $z(k)$ and $v(k)$ are bounded.

An illustrative example: An example is given as follows:

$$x(k+1) = \begin{bmatrix} 0 & 1 \\ -0.2 & 1.5 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$

$$+ \begin{bmatrix} 1 \\ 0 \end{bmatrix} f([1.5 \quad 0]x(k)) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} d(k)$$

$$y(k) = \begin{bmatrix} 0.3 & 1 \end{bmatrix} x(k) + d_0(k)$$

Reference model is given as follows:

$$x_m(k+1) = \begin{bmatrix} 0 & 1 \\ -0.15 & -0.8 \end{bmatrix} x_m(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r_m(k)$$

$$y_m(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x_m(k)$$

The responses of system are shown in Fig. 1. It can be concluded that output signal follows the reference even though disturbance existed in system.

CONCLUSION

We have presented model following control for nonlinear discrete system with boring chatter time delay. The illustrative example and the simulation results show the benefits of this proposed design methods. Topics of future include: the discrete control system for the time-delays and the predictive control of the nonlinear discrete system with boring chatter will be discussed.

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