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Research Article

Repetitive Group Sampling Plan Based on Truncated Tests for Weibull Models

[1]Muhammad Aslam, [2]Chi-Hyuck Jun, [3]Arturo J. Fernández, [4]Munir Ahmad and [1]Mujahid Rasool
[1]Department of Statistics, Forman Christian College University, Lahore, Pakistan
[2]Department of Industrial and Management Engineering, POSTECH, Pohang 790-784, Republic of Korea
[3]Department of Statistics and Operational Research, University of La Laguna, 38271 La Laguna, Tenerife, Spain
[4]National College of Business Administration and Economics, Lahore, Pakistan

Abstract: This study proposes a new repetitive group sampling plan by attributes based on truncated life test under the Weibull distributions with known shape parameter. The constrained optimization problem is formulated as a nonlinear integer programming to determine the sample size and the acceptance criterion of the proposed plan. The objective function to be minimized is the average sample number, whereas the constraints are related to the lot acceptance probabilities at the acceptable and limiting quality levels. Tables containing the optimal design parameters are constructed for various values of the Weibull shape parameter and the results are explained with several examples. Minimum producer's and consumer's risks are also determined. The proposed sampling plan reduces significantly the average sample number as compared with the traditional single sampling plan. The effect of the mis-specification of shape parameters is also discussed.

Keywords: Consumer's risk, producer's risk, repetitive group sampling plan, time-truncated tests

INTRODUCTION

Two types of censoring known as Type-I and Type-II censoring schemes are commonly used in medical sciences and in life testing experiments. Type-I censoring scheme fixes the test duration and Type-II censoring scheme fixes the number of failures. But, today in life testing, time truncated scheme is preferred. Usually, in life testing, the researcher truncates the experiment when the pre-defined test time (or experimental time) reaches or the pre-specified number of failures is observed. This scheme is more convenient than censoring schemes to minimize the cost and time of the experiment to reach the final decision.

There are many types of acceptance sampling plans such as attributes acceptance plan, variables acceptance plan, progressively acceptance plan, accelerated acceptance plan and group acceptance plan. But, the main purpose of these plans is to make a decision on the submitted lot with minimum sample size (or minimum number of sampled units) by providing the protection to producer and consumer. It is important to note that two risks are always attached whatever the type of the experiment is because all these schemes depend upon the random sample taken from a lot of products. Therefore, there is a chance to accept a bad lot or to reject a good lot. The acceptance of a bad lot is called the consumer's risk and the rejection of a good lot is called the producer's risk. So, the purpose of a sampling scheme is to minimize these risks with the minimum number of sample size.

In an acceptance sampling plan, the time truncated schemes have received increasing attention from the researcher. Recently, many authors including Kantam *et al*. (2001), Rosaiah *et al*. (2006), Balakrishnan *et al*. (2007), Tsai and Wu (2006) and Aslam and Kantam (2008) proposed the ordinary single sampling plans based on the time truncated life test for various life distributions. More recently, Lio *et al*. (2010a, b) developed single sampling plans based on truncated life tests for Birnbaum-Saunders distribution and Burr type XII distribution using percentiles, respectively, whereas Fernandez *et al*. (2011) discussed the progressively censored group sampling plans for Weibull distributions.

The attributes repetitive group sampling plan is proposed by Sherman (1965) for a normal distribution. The operation of this repetitive sampling is similar to a sequential sampling. According to him, this sampling scheme gives the minimum sample size as well as the required protection to consumer and producer. Furthermore, repetitive sampling is more efficient than single sampling plan but not as efficient as the sequential plan. Some authors including Balamurali and Jun (2006) discussed the variables repetitive group acceptance sampling plan and

Corresponding Author: Muhammad Aslam, Department of Statistics, Forman Christian College University, Lahore, Pakistan

compared the results with the single sampling plan. However, no attempt has been made to study attributes repetitive sampling plans based on truncated life tests for a Weibull distribution. In this study, we developed an attributes repetitive sampling plan for the truncated life tests assuming that the lifetime of products follows the Weibull distribution with known shape parameter and that the quality level is represented by the ratio of mean life to the specified life.

Repetitive group sampling plans based on truncated test: The proposed attributes repetitive group sampling plan based on truncated life tests can be described as follows:

Step 1: Take a random sample of size n from a lot and put it on life test for fixed time t_0.

Step 2: Accept the lot if the number of failures, D, is smaller than or equal to the acceptance number c_1. Truncate the test and reject the lot as soon as D exceeds c_2, where $c_2 \geq c_1$.

Step 3: If $c_1 < D \leq c_2$, then repeat Step 1.

The above attributes repetitive group sampling plan is characterized by three parameters, n, c_1 and c_2. It is important to note that the attributes repetitive group sampling plan is a generalization of the ordinary single sampling plan and reduces to it when $c_1 = c_2$. The probability of lot acceptance is determined by using the Operating Characteristic (OC) function, which is derived to be:

$$P_A(p) = \frac{P_a}{P_r + P_a}, \quad 0 < p < 1 \tag{1}$$

where, P_a is the probability of acceptance of a submitted lot with fraction defective p based on a given sample, whereas P_r is the corresponding probability of lot rejection. Since these probabilities are given by:

$$P_a = \Pr(D \leq c_1 \mid p) = \sum_{i=0}^{c_1} \binom{n}{i} p^i (1-p)^{n-i} \tag{2}$$

and

$$P_r = \Pr(D > c_2 \mid p) = 1 - \sum_{i=0}^{c_2} \binom{n}{i} p^i (1-p)^{n-i} \tag{3}$$

the OC function defined in Eq. (1) can be rewritten as:

$$P_A(p) = \frac{\sum_{i=0}^{c_1} \binom{n}{i} p^i (1-p)^{n-i}}{1 - \sum_{i=0}^{c_2} \binom{n}{i} p^i (1-p)^{n-i} + \sum_{i=0}^{c_1} \binom{n}{i} p^i (1-p)^{n-i}}, \quad 0 < p < 1 \tag{4}$$

Attributes repetitive group sampling plan for weibull models: The Weibull distribution is an important statistical model which has been widely used in many areas, including quality control, reliability analysis and life testing. Among others, Jun *et al*. (2006) and Aslam and Jun (2009) have constructed group acceptance plans using this distribution. Recently, Aslam *et al*. (2009) used the Weibull distribution to develop double acceptance sampling plans based on truncated life tests. The double sampling plan requires two sample sizes n_1 and n_2 as well as two acceptance numbers c_1 and c_2. But, they only considered the case of $c_1 = 0$ and $c_2 = 1$. They obtained the design parameters by only considering the consumer's risk. But, the present study is quite different from Aslam *et al*. (2009), which is a new sampling plan. Further, we considered producer's and consumer's risks at the same time when determining the design parameters.

We assume that the lifetime of a product follows the Weibull distribution, whose Cumulative Distribution Function (CDF) is given as:

$$F(t; \lambda, \gamma) = 1 - exp\left\{-(t/\lambda)^{\gamma}\right\}, t \geq 0, \tag{5}$$

where, $\gamma > 0$ is a known shape parameter and $\lambda > 0$ is an unknown scale parameter. The mean lifetime of the product, μ (unknown), is then given by:

$$\mu = (\lambda/\gamma)\Gamma(1/\gamma) \tag{6}$$

We may accept the lot if there is enough evidence that the true mean μ is larger than the specified mean μ_0. As stated by Aslam and Jun (2009), it would be convenient to express the termination time t_0 as a multiple of the specified length μ_0. Accordingly, we will consider that $t_0 = a\mu_0$ for a given constant a. It is important to note that for a known value of the shape parameter the cdf of the Weibull distribution depends only through t/λ. For the practical implementation of the proposed plan based on truncated life tests, if the shape parameter of the Weibull distribution is not known, it can be estimated from previous failure data of the products under investigation. Producers normally keep the estimated values of the Weibull shape parameter.

In order to find the design parameters of the proposed repetitive group sampling plan, we prefer the approach based on two points on the OC curve by considering the producer's and consumer's risks. Many authors, including Fertig and Mann (1980) and Aslam and Jun (2009), developed their sampling plans using this viewpoint. In our approach, the quality level is measured through the ratio of its mean lifetime to the specified length, μ/μ_0. These mean ratios are very helpful for the producer to enhance the quality of products. From the producer's perspective, the probability of lot acceptance should be at least 1-α at the Acceptable Quality Level (AQL), p_1. So the producer demands that a lot should be accepted at various levels, say $k = \mu/\mu_0 = 2, 4, 6, 8, 10.$ On the other hand, from the consumer's viewpoint, the lot rejection probability should be at most β at the Limiting Quality Level (LQL), p_2. In this way, the consumer considers that a lot should be rejected when μ/μ_0 = 1 Therefore, in order to find the design parameters, n, c_1 and c_2, the following two inequalities should be satisfied:

$$P_A(p_1) = \frac{\sum_{i=0}^{c_1}\binom{n}{i}p_1^i(1-p_1)^{n-i}}{1-\sum_{i=0}^{c_2}\binom{n}{i}p_1^i(1-p_1)^{n-i}+\sum_{i=0}^{c_1}\binom{n}{i}p_1^i(1-p_1)^{n-i}} \geq 1-\alpha \quad (7)$$

$$P_A(p_2) = \frac{\sum_{i=0}^{c_1}\binom{n}{i}p_2^i(1-p_2)^{n-i}}{1-\sum_{i=0}^{c_2}\binom{n}{i}p_2^i(1-p_2)^{n-i}+\sum_{i=0}^{c_1}\binom{n}{i}p_2^i(1-p_2)^{n-i}} \leq \beta, \quad (8)$$

Table 1: Design parameters of the proposed plan for the weibull distribution with γ = 1

		a = 0.5				a = 1.0			
β	μ/μ₀	c_1	c_2	n	P_A (p) (ASN)	c_1	c_2	n	P_A (p) (ASN)
0.25	2	7	9	25	0.9610 (34.8)	3	6	9	0.9565 (24.7)
	4	1	2	7	0.9534 (9.5)	1	2	4	0.9561 (5.9)
	6	0	1	4	0.9542 (6.2)	↑	↑	↑	0.9858 (5.9)
	8	↑	↑	↑	0.9746 (6.2)	↑	↑	↑	0.9937 (5.9)
	10	↑	↑	↑	0.9839 (6.2)	0	1	3	0.9668 (4.0)
0.10	2	11	13	40	0.9513 (47.2)	5	9	14	0.9705 (34.8)
	4	1	3	10	0.9677 (15.3)	0	3	5	0.9668 (12.5)
	6	1	2	9	0.9659 (10.8)	1	2	5	0.9669 (6.2)
	8	↑	↑	↑	0.9845 (10.8)	↑	↑	↑	0.9903 (6.2)
	10	0	1	6	0.9595 (7.4)	0	1	3	0.9668 (4.0)
0.05	2	15	18	56	0.9671 (64.5)	12	15	27	0.9641 (34.9)
	4	1	3	11	0.9514 (15.2)	1	3	6	0.9612 (9.4)
	6	0	2	8	0.9606 (11.6)	0	2	4	0.9757 (7.3)
	8	↑	↑	↑	0.9840 (11.6)	↑	↑	↑	0.9903 (7.3)
	10	↑	↑	↑	0.9920 (11.6)	↑	↑	↑	0.9952 (7.3)
0.01	2	16	20	65	0.9516 (71.3)	20	23	45	0.9528 (47.7)
	4	2	5	19	0.9708 (22.9)	1	4	8	0.9648 (11.9)
	6	0	3	11	0.9791 (15.9)	1	3	8	0.9654 (9.1)
	8	0	2	10	0.9651 (12.1)	0	2	5	0.9754 (6.7)
	10	↑	↑	↑	0.9826 (12.1)	↑	↑	↑	0.9879 (6.7)

The upward arrow (↑) indicates that the same value of the above cell applies

Table 2: Design parameters of the proposed plan for the weibull distribution with γ = 2

		a = 0.5				a = 1.0			
β	μ/μ₀	c_1	c_2	n	P_A (p) (ASN)	c_1	c_2	n	P_A (p) (ASN)
0.25	2	0	2	12	0.9695 (26.2)	0	2	4	0.9587 (9.4)
	4	0	1	10	0.9930 (14.4)	0	1	3	0.9923 (4.5)
	6	↑	↑	↑	0.9986 (14.4)	↑	↑	↑	0.9985 (4.5)
	8	↑	↑	↑	0.9996 (14.4)	↑	↑	↑	0.9995 (4.5)
	10	↑	↑	↑	0.9998 (14.4)	↑	↑	↑	0.9998 (4.5)
0.10	2	1	3	23	0.9688 (34.2)	1	3	7	0.9661 (11.0)
	4	0	1	13	0.9877 (16.7)	0	1	4	0.9845 (5.0)
	6	↑	↑	↑	0.9976 (16.7)	↑	↑	↑	0.9970 (5.0)
	8	↑	↑	↑	0.9993(16.7)	↑	↑	↑	0.9991 (5.0)
	10	↑	↑	↑	0.9997 (16.7)	↑	↑	↑	0.9996 (5.0)
0.05	2	0	3	20	0.9646 (39.3)	2	4	10	0.9731 (13.2)
	4	0	1	17	0.9784 (19.6)	0	1	5	0.9741 (5.7)
	6	↑	↑	↑	0.9958 (19.6)	↑	↑	↑	0.9951 (5.7)
	8	↑	↑	↑	0.9987 (19.6)	↑	↑	↑	0.9985 (5.7)
	10	↑	↑	↑	0.9995 (19.6)	↑	↑	↑	0.9994 (5.7)
0.01	2	0	4	27	0.9693(49.5)	1	4	10	0.9560 (13.7)
	4	0	1	24	0.9559 (25.2)	0	1	6	0.9610 (6.4)
	6	↑	↑	↑	0.9951 (25.2)	↑	↑	↑	0.9925 (6.4)
	8	↑	↑	↑	0.9973 (25.2)	↑	↑	↑	0.9977 (6.4)
	10	↑	↑	↑	0.9989 (25.2)	↑	↑	↑	0.9991 (6.4)

The upward arrow (↑) indicates that the same value of the above cell applies

Table 3: Design parameters of the proposed plan for the weibull distribution with γ = 3

		a = 0.5				a = 1.0			
β	μ/μ0	c1	c2	n	PA (p) (ASN)	c1	c2	n	PA (p) (ASN)
0.25	2	0	1	20	0.9752 (29.2)	0	1	3	0.9739 (4.7)
	4	↑	↑	↑	0.9996 (29.2)	↑	↑	↑	0.9996 (4.7)
	6	↑	↑	↑	1.0000 (29.2)	↑	↑	↑	1.0000 (4.7)
	8	↑	↑	↑	1.0000 (29.2)	↑	↑	↑	1.0000 (4.7)
	10	↑	↑	↑	1.0000 (29.2)	↑	↑	↑	1.0000 (4.7)
0.10	2	1	2	47	0.9834 (54.8)	1	2	7	0.9816 (8.3)
	4	0	1	29	0.9992 (36.4)	0	1	4	0.9992 (5.3)
	6	↑	↑	↑	0.9999 (36.4)	↑	↑	↑	0.9999 (5.3)
	8	↑	↑	↑	1.0000 (36.4)	↑	↑	↑	1.0000 (5.3)
	10	↑	↑	↑	1.0000 (36.4)	↑	↑	↑	1.0000 (5.3)
0.05	2	0	2	39	0.9860 (56.7)	0	2	6	0.9830 (8.7)
	4	0	1	36	0.9988 (41.7)	0	1	5	0.9987 (5.9)
	6	↑	↑	↑	0.9999 (41.7)	↑	↑	↑	0.9999 (5.9)
	8	↑	↑	↑	1.0000 (41.7)	↑	↑	↑	1.0000 (5.9)
	10	↑	↑	↑	1.0000 (41.7)	↑	↑	↑	1.0000 (5.9)
0.01	2	0	2	54	0.9613 (63.0)	0	2	7	0.9699 (8.8)
	4	0	1	53	0.9973 (55.4)	0	1	7	0.9973 (7.4)
	6	↑	↑	↑	0.9998 (55.4)	↑	↑	↑	0.9998 (7.4)
	8	↑	↑	↑	1.0000 (55.4)	↑	↑	↑	1.0000 (7.4)
	10	↑	↑	↑	1.0000 (55.4)	↑	↑	↑	1.0000 (7.4)

The upward arrow (↑) indicates that the same value of the above cell applies

Here, p_1 is defined by:

$$p_1 = 1 - \exp\left[-\left\{a\Gamma(1/\gamma)/(k\gamma)\right\}^{\gamma}\right] \quad (9)$$

where, $\mu = k\mu_0$ with $k>1$, whereas p_2 is given by:

$$p_2 = 1 - \exp\left[-\left\{a\Gamma(1/\gamma)/\gamma\right\}^{\gamma}\right]. \quad (10)$$

The determination of the optimal design parameters is an important issue in acceptance sampling. As suggested by Aslam and Jun (2009), the design parameters should be chosen to minimize the Average Sample Number (ASN) at the limiting quality level. The ASN of the proposed plan when p is the true fraction defective p is derived to be:

$$ASN(p) = \frac{n}{P_a + P_r} \quad (11)$$

So, the optimization problem to find the best plan (n, c_1, c_2) is a nonlinear integer programming, which may be formulated as follows:

Minimize ASN (P_2)
Subject to
$P_A(p_1) \geq 1-\alpha,$
$P_A(p_2) \leq \beta,\ n, c_1, c_2 \in Z, n > 0,\ 0 \leq c_1 \leq c_2,$

where, Z is the set of integers. The proposed methodology is flexible and can be used for any value of the shape parameter. We can solve the above optimization problem by using a grid search because the solution space is not too large.

Table 1 to 3 present the optimal design parameters of the proposed sampling plan for the Weibull distribution having $\gamma = 1, 2, 3$, respectively, for $\alpha = 0.05$ and some selected values of β, μ/μ_0 and a. The average sample number and the probability of acceptance are also reported.

From these tables, we can see that for the same values of γ, β and μ/μ_0, as the values of a increases from 0.5 to 1.0, the sample size n decreases. But, by fixing all other values, when the shape parameter changes from 1 to 3, we do not note any specific pattern in the design parameters. The above tables are also equipped with the probability of acceptance, P_a and the ASN for the optimal acceptance plans. In addition, we observe an increasing trend in the probability of acceptance as the mean ratio increases when the shape parameter is 2 or 3. It means that, as the producer increases the quality level of their product, the producer risks decrease.

Example 1: Consider, for example, that an experimenter wants to run a test for 1000 h and he/she knows that the lifetime of the product follows a Weibull distribution with $\gamma = 1$. The specified life of the submitted product is 1000 h and the termination ratio is $a = 1$. Assuming that $\alpha = 0.05$, $\beta = 0.05$ and $\mu/\mu_0 = 6$, it is obtained from Table 1 that the optimal design parameters are $n = 4$, $c_1 = 0$ and $c_2 = 2$. Thus, the sampling plan may be stated as follows: Take a random sample of size 4 from the lot and put it on test for 1000 h. Reject the lot if the number of failures reaches 3 and terminate the test. Accept the lot if no failure occurs during 1000 h. Repeat the experiments if the number of failures is 1 or 2.

Table 4: Sample sizes of the proposed plans and single sampling plans for the weibull distributions when α = 0.05 and α = 0.5

		$\gamma = 1$		$\gamma = 2$		$\gamma = 3$	
β	μ/μ_0	Proposed (ASN)	Single	Proposed (ASN)	Single	Proposed (ASN)	Single
0.25	2	34.8	40	26.2	50	29.2	70
	4	9.5	40	14.4	50	29.2	70
	6	6.2	40	14.4	50	29.2	70
	8	6.2	40	14.4	50	29.2	70
	10	6.2	40	14.4	50	29.2	70
0.10	2	47.2	63	34.2	50	54.8	70
	4	15.3	40	16.7	50	36.4	70
	6	10.8	40	16.7	50	36.4	70
	8	10.8	40	16.7	50	36.4	70
	10	7.4	40	16.7	50	36.4	70
0.05	2	64.5	78	39.3	64	56.7	72
	4	15.2	40	19.6	50	41.7	70
	6	11.6	40	19.6	50	41.7	70
	8	11.6	40	19.6	50	41.7	70
	10	11.6	40	19.6	50	41.7	70
0.01	2	71.3	113	49.5	93	63.0	115
	4	22.9	40	25.2	50	55.4	76
	6	15.9	40	25.2	50	55.4	70
	8	12.1	40	25.2	50	55.4	70
	10	12.1	40	25.2	50	55.4	70

Example 2: Suppose that a manufacturer adopts the proposed plan when the shape parameter is unknown. The specified life of the product is μ_0 = 1000 h and the test duration is 500 h Assuming that α = 0.05, β = 0.05 and μ/μ_0 = 2. It is known that the lifetime of the submitted product is well fitted by the Weibull distribution. In order to estimate γ, failure data were collected from 10 products of the previous lots as follows: 507, 720, 892, 949, 1031, 1175, 1206, 1428, 1538 and 2083, respectively. Then, the Maximum Likelihood Estimate (MLE) of the shape parameter is obtained by $\hat{\gamma}$ = 2.883. So, let us assume that γ = 3 now. From Table 3 we note that the optimal design parameters are n = 39, c_1 = 0 and c_2 = 2. Thus, the sampling plan may be stated as follows: Take a random sample of size 39 from the lot and put it on test for 500 h. Reject the lot if the number of failures reaches 3 and terminate the test. Accept the lot if no failure occurs during 500 h. If the number of failures is 1 or 2, the experiments should be repeated.

Evidently, in terms of the required sample sizes, an optimal attributes repetitive group sampling plan based on truncated life tests is preferable to the corresponding single sampling plan. Table 4 displays the average sample number of the proposed plan and the sample size required for the single sampling plan when $\alpha = 0.05$ and $a = 0.5$ for the selected values of β and μ/μ_0.

From Table 4, we can see that the proposed repetitive group sampling plan reduces the sample size for the life test. For instance, if β = 0.10, μ/μ_0 = 4, a = 0.5 and γ = 2, the ASN of the proposed plan is 16.7, whereas the single plan needs 50 units.

In general, due to the discreteness of Eq. (7) and (8), the producer's and consumer's risks corresponding to the optimal attributes repetitive group sampling plan do not coincide exactly with α and β. Therefore, we will find the minimum risks for the attribute plan for the selected sampling plans. Golub (1953) proposed a method to find the design parameters of the single sampling plan involving minimum producer and consumer risks. Govindaraju and Subramani (1990) proposed a method of selection of single plan for given AQL and LQL involving minimum risks. For the proposed plan, the minimum producer's risk and consumer's risk are determined for selected values of c_1, c_2 and n in such that Eq. (7) and (8) are satisfied simultaneously. Table 5 reports the minimum producer's and consumer's risks, a^* and β^*, of the optimal repetitive group sampling plans for a = 0.05 and some selected values of γ, β, μ/μ_0 and a.

From Table 5, it can be seen that, as the mean ratio increases, both producer's and consumer's risks decrease when γ is 2 or 3. Similarly, as the Weibull shape parameter increases from 1 to 3, both producer's and consumer's risks decrease. As illustration, if the analyst considers the case of γ = 1, α = 0.05, β = 0.05, μ/μ_0 = 4 and a = 1, the minimum producer's risk of the repetitive group sampling plan is only α^* = 0.0388, whereas the minimum consumer's risk is only β^* = 0.0439.

Effect of mis-specification of shape parameter: In this study, the shape parameter of Weibull distribution is assumed to be known. So, it may be interesting to see the effect of mis-specification of the shape parameter on the probability of acceptance. Let us denote γ_0 to be the true shape parameter, which is possibly different from the specified one γ used for designing the

Table 5: Minimum producer's and consumer's risks of the proposed sampling plans for the weibull distribution when α = 0.05

		γ = 1				γ = 2				γ = 3			
		a = 0.5		a = 1.0		a = 0.5		a = 1.0		a = 0.5		a = 1.0	
β	μ/μ0	α*	β*	α*	β*	α*	β*	α*	β*	α*	β*	α*	β*
0.25	2	0.0390	0.2362	0.0435	0.1857	0.0305	0.0206	0.0413	0.1018	0.0248	0.2458	0.0261	0.1868
	4	0.0466	0.2282	0.0439	0.2135	0.0070	0.2018	0.0077	0.1435	0.0004	0.2458	0.0004	0.1868
	6	0.0458	0.2086	0.0142	0.2135	0.0014	0.2018	0.0015	0.1435	0.0000	0.2458	0.0000	0.1868
	8	0.0254	0.2086	0.0063	0.2135	0.0004	0.2018	0.0005	0.1435	0.0000	0.2458	0.0000	0.1868
	10	0.0161	0.2086	0.0332	0.0670	0.0002	0.2018	0.0002	0.1435	0.0000	0.2458	0.0000	0.1868
0.10	2	0.0487	0.0980	0.0295	0.0844	0.0312	0.0974	0.0055	0.0604	0.0166	0.0956	0.0184	0.0669
	4	0.0323	0.0774	0.0332	0.0168	0.0123	0.0998	0.0155	0.0544	0.0008	0.0951	0.0013	0.0333
	6	0.0341	0.0913	0.0331	0.0807	0.0024	0.0998	0.0030	0.0544	0.0001	0.0951	0.0001	0.0333
	8	0.0155	0.0913	0.0149	0.0807	0.0007	0.0998	0.0009	0.0544	0.0000	0.0951	0.0000	0.0333
	10	0.0405	0.0618	0.0332	0.0670	0.0003	0.0988	0.0004	0.0544	0.0000	0.0951	0.0000	0.0333
0.05	2	0.0329	0.0399	0.0359	0.0468	0.0354	0.0387	0.0269	0.0396	0.9860	0.0452	0.0170	0.0203
	4	0.0486	0.0461	0.0388	0.0439	0.0216	0.0409	0.0259	0.0233	0.0012	0.0470	0.0013	0.0333
	6	0.0394	0.0266	0.0243	0.0333	0.0042	0.0409	0.0049	0.0233	0.0001	0.0470	0.0001	0.0333
	8	0.0160	0.0266	0.0097	0.0333	0.0013	0.0409	0.0015	0.0233	0.0000	0.0470	0.0000	0.0333
	10	0.0080	0.0266	0.0048	0.0333	0.0005	0.0409	0.0006	0.0233	0.0000	0.0470	0.0000	0.0333
0.01	2	0.0484	0.0100	0.0472	0.0084	0.0307	0.0091	0.0440	0.0069	0.0387	0.0095	0.0301	0.0086
	4	0.0292	0.0077	0.0352	0.0074	0.0441	0.0094	0.0390	0.0096	0.0027	0.0093	0.0027	0.0072
	6	0.0209	0.0059	0.0346	0.0056	0.0085	0.0094	0.0075	0.0096	0.0002	0.0093	0.0002	0.0072
	8	0.0349	0.0081	0.0246	0.0091	0.0027	0.0094	0.0023	0.0096	0.0000	0.0093	0.0000	0.0072
	10	0.0174	0.0081	0.0121	0.0091	0.0011	0.0094	0.0009	0.0096	0.0000	0.0093	0.0000	0.0072

Table 6: Acceptance probabilities corresponding to true shape parameters when γ = 2 and *a* = 0.5

		γ0 = 1.5		γ0 = 2.5		γ0 = 3.0	
β	μ/μ0	$P_A(p^0_1)$	$P_A(p^0_2)$	$P_A(p^0_1)$	$P_A(p^0_2)$	$P_A(p^0_1)$	$P_A(p^0_2)$
0.25	2	0.7130	0.0410	0.9970	0.5420	0.9997	0.8191
	4	0.9320	0.0602	0.9992	0.4333	0.9999	0.6647
	6	0.9801	↑	0.9999	↑	1.0000	↑
	8	0.9917	↑	1.0000	↑	↑	↑
	10	0.9958	↑	1.0000	↑	↑	↑
0.10	2	0.3075	0.0011	0.9971	0.1364	0.9999	0.5040
	4	0.8836	0.0222	0.9987	0.2722	0.9998	0.5075
	6	0.9654	↑	0.9998	↑	1.0000	↑
	8	0.9856	↑	1.0000	↑	↑	↑
	10	0.9927	↑	1.0000	↑	↑	↑
0.05	2	0.4691	0.0031	0.9984	0.2442	0.9999	0.6654
	4	0.8048	0.0063	0.9977	0.1449	0.9997	0.3380
	6	0.9395	↑	0.9997	↑	1.0000	↑
	8	0.9747	↑	0.9999	↑	↑	↑
	10	0.9872	↑	1.0000	↑	↑	↑
0.01	2	0.3057	0.0003	0.9994	0.1114	1.0000	0.5476
	4	0.6479	0.0007	0.9952	0.0503	0.9995	0.1604
	6	0.8792	↑	0.9994	↑	1.0000	↑
	8	0.9484	↑	0.9999	↑	↑	↑
	10	0.9738	↑	1.0000	↑	↑	↑

The upper arrow (↑) represents the same value applied to its cell

sampling plan. Let p^0_1 and p^0_2 be the AQL and LQL, respectively, corresponding to the true shape parameter. Then,

$$p_1^0 = 1 - \exp\left[-\{a\Gamma(1/\gamma_0)/(k\gamma_0)\}^{\gamma_0}\right]$$

$$p_2^0 = 1 - \exp\left[-\{a\Gamma(1/\gamma_0)/\gamma_0\}^{\gamma_0}\right]$$

Therefore, the probabilities of acceptance at AQL and LQL will be $P_A(p^0_1)$ and $P_A(p^0_2)$, respectively, based on Eq. (7) and (8), where parameters (n, c_1, c_2) are the ones for the specified shape parameter γ. There may be no problems from the mis-specification of the shape parameter if they still satisfy the producer's and the consumer's risks (i.e., $P_A(p_1^0) \geq 1-\alpha$; $P_A(p_2^0) \leq \beta$).

Suppose for illustration purpose that we are using the shape parameter $\gamma = 2$ when designing the proposed plan but that the true value is different from this. We would like to know the effect of the mis-specification of shape parameter on the lot acceptance probabilities at the true value. Table 6 shows the acceptance probabilities at AQL and LQL for the proposed sampling plan with a = 0.5 when the true shape parameter (assumed as 1.5, 2.5 or 3.0, respectively) is different from the specified one ($\gamma = 2$). Table 7 reports

Table 7: Acceptance probabilities corresponding to true shape parameters when $\gamma = 2$ and $a = 1.0$

		$\gamma_0 = 1.5$		$\gamma_0 = 2.5$		$\gamma_0 = 3.0$	
β	μ/μ_0	$P_A(p^0_1)$	$P_A(p^0_2)$	$P_A(p^0_1)$	$P_A(p^0_2)$	$P_A(p^0_1)$	$P_A(p^0_2)$
0.25	2	0.8427	0.0726	0.9888	0.1288	0.9967	0.1506
	4	0.9624	0.1142	0.9983	0.1681	0.9996	0.1868
	6	0.9893	↑	0.9998	↑	1.0000	↑
	8	0.9956	↑	0.9999	↑	↑	↑
	10	0.9978	↑	1.0000	↑	↑	↑
0.10	2	0.8415	0.0399	0.9927	0.0808	0.9983	0.0980
	4	0.9251	0.0408	0.9967	0.0666	0.9992	0.0763
	6	0.9784	↑	0.9996	↑	0.9999	↑
	8	0.9911	↑	0.9999	↑	1.0000	↑
	10	0.9955	↑	1.0000	↑	↑	↑
0.05	2	0.8476	0.0243	0.9954	0.0556	0.9992	0.0697
	4	0.8770	0.0158	0.9944	0.0284	0.9987	0.0333
	6	0.9638	↑	0.9993	↑	0.9999	↑
	8	0.9850	↑	0.9998	↑	1.0000	↑
	10	0.9924	↑	0.9999	↑	↑	↑
0.01	2	0.7142	0.0038	0.9937	0.0105	0.9990	0.0140
	4	0.8206	0.0064	0.9916	0.0127	0.9981	0.0153
	6	0.9457	↑	0.9989	↑	0.9998	↑
	8	0.9774	↑	0.9997	↑	1.0000	↑
	10	0.9886	↑	0.9999	↑	↑	↑

The upper arrow (↑) represents the same value applied to its cell

similar results for the proposed sampling plan with $a = 1.0$. It is observed that producer's risks are still satisfied when the true parameter is larger than the specified one but that they may not be satisfied at lower mean ratios when the true parameter is smaller than the specified one. It is also seen that consumer's risks are still satisfied when the true parameter is smaller than the specified one but that they may not when the true parameter is larger than the specified one. We observe from Table 7 that the mis-specification of shape parameters may not be serious when $a = 1.0$.

CONCLUSION

An attributes repetitive group sampling plan based on truncated life tests has been proposed when the life time of the submitted product follows a Weibull distribution with known shape parameter. The design parameters of the proposed plan were determined by applying the two-point approach considering the producer's and the consumer's risks simultaneously. The quality level was considered in terms of the mean ratio to the specified life. We investigated the effect of mis-specification of shape parameter on the lot acceptance probabilities at AQL and LQL.

The proposed plan is quite flexible and outperforms the ordinary single sampling plan in terms of the sample size required. So, it is strongly recommended that industrial practitioners can use the proposed plan for the testing of electronic components such as mobile charger, electronic devices etc. when they have strong evidence that the failure time of products follows the Weibull distribution. The researchers can extend our approach to other statistical distributions such as gamma distribution, generalized exponential distribution, Pareto distribution and Burr type XII distributions. The comparison between several distributions may also be a fruitful area for future research.

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REFERENCES

Aslam, M. and C.H. Jun, 2009. A group acceptance sampling plan for truncated life test having Weibull distribution. J. Appl. Stat., 36(9): 1021-1027.

Aslam, M. and R.R.L. Kantam, 2008. Economic acceptance sampling based on truncated life tests in the Birnbaum-Saunders distribution. Pak. J. Stat., 24(4): 269-276.

Aslam, M., C.H. Jun and M. Ahmad, 2009. Double acceptance sampling plans based on truncated life tests in the Weibull model. J. Stat. Theor. Appl., 8(2): 191-206.

Balakrishnan, N., V. Leiva and J. Lopez, 2007. Acceptance sampling plans from truncated life tests based on the generalized Birnbaum-Saunders distribution. Commun. Stat-Simul. C., 36: 643-656.

Balamurali, S. and C.H. Jun, 2006. Repetitive group sampling procedure for variables inspection. J. Appl. Stat., 33(3): 327-338.

Fernandez, A.J., C.J. Perez-Gonzalez, M. Aslam and C.H. Jun, 2011. Design of progressively censored group sampling plans for Weibull distributions: An optimization problem. Eur. J. Oper. Res., 211(3): 525-532.

Fertig, F.W. and N.R. Mann, 1980. Life-test sampling plans for two-parameter Weibull populations. Technometrics, 22(2): 165-177.

Golub, A., 1953. Designing single-sampling inspection plans when the sample size is fixed. J. Appl. Stat. Assoc., 48: 278-288.

Govindaraju, K. and K. Subramani, 1990. Selection of single sampling attributes plan for given acceptable quality level and limiting quality level involving minimum risks. Commun. Stat-Simul. C., 19(4): 1293-1302.

Jun, C.H., S. Balamurali and S.H. Lee, 2006. Variables sampling plans for Weibull distributed lifetimes under sudden death testing. IEEE T. Reliab., 55(1): 53-58.

Kantam, R.R.L., K. Rosaiah and G.S. Rao, 2001. Acceptance sampling based on life tests: Log-logistic models. J. Appl. Stat., 28(1): 121-128.

Lio, Y.L., T.R. Tsai and S.J. Wu, 2010a. Acceptance sampling plans from truncated life tests based on the Birnbaum-Saunders distribution for percentiles. Commun. Stat-Simul. C., 39: 119-136.

Lio, Y.L., T.R. Tsai and S.J. Wu, 2010b. Acceptance sampling plans from truncated life tests based on the Burr type XII percentiles. J. Chin. Inst. Ind. Eng., 27(4): 270-280.

Rosaiah, K., R.R.L. Kantam and S. Kumar, 2006. Reliability of test plans for exponentiated log-logistic distribution. Econ. Qual. Cont., 21(2): 165-175.

Sherman, R.E., 1965. Design and evaluation of repetitive group sampling plan. Technometrics, 7: 11-21.

Tsai, T.R. and S.J. Wu, 2006. Acceptance sampling based on truncated life tests for generalized Rayleigh distribution. J. Appl. Stat., 33(6): 595-600.