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Research Article

Decay of Temperature Fluctuations in Dusty Fluid Homogeneous Turbulence Prior to the Ultimate Period in Presence of Coriolis Force

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Abstract: Using deissler's method we have studied the decay of temperature fluctuations in dusty fluid homogeneous turbulence before the final period in presence of coriolis force and have considered correlations between fluctuating quantities at two- and three-point. The equations for two- and three-point correlation is obtained and the set of equations is made to determinate by neglecting the forth-order correlation in comparison to the second- and third-order correlations. For solving the correlation equations are converted to spectral form by taking their Fourier transform. Finally, integrating the energy spectrum over all wave numbers, the energy decay law of temperature fluctuations in homogeneous dusty fluid turbulence before the final period in presence of coriolis force is obtained.

Keywords: Coriolis force, dust particles, homogeneous turbulence, temperature fluctuations

INTRODUCTION

Interest in motion of dusty viscous fluid has developed rapidly in recent years. Such situations occur in movement of dust-laden air, in problems of fluidization, in the use of dust in gas cooling system and in sedimentation problem in tidal rivers.

In geophysical flows, the system is usually rotating with a constant angular velocity. Such large-scale flows are generally turbulent. When the motion is referred to axes, which rotate steadily with the bulk of the fluid, the coriolis and centrifugal force must be supposed to act on the fluid. On a rotating earth the coriolis force acts to change the direction of a moving body to the right in the Northern Hemisphere and to the left in the Southern Hemisphere. This force plays an important role in a rotating system of turbulent flow, while centrifugal force with the potential is incorporated in to the pressure.

Taylor (1935) has been pointed out that the equation of motion of turbulence relates the pressure gradient and the acceleration of the fluid particles and the mean-square acceleration can be determined from the observation of the diffusion of marked fluid particles. The behavior of dust particles in a turbulent flow depends on the concentration of the particles and the size of the particles with respect to the scale of turbulent fluid. Saffman (1962) derived and equation that describe the motion of a fluid containing small dust particle, which is applicable to laminar flows as well as turbulent flow. Kishore and Sarker (1990) studied the rate of change of vorticity covariance in MHD turbulent flow of dusty incompressible fluid. Also Rahman (2010) studied the Rate of change of vorticity covariance in MHD turbulent flow of dusty fluid in a rotating system. Kishore and Sinha (1988) also studied the rate of change of vorticity covariance of dusty fluid turbulence. Corrsin (1951b) had made an analytical attack on the problem of turbulent temperature fluctuations using the approaches employed in the statistical theory of turbulence. His results pertain to the final period of decay and for the case of appreciable convective effects, to the "energy" spectral from in specific wave-number ranges. Deissler (1958, 1960) developed a theory for homogeneous turbulence, which was valid for times before the final period. Following Deissler's theory Loeffler and Deissler (1961) studied the decay of temperature fluctuations in homogeneous turbulence before the final period. Sarker and Azad (2006), Azad and Sarker (2006), Azad and Sarker (2008), Azad *et al*. (2006), Azad and Sarker (2009) and Azad *et al*. (2007-2008) also studied the decay of temperature fluctuations in homogeneous and MHD dusty fluid turbulence. Azad *et al*. (2012) studied the transport equatoin for the joint distribution function of velocity, temperature and concentration in convective tubulent flow in presence of dust particles. Molla *et al*. (2012) also studied decay of temperature fluctuations in homogeneous turbulenc before the final period in a rotating system. Bkar Pk *et al*. (2012) studied first-order reactant in homogeneou dusty fluid turbulence prior to the ultimate phase of decay for four-point correlation in a rotating system.

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They considered dust particles and Coriolis force on their won works. In their study, they considered two- and three-point correlations and neglecting fourth- and higher-order correlation terms compared to the second- and third-order correlation terms. Sinha (1988) had considered the effect of dust particles on the acceleration of ordinary turbulence. Kishore and Singh (1984) had studied the statistical theory of decay process of homogeneous hydro-magnetic turbulence. Dixit and Upadhyay (1989a) also had deliberated the effect of coriolis force on acceleration covariance in MHD turbulent dusty flow with rotational symmetry. Kishore and Golsefied (1988) considered the effect of coriolis force on acceleration covariance in MHD turbulent flow of a dusty incompressible fluid. Shimomura and Yoshizawa (1986) discussed the statistical analysis of an isotropic turbulent viscosity in a rotating system.

In the present study, by analyzing the above theories we have studied the decay of temperature fluctuations in dusty fluid homogeneous turbulence prior to the final period in presence of coriolis force considering the correlations between fluctuating quantities at two- and -three point and single time. In this study, Deissler's method is used to solving the problem. Throughout the study we have obtained the energy decay law of temperature fluctuations in homogeneous dusty fluid turbulence prior to the final period due to corilis force. In result, it is shown that the energy decays more rapidly than non rotating clean fluid.

CORRELATION AND SPECTRAL EQUATIONS

For an incompressible fluid with constant properties and for negligible frictional heating, the energy equation may be written at the point P':

$$\left[\frac{\partial \tilde{T}}{\partial t}+\tilde{u}_i\frac{\partial \tilde{T}}{\partial x_i}\right]=\frac{k}{\rho c_p}\frac{\partial^2 \tilde{T}}{\partial x_i\partial x_i} \tag{1}$$

where,

$\tilde{T}$ = Instantaneous values of temperature
$\tilde{u}_i$ = Instantaneous velocity
ρ = Fluid density
c_p = Heat capacity at constant pressure
k = Thermal conductivity
x_i = Space co-ordinate
t = Time

Separate these instantaneous values into time average and fluctuating components as $\tilde{T}=\overline{T}+T$ and $\tilde{u}_i=\overline{u}_i+u_i$ Eq. (1) may be written:

$$\left[\frac{\partial \overline{T}}{\partial t}+\frac{\partial T}{\partial t}+\overline{u}_i\frac{\partial \overline{T}}{\partial x_i}+\overline{u}_i\frac{\partial T}{\partial x_i}+u_i\frac{\partial \overline{T}}{\partial x_i}+u_i\frac{\partial T}{\partial x_i}\right.$$
$$=\gamma\left[\frac{\partial^2 \overline{T}}{\partial x_i\partial x_i}+\frac{\partial^2 T}{\partial x_i\partial x_i}\right] \tag{2}$$

where,

$$\gamma=\frac{k}{\rho c_p}$$

From the case of homogeneity it follows that $\frac{\partial \overline{T}}{\partial x_i}=0$ and in addition the usual assumption is made that $\overline{T}$ is independent of time and that $u_i=0$ Thus Eq. (2) simplifies to:

$$\left[\frac{\partial T}{\partial t}+u_i\frac{\partial T}{\partial x_i}\right]=\frac{\nu}{\mathrm{P}_r}\left[\frac{\partial^2 T}{\partial x_i\partial x_i}\right] \tag{3}$$

where,
$P_r=\frac{\nu}{\gamma}$, Prandtl number
v = Kinematic Viscosity

Equation (3) holds at the arbitrary point P. For the point P' the corresponding equation can be written:

$$\left[\frac{\partial T'}{\partial t}+u_i'\frac{\partial T'}{\partial x_i'}\right]=\frac{\nu}{\mathrm{P}_r}\left[\frac{\partial^2 T'}{\partial x_i'\partial x_i'}\right] \tag{4}$$

Multiplying Eq. (3) by T', Eq. (4) by T and taking time average and adding the two equations gives:

$$\left[\frac{\partial \overline{TT'}}{\partial t}+u_i\frac{\partial \overline{TT'}}{\partial x_i}+u_i'\frac{\partial \overline{TT'}}{\partial x_i'}\right]=\frac{\nu}{\mathrm{P}_r}\left[\frac{\partial^2 \overline{TT'}}{\partial x_i\partial x_i}+\frac{\partial^2 \overline{TT'}}{\partial x_i'\partial x_i'}\right] \tag{5}$$

The continuity equation is:

$$\frac{\partial u_i}{\partial x_i}=\frac{\partial u_i'}{\partial x_i'}=0 \tag{6}$$

Substitution of Eq. (6) into (5) yields:

$$\frac{\partial \overline{TT'}}{\partial t}+u_i\frac{\partial \overline{u_iTT'}}{\partial x_i}+\frac{\partial \overline{u_i'TT'}}{\partial x_i'}=\frac{\nu}{\mathrm{P}_r}\left[\frac{\partial^2 \overline{TT'}}{\partial x_i\partial x_i}+\frac{\partial^2 \overline{TT'}}{\partial x_i'\partial x_i'}\right] \tag{7}$$

By use of a new independent variable:

$$r_i=x_i'-x_i \text{ i.e., } \frac{\partial}{\partial x_i}=-\frac{\partial}{\partial r_i},\quad \frac{\partial}{\partial x_i'}=\frac{\partial}{\partial r_i}$$

$$\frac{\partial \overline{TT'}}{\partial t}-\frac{\partial \overline{u_iTT'}}{\partial r_i}+\frac{\partial \overline{u_i'TT'}}{\partial r_i}=\frac{2\nu}{\mathrm{P}_r}\frac{\partial^2 \overline{TT'}}{\partial r_i\partial r_i} \tag{8}$$

This equation is converted into spectral form by use of the following three dimensional Fourier transforms:

$$\overline{TT'}(\hat{r})=\int_{-\infty}^{\infty}\overline{\tau\tau'}(\hat{K})\,\exp\left[i\left(\hat{K}.\hat{r}\right)\right]d\hat{K} \tag{9}$$

$$\overline{u_i TT'}(\hat{r}) = \int_{-\infty}^{\infty} \overline{\phi_i \tau\tau'}(\hat{K}) \exp\left[i(\hat{K}.\hat{r})\right] d\hat{K} \tag{10}$$

And by interchanging P and P':

$$\overline{u_i' TT'}(\hat{r}) = \overline{u_i TT'}(-\hat{r})$$

$$\overline{u_i TT'}(\hat{r}) = \int_{-\infty}^{\infty} \phi_i \tau\tau'(-\hat{K}) \exp\left[i(\hat{K}.\hat{r})\right] d\hat{K} \tag{11}$$

$$\overline{u_i T'}(\hat{r}) = \int_{-\infty}^{\infty} \overline{\phi_i \tau'}(-\hat{K}) \exp\left[i(\hat{K}.\hat{r})\right] d\hat{K} \tag{12}$$

$$\overline{u_i' T}(\hat{r}) = \int_{-\infty}^{\infty} \overline{\phi_i \tau}(\hat{K}) \exp\left[i(\hat{K}.\hat{r})\right] d\hat{K} \tag{13}$$

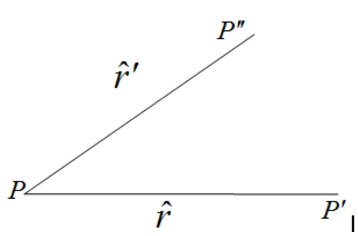
Substitution of Eq. (9)-(13) into Eq. (8) leads to the spectral equation:

$$\frac{\partial \overline{\tau\tau'}(\hat{K})}{\partial t} + ik_i\left[\overline{\phi_i \tau\tau'}(-\hat{K}) - \overline{\phi_i \tau\tau}(\hat{K})\right] = -\frac{2\nu}{P_r} k^2\, \overline{\tau\tau'}(\hat{K}) \tag{14}$$

Equation (14) is analogous to the two-point spectral equation governing the decay of velocity fluctuations and therefore the quantity $\tau\tau'(k)$ may be interpreted as a temperature fluctuation "energy" contribution of thermal eddies of size 1/k. Equation (14) expresses the time derivative of this "energy" as a function of the convective transfer to other wave numbers and the "dissipation" due to the action of thermal conductivity. The second term on the left hand side of Eq. (14) is the so called transfer to term while the term on the right hand side is "dissipation" term.

THREE POINTS CORRELATION AND SPECTRAL EQUATIONS

In order to obtain single time and three-point correlation and spectral equation we consider three points P, P' and P" with position vectors $\hat{r}$ and $\hat{r}'$ are considered:



For the two points P' and P" we can write a relation according to Eq. (7):

$$\frac{\partial (T'T'')}{\partial t} + u_i \frac{\partial (u_i' T'T'')}{\partial x_i'} + \frac{\partial (u_i'' T'T'')}{\partial x_i''} = \frac{\nu}{P_r}\left[\frac{\partial^2 (T'T'')}{\partial x_i' \partial x_i'} + \frac{\partial^2 (T'T'')}{\partial x_i'' \partial x_i''}\right] \tag{15}$$

Equation (15) multiplied through by u_j the j-th velocity fluctuation component at point P. Then the equation can be written in a rotating system at the point P:

$$\frac{\partial (u_j T'T'')}{\partial t} + u_i \frac{\partial (u_j u_i' T'T'')}{\partial x_i'} + \frac{\partial (u_j u_i'' T'T'')}{\partial x_i''} = \frac{\nu}{P_r}\left[\frac{\partial^2 (u_j T'T'')}{\partial x_i' \partial x_i'} + \frac{\partial^2 (u_j T'T'')}{\partial x_i'' \partial x_i''}\right] + T'T''\frac{\partial u_j}{\partial t} \tag{16}$$

The momentum equation at point P in presence of dust particles:

$$\frac{\partial u_j}{\partial t} + \frac{\partial (u_j u_i)}{\partial x_i} = -\frac{1}{\rho}\frac{\partial P}{\partial x_j} + \nu \frac{\partial^2 u_j}{\partial x_i \partial x_i} - 2\varepsilon_{mij}\Omega_m + f(u_j - v_j)$$

$$\Rightarrow \frac{\partial u_j}{\partial t} = -\frac{\partial (u_j u_i)}{\partial x_i} - \frac{1}{\rho}\frac{\partial P}{\partial x_j} + \nu \frac{\partial^2 u_j}{\partial x_i \partial x_i} - 2\varepsilon_{mij}\Omega_m + f(u_j - v_j) \tag{17}$$

where,

u_j : Turbulent velocity component
v_j : Dust velocity component
$f = \frac{kN}{\rho}$: (Dimension of frequency)
ε_{mij} : Alternating tensor
Ω_m : Angular velocity of a uniform rotation
N : Constant number density of dust particle

Substituted Eq. (17) into (16) the result on taking time averages is:

$$\begin{aligned}
&\frac{\partial \left(\overline{u_j T'T''}\right)}{\partial t} + u_i \frac{\partial \left(\overline{u_j u_i' T'T''}\right)}{\partial x_i'} + \frac{\partial \left(\overline{u_j u_i'' T''T'}\right)}{\partial x_i''} \\
&= \frac{\nu}{P_r}\left[\frac{\partial^2 \left(\overline{u_j T'T''}\right)}{\partial x_i' \partial x_i'} + \frac{\partial^2 \left(\overline{u_j T'T''}\right)}{\partial x_i'' \partial x_i''}\right] \\
&- \frac{\partial \left(\overline{u_j u_i T'T''}\right)}{\partial x_i} - \frac{1}{\rho}\frac{\partial \left(\overline{PT'T''}\right)}{\partial x_j} + \nu \frac{\partial^2 \left(\overline{u_j T'T''}\right)}{\partial x_i \partial x_i} \\
&+ f\left(\overline{u_j T'T''} - \overline{v_j T'T''}\right) - 2\varepsilon_{mij}\Omega_m \left(\overline{u_j T'T''}\right)
\end{aligned} \tag{18}$$

Making use of the relations $r_i = x_i' - x_i$ *and* $r_i' = x_i'' - x_i'$ allows Eq. (18) can be written as:

$$\frac{\partial\left(\overline{u_jT'T''}\right)}{\partial t}-\frac{\nu}{P_r}\left\{\begin{aligned}&(1+P_r)\frac{\partial^2\left(\overline{u_jT'T''}\right)}{\partial r_i\partial r_i}+2P_r\frac{\partial^2\left(\overline{u_jT'T''}\right)}{\partial r_i\partial r_i'}\\&+(1+P_r)\frac{\partial^2\left(\overline{u_jT'T''}\right)}{\partial r_i'\partial r_i'}\end{aligned}\right\}=$$
$$-\frac{\partial\left(\overline{u_ju_i'T'T''}\right)}{\partial r_i}-\frac{\partial\left(\overline{u_ju_i''T''T'}\right)}{\partial r_i'}+\frac{\partial\left(\overline{u_ju_iT'T''}\right)}{\partial r_i'}$$
$$+\frac{\partial\left(\overline{u_ju_iT'T''}\right)}{\partial r_i}+\frac{1}{\rho}\frac{\partial\left(\overline{PT'T''}\right)}{\partial r_j'}+\frac{1}{\rho}\frac{\partial\left(\overline{PT'T''}\right)}{\partial r_j}$$
$$+f(\overline{u_jT'T''}-\overline{v_jT'T''})-2\varepsilon_{mij}\Omega_m\left(\overline{u_jT'T''}\right) \tag{19}$$

Six-dimensional Fourier transforms for quantities this equation may be defined as:

$$\overline{u_jT'T''}=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\overline{\beta_j\theta'\theta''}\exp\left[i\left(\hat{K}\cdot\hat{r}+\hat{K}'\cdot\hat{r}'\right)\right]d\hat{K}\,d\hat{K} \tag{20}$$

$$\overline{u_ju_i'T'T''}=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\overline{\beta_j\beta_i'\theta'\theta''}\exp\left[i\left(\hat{K}\cdot\hat{r}+\hat{K}'\cdot\hat{r}'\right)\right]d\hat{K}\,d\hat{K} \tag{21}$$

$$\overline{pT'T''}=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\overline{\alpha\theta'\theta''}\exp\left[i\left(\hat{K}\cdot\hat{r}+\hat{K}'\cdot\hat{r}'\right)\right]d\hat{K}\,d\hat{K} \tag{22}$$

Interchanging the points P' and P" shows that:

$$\overline{v_ju_i''T''T'}=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\overline{\gamma_j\theta'\theta''}\exp\left[i\left(\hat{K}\cdot\hat{r}+\hat{K}'\cdot\hat{r}'\right)\right]d\hat{K}\,d\hat{K} \tag{23}$$

$$\overline{u_ju_i''T''T'}=\overline{u_ju_i'T'T''}=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\overline{\beta_j\beta_i'\theta'\theta''}\exp\left[i\left(\hat{K}\cdot\hat{r}+\hat{K}'\cdot\hat{r}'\right)\right]d\hat{K}\,d\hat{K} \tag{24}$$

Using Eq. (20)-(24) into Eq. (19) then the transformed equation can be written as:

$$\frac{\partial\left(\overline{\beta_j\theta'\theta''}\right)}{\partial t}+\frac{\nu}{P_r}\left\{\begin{aligned}&(1+P_r)k^2+2P_rk_ik_i'+(1+P_r)k'^2\\&+\frac{p_r}{\upsilon}(2\varepsilon_{mij}\Omega_m-f)\end{aligned}\right\}$$
$$-i(k_i+k_i')\overline{\beta_j\beta_i'\theta'\theta''}+i(k_i'+k_i)\overline{\beta_j\beta_i\theta'\theta''} \tag{25}$$
$$+\frac{1}{\rho}i(k_j+k_j')\overline{\alpha\theta'\theta''}-f\overline{\gamma_j\theta'\theta''}$$

If the derivative with respect to x_j is taken of the momentum Eq. (18) for point P and taking time average the resulting equation is:

$$\frac{\partial^2\left(\overline{u_ju_iT'T''}\right)}{\partial x_j\,\partial x_i}=-\frac{1}{\rho}\frac{\partial^2\left(\overline{PT'T''}\right)}{\partial x_j\,\partial x_j} \tag{26}$$

In terms of the displacement vectors $\hat{r}$ and $\hat{r}'$ this becomes:

$$\left[\frac{\partial^2}{\partial r_j'\,\partial r_i'}+2\frac{\partial^2}{\partial r_j'\,\partial r_i}+\frac{\partial^2}{\partial r_j\,\partial r_i}\right]\overline{u_ju_iT'T''}$$
$$=-\frac{1}{\rho}\left[\frac{\partial^2}{\partial r_j'\,\partial r_j'}+2\frac{\partial^2}{\partial r_j'\,\partial r_j}+\frac{\partial^2}{\partial r_j\,\partial r_j}\right]\overline{PT'T''} \tag{27}$$

Taking the Fourier transform of Eq. (27) and then solving for $\overline{\alpha\theta'\theta''}$ we get:

$$\overline{\alpha\theta'\theta''}=\frac{-\rho\left[k_j'k_i'+2k_j'k_i+k_jk_i\right]}{\left[k_j'k_j'+2k_j'k_j+k_jk_j\right]}\overline{\beta_j\beta_i\theta'\theta''} \tag{28}$$

Equation (28) can be used to eliminate $\overline{\alpha\theta'\theta''}$ from Eq. (25):

SOLUTION FOR TIMES PRIOR TO THE ULTIMATE PERIOD

To obtain the equation for final period of decay the third-order fluctuation terms are neglected compared to the second-order terms. Analogously, it would be anticipated that for times before but sufficiently near to the final period the fourth-order fluctuation terms should be negligible in comparison with the third-order terms. If this assumption is made then Eq. (28) shows that the term $\overline{\alpha\theta'\theta''}$ associated with the pressure fluctuations, should also be neglected. Thus Eq. (25) simplifies to:

$$\frac{\partial\left(\overline{\beta_j\theta'\theta''}\right)}{\partial t}+\frac{\nu}{P_r}\left\{\begin{aligned}&(1+P_r)k^2+2P_rk_ik_i'\\&+(1+P_r)k'^2+\frac{p_r}{\upsilon}(2\varepsilon_{mij}\Omega_m-fs)\end{aligned}\right\}\overline{\beta_j\theta'\theta''}=0 \tag{29}$$

where, $R\overline{\beta_j\theta'\theta''}=\overline{\gamma_j\theta'\theta''}$ and 1-R = S, R and S are arbitrary constant.

Inner multiplication of Eq. (29) by k_j and integrating between t_0 and t gives:

$$k_j\overline{\beta_j\theta'\theta''}=\left[k_j\overline{\beta_j\theta'\theta''}\right]_0\exp\left\{-\frac{\nu}{P_r}\left[\begin{aligned}&(1+P_r)k^2+2P_rk_ik_i'\cos\xi\\&+(1+P_r)k'^2+\frac{P_r}{\upsilon}(2\varepsilon_{mij}\Omega_m-fs)\end{aligned}\right](t-t_0)\right\} \tag{30}$$

Now, letting r' = 0 in Eq. (20) and comparing the result with the Eq. (10) shows that:

$$k_i\overline{\phi_i\tau\tau'}\left(\hat{K}\right)=\int_{-\infty}^{\infty}k_i\overline{\beta_i\theta'\theta''\hat{K}\hat{K}'}\,d\hat{K}' \tag{31}$$

Substituting of Eq. (30) and (31) into Eq. (14), we obtain:

$$\frac{\partial\overline{\tau\tau'}(\hat{K})}{\partial t}+\frac{2\nu}{P_r}k^2\,\overline{\tau\tau'}(\hat{K})=\int_{-\infty}^{\infty}ik_i\left[\overline{\beta_i\theta'\theta''}-\beta_i\theta'(-\hat{K})\theta''(-\hat{K}')\right]_0 \times\exp\left\{-\frac{\nu(t-t_0)}{P_r}\begin{bmatrix}(1+P_r)(k^2+k'^2)+2P_r k k'\cos\xi\\ +\frac{P_r}{\upsilon}(2\varepsilon_{mij}\Omega_m-fs)\end{bmatrix}\right\}d\hat{K}' \quad (32)$$

Now, $d\hat{K}'(\equiv dk_1' dk_2' \, dk_3')$ can be expressed in terms of k' and ξ as:

$$d\hat{K}'=-2\pi\,k'^2\,d(\cos\xi)dk' \quad (33)$$

Deissler (1958)
Substituting Eq. (33) into (32) yields:

$$\frac{\partial\overline{\tau\tau'}(\hat{K})}{\partial t}+\frac{2\nu}{P_r}k^2\,\overline{\tau\tau'}(\hat{K})= 2\int_{-\infty}^{\infty}2i\pi k_i\left[\overline{\beta_i\theta'\theta''}(\hat{K},\hat{K}')-\overline{\beta_i\theta'\theta''}(-\hat{K},-\hat{K}')\right]_0 \times\left[\int_{-1}^{1}\exp\left\{-\frac{\nu(t-t_0)}{P_r}\begin{bmatrix}(1+P_r)(k^2+k'^2)\\ +2P_r k k'\cos\xi+\frac{P_r}{\upsilon}(\varepsilon_{mij}\Omega_m-fs)\end{bmatrix}\right\}d(\cos\xi)\right]dk' \quad (34)$$

In order to find the solutions completely and following Loeffler and Deissler (1961), we assume that:

$$ik_i\left[\overline{\beta_i\theta'\theta''}(\hat{K},\hat{K}')-\overline{\beta_i\theta'\theta''}(-\hat{K},-\hat{K}')\right]_0 =-\frac{\delta_0}{(2\pi)^2}\left(k^2k'^4-k^4k'^2\right) \quad (35)$$

where, δ_0 constant depending on the initial condition. The negative sign is placed in front of δ_0 in order to make the transfer of energy from small to large wave no. for positive value of δ_0. Substituting Eq. (35) into (34):

$$\frac{\partial\overline{\tau\tau'}(\hat{K})}{\partial t}+\frac{2\nu}{P_r}\cdot 2\pi k^2\,\overline{\tau\tau'}(\hat{K})=-2\delta_0\int_0^{\infty}\left(k^2k'^4-k^4k'^2\right) \times\left[\int_{-1}^{1}\exp\left\{-\frac{\nu(t-t_0)}{P_r}\begin{bmatrix}(1+P_r)(k^2+k'^2)\\ +2P_r k k'\cos\xi\\ +\frac{P_r}{\upsilon}(2\varepsilon_{mij}\Omega_m-fs)\end{bmatrix}\right\}d(\cos\xi)\right]dk' \quad (36)$$

Multiplying both sides of Eq. (36) by k^2 and defining the spectral energy function:

$$E=2\pi k^2\,\overline{\tau\tau'}(\hat{K}) \quad (37)$$

And the resulting equation is:

$$\frac{\partial E}{\partial t}+\frac{2\nu}{P_r}k^2\,E=w \quad (38)$$

where,

$$w=-2\delta_0\int_0^{\infty}\left(k^2k'^4-k^4k'^2\right)k^2k'^2 \times\left[\int_{-1}^{1}\exp\left\{-\frac{\nu(t-t_0)}{P_r}\begin{bmatrix}(1+P_r)(k^2+k'^2)\\ +2P_r k k'\cos\xi\\ +\frac{P_r}{\upsilon}(2\varepsilon_{mij}\Omega_m-fs)\end{bmatrix}\right\}d(\cos\xi)\right]dk' \quad (39)$$

Integrating Eq. (39) w.r.t. ξ, we have:

$$w=-\frac{\delta_0}{2\upsilon(t-t_0)}\int_0^{\infty}\left(k^3k'^5-k^5k'^3\right) \times\left[\exp\left\{-\frac{\nu(t-t_0)}{P_r}\begin{bmatrix}(1+P_r)(k^2+k'^2)\\ -2P_r k k'+\frac{P_r}{\upsilon}(2\varepsilon_{mij}\Omega_m-fs)\end{bmatrix}\right\}\right] dk'+\frac{\delta_0}{\upsilon(t-t_0)}\int_0^{\infty}\left(k^3k'^5-k^5k'^3\right) \times\left[\exp\left\{-\frac{\nu(t-t_0)}{P_r}\begin{bmatrix}(1+P_r)(k^2+k'^2)\\ +2P_r k k'+\frac{P_r}{\upsilon}(2\varepsilon_{mij}\Omega_m-fs)\end{bmatrix}\right\}\right]dk' \quad (40)$$

Again integrating Eq. (40) w.r.t. k' we have:

$$w=-\frac{\delta_0\sqrt{\pi}\,P_r^{\frac{5}{2}}}{2\upsilon^{\frac{3}{2}}(t-t_0)^{\frac{3}{2}}(1+P_r)^{\frac{5}{2}}} \exp\left\{-\frac{P_r}{\upsilon}(2\varepsilon_{mij}\Omega_m-fs)(t-t_0)\right\} \times\exp\left\{\frac{-k\upsilon(1+2P_r)(t-t_0)}{P_r(1+P_r)}\right\} \times\left\{\frac{15P_r k^4}{4\upsilon^2(t-t_0)^2(1+P_r)}+\left[\frac{5P_r^2}{(1+P_r)^2}-\frac{3}{2}\right]\frac{k^6}{\upsilon(t-t_0)} +\left[\frac{P_r^3}{(1+P_r)^3}-\frac{P_r}{(1+P_r)}\right]k^8\right\} \quad (41)$$

The Eq. (38) indicates that w must begin as k^4 for small k. The condition of w is fulfilled by the Eq. (41). It can be shown, using Eq. (41) that:

$$\int_0^{\infty}w\,dk=0 \quad (42)$$

It was to be expected physically since w is a measure of the transfer of "energy" and the total energy transferred to all wave numbers must be zero.

The necessity for Eq. (41) to hold can be shown as follows if Eq. (10) is written for both $\hat{k}$ $and - \hat{k}$ and resulting equations differentiated with respect to r_i and added, the result is, for:

$$\hat{r} = 0 \quad \left(\frac{\partial}{\partial r_i} = -\frac{\partial}{\partial x_i} \right)$$

$$-2\frac{\partial}{\partial x_i}\overline{u_i TT} = \int_{-\infty}^{\infty} ik_i \left[\overline{\phi_i \tau\tau'(\hat{K})} - \overline{\phi_i \tau\tau'(-\hat{K})} \right] d\hat{K} \quad (43)$$

Since according to the Eq. (36), (38) and (14):

$$w \equiv 2\pi i k^2 k_i \left[\overline{\phi_i \tau\tau'(-\hat{K})} - \overline{\phi_i \tau\tau'(\hat{K})} \right]$$

So the Eq. (43) can be written as:

$$-2\frac{\partial}{\partial x_i}\overline{u_i \tau\tau'} = \int_{-\infty}^{\infty} \frac{w}{2\pi k^2} dk$$

as $d\hat{K} = 4\pi k^2 dk$ for $w = w(k, t)$ Then the Eq. (42) becomes:

$$\int_0^{\infty} w\, dk = -\frac{\partial}{\partial x_i}\overline{u_i \tau\tau'} = 0$$

The linear Eq. (38) can be solved for w as:

$$E = \exp\left[-\frac{2\upsilon k^2(t-t_0)}{P_r}\right]\int w \exp\left[\frac{2\upsilon k^2(t-t_0)}{P_r}\right]dt + J(k)\exp\left[-\frac{2\upsilon k^2(t-t_0)}{P_r}\right] \quad (44)$$

where, J (k) is an arbitrary function of k.

For large times, Corrsin (1951b) has shown the correct form of the expression for E to be:

$$E = \frac{N_0}{\pi} k^2 \exp\left[-\frac{2\upsilon k^2(t-t_0)}{P_r}\right] \quad (45)$$

where, N_0 is an constant which depends on the initial conditions. Using Eq. (45) to evaluate J (k) in Eq. (44) yields:

$$J(k) = \frac{N_0 K^2}{\pi} \quad (46)$$

Now, substituting the values of w and J (k) as given by the Eq. (41) and (46) into Eq. (44) gives the equation:

$$\begin{aligned} E(k,t) &= \frac{N_0 k^2}{\pi}\exp\left[-\frac{2\upsilon k^2(t-t_0)}{P_r}\right] + \frac{\delta_0\sqrt{\pi}\, P_r^{\frac{5}{2}}}{2\upsilon^{\frac{3}{2}}(1+P_r)^{\frac{7}{2}}} \\ &\exp\{-(2\varepsilon_{mij}\Omega_m - fs)(t-t_0)\} \times \exp\left\{\frac{-k^2\upsilon(1+2P_r)(t-t_0)}{P_r(1+P_r)}\right\} \\ &\times \left[\frac{3P_r k^4}{2\upsilon^2(t-t_0)^{\frac{5}{2}}} + \frac{P_r(7P_r-6)k^6}{3\upsilon(1+P_r)(t-t_0)^{\frac{3}{2}}} - \frac{4(3P_r^2-2P_r+3)k^8}{3(1+P_r)^2(t-t_0)^{\frac{1}{2}}} + \frac{8\sqrt{\upsilon}(3P_r^2-2P_r+3)k^9F(\eta)}{3(1+P_r)^{\frac{5}{2}}P_r^{\frac{1}{2}}}\right] \\ &+ \frac{\delta_0\sqrt{\pi}\, P_r^{\frac{5}{2}}}{4\upsilon^{\frac{3}{2}}(1+P_r)^{\frac{7}{2}}} \times \exp\{-(2\varepsilon_{mij}\Omega_m - fs)\} \times \\ &\exp\left\{\frac{-k^2\upsilon(1+2P_r)(t-t_0)}{P_r(1+P_r)}\right\} \\ &\times \left[\frac{3P_r k^4}{2\upsilon^2(t-t_0)^{\frac{5}{2}}} + \frac{P_r(7P_r-6)k^6}{3\upsilon(1+P_r)(t-t_0)^{\frac{3}{2}}} - \frac{4(3P_r^2-2P_r+3)k^8}{3(1+P_r)^2(t-t_0)^{\frac{1}{2}}} + \frac{8\sqrt{\upsilon}(3P_r^2-2P_r+3)k^9F(\eta)}{3(1+P_r)^{\frac{5}{2}}P_r^{\frac{1}{2}}}\right] \end{aligned} \quad (47)$$

where,

$$F(\eta) = e^{-\eta^2 \int_0^{\eta} e^{x^2} dx} \quad (48)$$

$$\eta = k\sqrt{\frac{\upsilon(t-t_0)}{P_r(1+P_r)}} \quad (49)$$

Putting $\hat{r} = 0$ in Eq. (9) and we use the definition of E given by the Eq. (47), the result is:

$$\frac{\overline{TT'}}{2} = \frac{\overline{T^2}}{2} = \int_0^{\infty} E(k)\, dk \quad (50)$$

Substituting Eq. (47) into (50) gives:

$$\begin{aligned} \frac{\overline{T}^2}{2} &= \frac{N_0 (P_r)^{\frac{3}{2}}}{8\sqrt{(2\pi)}\,\upsilon^{\frac{3}{2}}(t-t_0)^{\frac{3}{2}}} + \frac{\delta_0 R}{\upsilon^6(t-t_0)^5} \\ &\exp[-(2\varepsilon_{mij}\Omega_m - fs)] \\ &\Rightarrow \overline{T}^2 = A(t-t_0)^{-\frac{3}{2}} + B\exp[-(2\varepsilon_{mij}\Omega_m - fs)] \times (t-t_0)^{-5} \end{aligned} \quad (51)$$

where,

$$A = \frac{N_0 (P_r)^{\frac{3}{2}}}{4\sqrt{(2\pi)}\,\upsilon^{\frac{3}{2}}}, \qquad B = \frac{2\delta_0 R}{\upsilon^6}$$

and

$$R=\frac{\pi\left(\mathrm{P}_r\right)^6}{2\left(1+\mathrm{P}_r\right)\left(1+2\mathrm{P}_r\right)^{\frac{5}{2}}} \tag{52}$$

$$\left\{\frac{9}{16}+\frac{5\mathrm{P}_r\left(7\mathrm{P}_r-6\right)}{16\left(1+2\mathrm{P}_r\right)}-\frac{35\ \mathrm{P}_r\left(3\mathrm{P}_r^2-2\mathrm{P}_r+3\right)}{8\left(1+2\mathrm{P}_r\right)^2}\right.$$
$$\left.+\frac{1.5422\ \mathrm{P}_r\left(3\mathrm{P}_r^2-2\mathrm{P}_r+3\right)\left(1+2\mathrm{P}_r\right)^{\frac{5}{2}}}{\sqrt{(\pi)}\left(1+\mathrm{P}_r\right)^{\frac{11}{2}}}\left[1+\sum_{n=1}^{\infty}\frac{(11)\cdots\cdots[11+2(n-1)]}{(2n+1)n!\,(2)^{2n}\left(1+\mathrm{P}_r\right)^n}\right]\right\}$$

R is a function of Prandtl no.

Equation (51) is the decay law of temperature fluctuation in homogeneous dusty fluid turbulence prior to the ultimate period in presence of coriolis force. The first term of the right side of Eq. (51) corresponds to the temperature energy for two- point correlation and the second terms represents the energy for the three-point correlation. This second term becomes negligible at large times leaving the final period decay law previously found by Corrsin (1951b). $\bar{T}^2$ Is the total "energy" (the mean square of the temperature fluctuations).

RESULTS AND DISCUSSION

Equation (51) is the decay law of temperature fluctuation in homogenous dusty fluid turbulence before the final period in presence of coriolis force. In the absence of the dust particle and coriolis force, i.e., $f = 0$ then the Eq. (51) becomes:

$$\frac{\overline{T}^2}{2}=\frac{N_0\left(\mathrm{P}_r\right)^{\frac{3}{2}}}{8\sqrt{(2\pi)}\,\upsilon^{\frac{3}{2}}\left(t-t_0\right)^{\frac{3}{2}}}+\frac{\delta_0 R}{\upsilon^6\left(t-t_0\right)^5}\exp\left[-2\varepsilon_{mij}\Omega_m\right] \tag{53}$$

$$\Rightarrow \overline{T}^2=A\left(t-t_0\right)^{-\frac{3}{2}}+B\exp\left[-2\varepsilon_{mij}\Omega_m\right]\times\left(t-t_0\right)^{-5}$$

which was obtained earlier by Molla *et al*. (2012). In this study, they had studied the decay of temperature fluctuation in homogeneous turbulenc before the final period in a Rotating System. They considered two-and three-point correlations and neglecting fourth-and higher-order correlation terms compared to the second- and third-order correlation terms and derived the above equation.

If $\Omega_m = 0$, then the Eq. (51) becomes:

$$\frac{\overline{T}^2}{2}=\frac{N_0\left(\mathrm{P}_r\right)^{\frac{3}{2}}}{8\sqrt{(2\pi)}\,\upsilon^{\frac{3}{2}}\left(t-t_0\right)^{\frac{3}{2}}}+\frac{\delta_0 R}{\upsilon^6\left(t-t_0\right)^5}\exp[fs] \tag{54}$$

$$=A\left(t-t_0\right)^{-\frac{3}{2}}+B\exp[fs]\times\left(t-t_0\right)^{-5}$$

which was obtained earlier by Azad and Mumtahinah (2013).

In the absence of the dust particle and the coriolis force i.e., $f = 0$ and $\Omega_m = 0$, the Eq. (51) becomes:

$$\frac{\overline{T^2}}{2}=\frac{N_0\left(\mathrm{P}_r\right)^{\frac{3}{2}}\upsilon^{-\frac{3}{2}}}{8\sqrt{(2\pi)}\left(t-t_0\right)^{\frac{3}{2}}}+\frac{\delta_0 R}{\upsilon^6\left(t-t_0\right)^5} \tag{55}$$

$$=A\left(t-t_0\right)^{-\frac{3}{2}}+B\left(t-t_0\right)^{-5}$$

which was obtained earlier by Loeffler and Deissler (1961). Here,

$$A=\frac{N_0\left(\mathrm{P}_r\right)^{\frac{3}{2}}\upsilon^{-\frac{3}{2}}}{8\sqrt{(2\pi)}}\quad and \quad B=\frac{\delta_0 R}{\upsilon^6}$$

Due to the effect of coriolis force in homogeneous dusty fluid turbulence, the temperature energy fluctuations decays more rapidly than the energy for non rotating clean fluid prior to the ultimate period. For large times, the second term in the Eq. (51) becomes negligible leaving the -3/2 power decay law for the ultimate period.

In their study, they considered two- and three-point correlations and neglecting fourth- and higher-order correlation terms compared to the second- and third-order correlation terms.

In the present study, I have studied the decay of temperature fluctuations in homogeneous turbulence prior to the final period taking dust particle and coriolis force considering the correlations between fluctuating quantities at two- and three-point and single time. In this study, we have used Deissler (1958) method to solving the problem. Through the study we have obtained the Eq. (51) for energy decay law of temperature fluctuations in homogeneous dusty fluid turbulence prior to the final period in a rotating system. In this result, it is shown that the energy decays more rapidly than clean fluid and non rotating system.

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REFERENCES

Azad, M.A.K. and M.S.A. Sarker, 2006. Decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time in presence of dust particle. Rajshahi Univ., Stud. Part-B. J. Sci., 34: 37-50.

Azad, M.A.K. and M.S.A. Sarker, 2008. Decay of temperature fluctuations in homogeneous turbulence before the final period for the case of multi-point and multi-time in a rotating system in presence of dust particle. J. Appl. Sci. Res., 4(7): 793- 802.

Azad, M.A.K. and M.S.A. Sarker, 2009. Decay of temperature fluctuations in MHD turbulence before the final period in a rotating system. Bangladesh J. Sci. Ind. Res., 44(4): 407-414.

Azad, M.A.K. and Mst. Mumtahinah, 2013. Decay of temperature fluctuations in dusty fluid homogeneous turbulence prior to final period. Res. J. Appl. Sci. Eng. Tech., 6(8): 1490-1496.

Azad, M.A.K., M.S.A. Sarker and N.I. Mondal, 2006. Decay of temperature fluctuations in dusty fluid MHD turbulence before the final period in a rotating system. J. Eng. Appl. Sci., 1(2): 187-194.

Azad, M.A.K., M.S.A. Sarker and M.A. Aziz, 2007-2008. Decay of temperature fluctuations in dusty fluid MHD turbulence before the final period. Math. Forum, 20: 32-48.

Azad, M.A.K., M.H.U. Molla and M.Z. Rahman, 2012. Transport equatoin for the joint distribution function of velocity, temperature and concentration in convective tubulent flow in presence of dust particles. Res. J. Appl. Sci. Eng. Tech., 4(20): 4150-4159.

Bkar Pk, M.A., M.A.K. Azad and M.S. Alam Sarker, 2012. First-order reactant in homogeneou dusty fluid turbulence prior to the ultimate phase of decay for four-point correlation in a rotatingsystem. Res. J. Math. Stat., 4(2): 30-38.

Corrsin, S., 1951b. On the spectrum of isotropic temperature fluctuations in isotropic turbulence. J. Appl. Phys., 22: 469- 473.

Deissler, R.G., 1958. On the decay of homogeneous turbulence before the final period. Phys. Fluids, 1: 111-121.

Deissler, R.G., 1960. A theory of decaying homogenous turbulence. Phys. Fluids, 3: 176-187.

Dixit, T. and B.N. Upadhyay, 1989a. The effect of coriolis force on acceleration covariance in MHD turbulent dusty flow with rotational symmetry. Astrophys. Space Sci., 153: 257-268.

Kishore, N. and A. Sinha, 1988. Rate of change of vorticity covariance in dusty fluid turbulence. Astrophys. Space Sci., 146(1): 53-58.

Kishore, N. and S.R. Singh, 1984. Statistical theory of decay process of homogeneous hydromagnetic turbulence. Astrophys. Space Sci., 104: 121-125.

Kishore, N. and Y.T. Golsefied, 1988. Effect of coriolis force on acceleration covariance in MHD turbulent flow of a dusty incompressible fluid. Astrophys. Space Sci., 150(1): 89-101.

Kishore, N. and M.S.A. Sarker, 1990. Rate of change of vorticity covariance in MHD turbulent flow of dusty incompressible fluid. Int. J. Energ. Res., 14(5): 573-577.

Loeffler, A.L. and R.G. Deissler, 1961. Decay of temperature fluctuations in homogeneous turbulence before the final period. Int. J. Heat Mass Tran., 1: 312-324.

Molla, M.H.U., M.A.K. Azad and M.Z. Rahman, 2012. Decay of temperature fluctuations in homogeneous turbulenc before the final period in a rotating system. Res. J. Math. Stat., 4(2): 45-51.

Rahman, M.L., 2010. Rate of vorticity covariance in MHD turbulent flow of dusty fluid in a rotating system. J. Mech. Cont. Math. Sci., 5(1): 570-578.

Saffman, P.G., 1962. On the stability of laminar flow of a dusty gas. J. Fluid Mech., 13: 120-128.

Sarker, M.S.A. and M.A.K. Azad, 2006. Decay of temperature fluctuations in homogeneous turbulence before the final period for the case of Multi-point and Multi-time in a rotating system. Bangladesh J. Sci. Ind. Res., 41(3-4): 147-158.

Shimomura, Y. and Y. Yoshizawa, 1986. Statistical analysis of an isotropic turbulent viscosity in a rotating system. J. Phys. Soc. Jpn., 55: 1904-1917.

Sinha, A., 1988. Effect of dust particles on the acceleration of ordinary turbulence. J. Sci. Res., 38: 7.

Taylor, G.I., 1935. Statistical theory of turbulence. I-IV, Proc. Roy. Soc. London, A151(874): 421-478.