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Research Article

Mixed-integer Formulation for Integration of Lot Sizing and Lot Streaming Problem with Scheduled Preventive Maintenance

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Abstract: In this study, a mathematical model for the integration of lot sizing and flow shop scheduling with lot streaming was proposed. A mixed-integer linear model for multi-product lot sizing and lot streaming problem was developed. Mixed-integer programming formulation is proposed which will enable user to identify optimal production quantities, inventory levels, sub lot sizes and sequences simultaneously. Two situations where considered: 1) all machines are available and 2) all machines need preventive maintenance tasks. For both situations a new mixed-integer formulation is developed. To demonstrate the practicality of the proposed model, numerical example was used. It is shown that, the best makespan can be achieved through consistent sublots with intermingling cases as compared to non-intermingling cases.

Keywords: Flow shop scheduling, lot sizing, lot streaming, mixed integer programming, preventive maintenance

INTRODUCTION

In the manufacturing industries, the used planning and scheduling decision-making strategy usually follows a hierarchical approach, in which the planning problem is solved first to determine the production plans and the scheduling problem is solved next to meet these plans. However, this traditional strategy presents a major defect, since there is no interaction between the two decision levels; the planning decisions generated might cause infeasible scheduling sub-problems (Li and Ierapetritou, 2010). Since the production planning model ignores detailed scheduling constraints, there is no guarantee that a feasible production schedule exists for the generated production plan (Li and Ierapetritou, 2010). Therefore, considering production planning and scheduling in an integrated model is recommendable to prevent an infeasible solution. We summarize some works of research regarding integration of lot sizing and flow shop scheduling as follows. Yan and Zhang (2007) developed an integrated model for production planning and scheduling in a three-stage manufacturing system. Yan *et al*. (2003) developed an integrated production planning and scheduling model for automobile assembly lines. They solved first production planning model and then solved the scheduling problem by dispatching rules (earliest due date and smallest lots); if they are infeasible solutions, the neighbor plan and neighbor schedule definition will be used to find feasible initial solutions (Yan *et al*., 2003). Palaniappan and Jawahar (2011) proposed a model for simultaneous optimization of lot sizing and scheduling in a flow line assembly. The objective of their model was the minimization of total cost, which includes assembly, procurement, switchover and inventory and order backlog costs. However, none of the above scholars considered the lot streaming problem within their models.

Lot streaming is a technique for splitting jobs, each consisting of identical items, into sublots to allow their overlapping on consecutive machines in multi-stage production systems (Chang and Chiu, 2005). Through lot streaming, production can be accelerated and a significant decrease of makespan and improved timeliness are with inreach (Kalir and Sarin, 2000; Zhang *et al*., 2005; Sarin and Jaiprakash, 2007; Feldmann and Biskup, 2008). The sublots of a lot are assumed to take real-valued and integer-valued sizes. Integersublot sizes are more appropriate for the manufacturing facilities (Kalir and Sarin, 2003). Two types of problem exist for multi-product lot streaming problems:

- Intermingled sublots
- Non-intermingled sublots

In intermingled sublots, the sequence of sublots of job j can be interrupted by the sublots of job v. Otherwise, it is non-intermingled sublots. In the following section we summarize research on lot streaming problems and focus on the flow shop environment.

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Biskup and Feldmann (2005) presented the first integer programming formulation for the single product lot streaming problem with variable sublots. Trietsch and Baker (1993) developed a linear formulation for a single-product lot streaming problem with consistent sublots. Chiu *et al*. (2004) and Chang and Chiu (2005) found that, under the same sublot type, although growing number of sub lots will diminish makespan, the marginal decrease in makespan will reduce with the increase of the number of sublots. Glass and Possani (2011) indicated that, for products with identical number of sublots and processing times, no advantage can be achieved by using variable sublot in these successive products. Feldmann and Biskup (2008) developed a mathematical model for multi-product lot streaming problem. They revealed that, the benefit of lot streaming grows not only with the number of sub lots but also with an increasing number of stages. However, all lot streaming research assumes that the number of identical items of the product on each machine is given in advance. In other words, the a lot sizing problem is not integrated into lot streaming problem. Also they assumed that machines are always available, in the other words; any breakdowns and scheduled maintenance are not allowed which it is not realistic. Ramezanian *et al*. (2013) presented a mixed-integer programming model for lot sizing and scheduling problems with availability constraints; however, they did not consider lot streaming within their model. In this research, we will develop a mixed-integer linear mathematical model for the integration of lot sizing and scheduling with lot streaming problems where machines are unavailable due to the performance of preventive maintenance tasks.

INTEGRATED MODEL FOR LOT SIZING AND SCHEDULING WITH LOT STREAMING

With the following model, the four goals of the problem, determining the size of the each lot, the sequence among the sub lots, inventory levels and the size of the individual sublots, are solved simultaneously. The model assumptions are as follows:

- The machine configuration considered constitutes a flow shop.
- Break downs and preventive maintenance are allowed (machines can be unavailable).
- Completion of preventive maintenance happens in predefined time window.
- Set up times are negligible or include processing times.
- Back logging is not allowed.
- There is an external demand for finished products which processed by last machine.
- All machines have capacity restrictions.
- Planning horizon is a single period (i.e., a day).
- All parameters are deterministic.
- Intermittent idling is allowed.
- Consistent sublots type is considered.
- Before doing lot streaming, the number of sublots for all lots is known.

This problem with above-mentioned assumptions can be formulated as follows:

Indices and notions:

N : The number of jobs
M : The number of non-identical machines
S : The number of sub lots
l : Index of maintenance tasks
j, v: Indices for jobs j, v = 1, 2, .., N
k : k′th machine k = 1, 2, .., M
s, z: Indices for sublotss, z = 1, 2, .., S

Decision variables:

x_{sjzv} : Binary variable, which takes the value 1 if sublot s of product j is sequenced prior to sublot z of product v, 0 otherwise
Y_{sjk} : A binary variable that is equal 1 if sublot s of job j processed before maintenance task L when processing on machine k and 0 otherwise
c_{sjk} : Completion time of sublot s of product j on machine k
ST_{sjk} : Starting time of sublot s of product j on machine k
cm_{lk} : Completion time of l^{th} preventive maintenance task on machine k
p_{jk} : Quantity of product j produced in machine k
s_{jk} : Stock of product j after operation in machine k
c_{max} : Maximum completion time on machine M (makespan)
u_{sjk} : Sublot size of sthsublot of product j on machine k

Parameters and constants:

bi_M : Beginning inventory of product on machine M
cp_{jk} : Production cost of product j in machine k
h_{jk} : Holding cost of product j
D : Used to convert the makes pan in to a costs (cost per unit time)
Ac_k : Available capacity of machine k (measured in time units)
d_j : External demand for product j at the end of period (a day)
pt_{jk} : Processing time for one unit of product j on machine k
L_k : Number of different preventive maintenance task on machine k (set of maintenance tasks)
t_{lk} : Duration of l^{th} preventive maintenance task on machine k
Em_{lk} : The early completion time of l^{th} preventive maintenance task on machine k
Lm_{lk} : The late completion time of l^{th} preventive maintenance task on machine k

R : Large number

$$Z=\sum_{j=1}^{N}\sum_{k=1}^{M}cp_{jk}\ p_{jk}+\sum_{j=1}^{N}\sum_{k=1}^{M}h_{jk}\ s_{jk}+D\times c_{\max} \quad (1)$$

$$d_j = p_{jM} - s_{jM} + bi_M \qquad j=1,..,N \quad (2)$$

$$\sum_{j=1}^{N} pt_{jk} p_{jk} \le Ac_k \qquad k=1,...,M \quad (3)$$

$$c_{\max} \ge c_{sjM} \qquad s=1,...,S, j=1,...,N \quad (4)$$

$$\sum_{s=1}^{S} u_{sjk} = p_{jk} \qquad j=1,..,N,\ k=1,...,M \quad (5)$$

$$u_{sjk} = u_{sj(k+1)} \quad s=1,...,S, j=1,...,N, k=1,...,M-1 \quad (6)$$

$$c_{sjk} - pt_{jk} \times u_{sjk} \ge c_{sj(k-1)}$$
$$s=1,...,S, j=1,...,N, k=2,....,M \quad (7)$$

$$c_{sjk} - pt_{jk} \times u_{sjk} \ge c_{(s-1)jk}$$
$$s=2,...,S, j=1,...,N, k=1,....,M \quad (8)$$

$$c_{zvk} - pt_{vk} \times u_{vzk} - c_{sjk} + R(1-x_{sjzv}) \ge 0 \ j, v=1,...,$$
$$N\ s, z=1,...,S\ if\ v=j, then\ z \ne s,\ k=1,...,M \quad (9)$$

$$c_{sjk} - pt_{jk} \times u_{sjk} - c_{zvk} + R \times x_{sjzv} \ge 0 \ j, v=1,...,$$
$$N\ s, z=1,...,S\ if\ v=j, then\ z \ne s,\ k=1,...,M \quad (10)$$

$$c_{sj1} - pt_{j1} \times u_{sj1} \ge 0 \quad s=1,...,S,\ j=1,...,N \quad (11)$$

$$ST_{sjk} \ge c_{sj(k-1)} \quad j=1,...,N, s=1,...,S, k=2,...,M \quad (12)$$

$$c_{sjk} = ST_{sjk} + pt_{jk} \times u_{sjk}$$
$$j=1,...,N, s=1,...,S, k=1,...,M \quad (13)$$

$p_{jk}, s_{jk}, c_{sjk}, u_{sjk} \ge 0, and\ u_{sjk}\ are$ Integer, x_{sjzv} are binary, $s=1,..,S,\ j=1,...,N,\ k=1,...,M$ (14)

The objective function (expression (1)) minimizes the sum of production costs, holding costs and makespan costs. Constraints (2) ensure that demand for each product is equal to production quantity plus beginning inventory minus ending inventory. Constraints (3) make sure that the production time of each machine does not overpass its available capacity. In (4) the maximum of completion time of sublots on the last machine is used to define the makespan (c_{max}). Restrictions (5) make sure that in sum p_{jk}items are produced of product j on machine k. Constraints (6) make sure that sublots type is consistent. Constraints (7) and (8) ensure that the sublots of the same products do not overlap. Since sublots are allowed to intermingle, constraints (9) and (10) specify the sequence of sublots. Restrictions (11) ensure that any sublot s of any job j begins processing on machine 1 after time zero. Constraints (12) ensure that start the processing of sublot s of product j only when it has been completed on the precedent machine. Constraints (13) are related to the calculation of completion time sublots. In reality machines maybe stopped for many reasons, such as maintenances. By adding Eq. (15) to (18), the proposed model is adapted to these situations:

$$ST_{sjk} - cm_{lk} + R \times Y_{sjk} \ge 0$$
$$j=1,...,N, s=1,...,S, k=1,...,M,\ l=1,...,l_k \quad (15)$$

$$cm_{lk} - t_{lk} - c_{sjk} + R \times (1 - Y_{sjk})$$
$$\ge 0,\ j=1,...,N, s=1,...,S, k=1,...,M, l=1,...,l_k \quad (16)$$

$$Em_{lk} \le cm_{lk} \le lm_{lk} \qquad l=1,...,l_k, k=1,...,M \quad (17)$$

$$cm_{lk} \ge 0\ and\ Y_{sjk}\ are\ binary \quad (18)$$

No intermingling between the sub lots: A fast way for non-intermingling setting is to use the model Eq. (1) to (18) and equate the binary variables for the sublots of the products which are not allowed to intermingle (Feldmann and Biskup, 2008). If all products are not allowed to intermingle, for a three-product example, this would be:

$x_{sj11} = x_{sj21} = x_{sj31}$ s = 1, …, S, j = 2, 3 for first product (19)

$x_{sj12} = x_{sj22} = x_{sj32}$ s = 1,…, S, j = 1, 3 for second product (20)

$x_{sj13} = x_{sj23} = x_{sj33}$ s = 1, …, S, j = 1, 2 for third product (21)

NUMERICAL EXAMPLE

In order to evaluate this model's performance, we use the model to test the following randomly generated problem: we have three types of products being processed on four machines. The number of sublots per product is three. Demands are 20, 20 and 15 for products 1, 2 and 3, respectively. Production costs are 10, 15 and 12 for products 1, 2 and 3, respectively. Holding costs are 3, 4, and 3 for products 1, 2 and 3,

Table 1: Processing times of jobs on machines

	Machine number			
Product	1	2	3	4
1	2	1	2	2
2	2	4	1	1
3	4	2	2	3

Table 2: Input data on different machines

	Machine 1	Machine 2	Machine 3	Machine 4
Duration of maintenance tasks	20	30	40	40
Early completion time of maintenance tasks	50	110	100	120
Late completion time of maintenance tasks	80	120	120	140

Table 3: Sublot completion times on different machines in consistent sublot with intermingling setting

Machine number	C_{11}	C_{21}	C_{31}	C_{12}	C_{22}	C_{32}	C_{13}	C_{23}	C_{33}
1	10	68	110	26	82	140	50	98	130
2	15	82	126	58	110	160	70	118	140
3	25	100	138	66	117	165	82	126	150
4	35	119	150	74	126	170	100	138	165

Table 4: Sublot completion times on different machines in consistent sublot with intermingling setting and maintenance tasks

Machine number	C_{11}	C_{21}	C_{31}	C_{12}	C_{22}	C_{32}	C_{13}	C_{23}	C_{33}
1	14	40	100	26	56	161	88	124	148
2	26	57	131	50	89	185	125	143	160
3	50	71	143	56	125	191	131	155	172
4	64	85	155	70	133	197	143	173	191

Table 5: Results of lot streaming problems with and without maintenance tasks

Classification	Optimal sequence	Makespan	Total cost	Comparison of total cost (%)	Comparison of makespan (%)
Consistent sub lots with intermingling and without maintenance tasks	11-12-13-21-22-23-31-33-32	170	3570	-	-
Consistent sub lots without intermingling and maintenance tasks	2-3-1	189	3665	2/66	11
Consistent sub lots with intermingling and maintenance tasks	11-12-21-22-13-31-23-33-32	197	3705	3/8	16
Consistent sub lots without intermingling and with maintenance tasks	2-3-1	217	3805	6/5	27

respectively. The maximum available capacity of machines is 400 time units for machines 1 to 4. The beginning inventory is zero. Cost per unit time (D) is equal to 5. Table 1 summarizes the processing times of products on machines. Other input data is summarized in Table 2. We consider two situations:

- None of machines needs maintenance.
- All machines need preventive maintenance.

The example has been solved using LINGO 12.0, on a laptop computer with Intel core i5-2410 m processor 2.3 GHz with 4 GB of RAM.

RESULTS OF THE PROBLEM

The resulting formulation has a total of 169 variables and 691 constraints for consistent sublots with intermingling case where all machines are available. The solution was achieved after running the solver for 146 sec. The results of the consistent sublots with intermingling case are as follows. Total cost is 3570 and makespan is equal to 170. Sublot sizes are as follows:

$u_{111} = u_{112} = u_{113} = u_{114} = 5$, $u_{211} = u_{212} = u_{213} = u_{214} = 9$, $u_{311} = u_{312} = u_{313} = u_{314} = 6$, $u_{121} = u_{122} = u_{123} = u_{124} = 8$, $u_{221} = u_{222} = u_{223} = u_{224} = 7$, $u_{321} = u_{322} = u_{323} = u_{324} = 5$, $u_{131} = u_{132} = u_{133} = u_{134} = 6$, $u_{231} = u_{232} = u_{233} = u_{234} = 4$ and $u_{331} = u_{332} = u_{333} = u_{334} = 5$

Product quantities are:

$p_{11} = p_{12} = p_{13} = p_{14} = p_{21} = p_{22} = p_{23} = p_{24} = 20$ and $p_{31} = p_{32} = p_{33} = p_{34} = 15$, with all inventory level or $S_{jk} = 0$

Table 3 summarizes the completion times of each sublot. Figure 1 demonstrated the Gantt chart of this problem. The makespan is equal to total processing time on the last machine plus total idle time on the last machine (Trietsch and Baker, 1993). For this example, in consistent sublots with intermingling setting, as demonstrated in Fig. 1, makespan is equal to total idle time on machine number four which is 65 min plus total processing time on machine number four or 105 min, which will be equal to 170 min.

The resulting formulation includes a total of 245 variables and 834 constraints for consistent sublots with intermingling case where all machines are unavailable. The solution was achieved after running the solver for 85 min. Table 4 summarizes the completion times of each sublot where all machines are unavailable due to performing maintenance tasks. Sublot sizes are as follows:

$u_{111} = u_{112} = u_{113} = u_{114} = 7$, $u_{211} = u_{212} = u_{213} = u_{214} = 7$, $u_{311} = u_{312} = u_{313} = u_{314} = 6$, $u_{121} = u_{122} = u_{123} = u_{124} = 6$, $u_{221} = u_{222} = u_{223} = u_{224} = 8$, $u_{321} = u_{322} = u_{323} = u_{324} = 6$, $u_{131} = u_{132} = u_{133} = u_{134} = 3$, $u_{231} = u_{232} = u_{233} = u_{234} = 6$ and $u_{331} = u_{332} = u_{333} = u_{334} = 6$

Product quantities are:

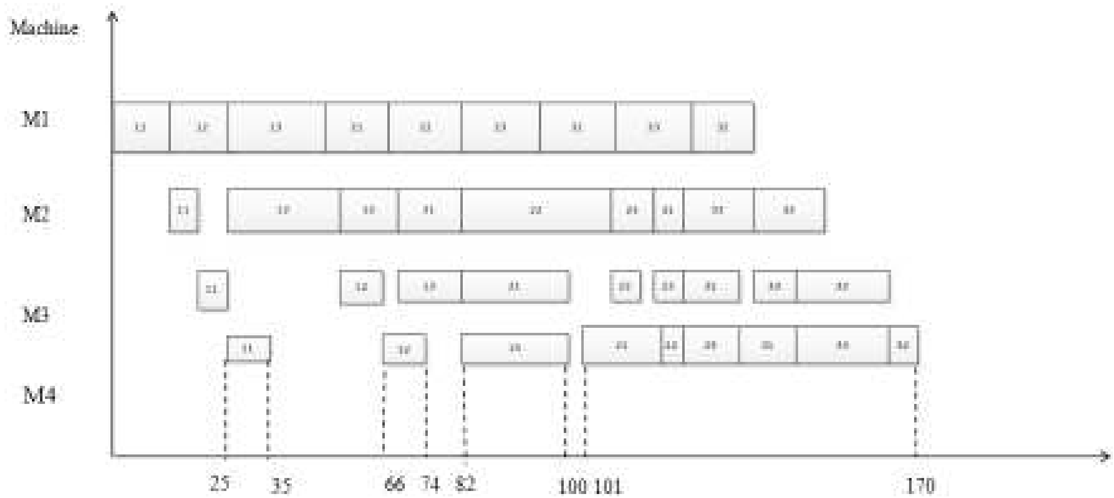

Fig. 1: Optimal solutions of example with intermingling integer consistent sublots

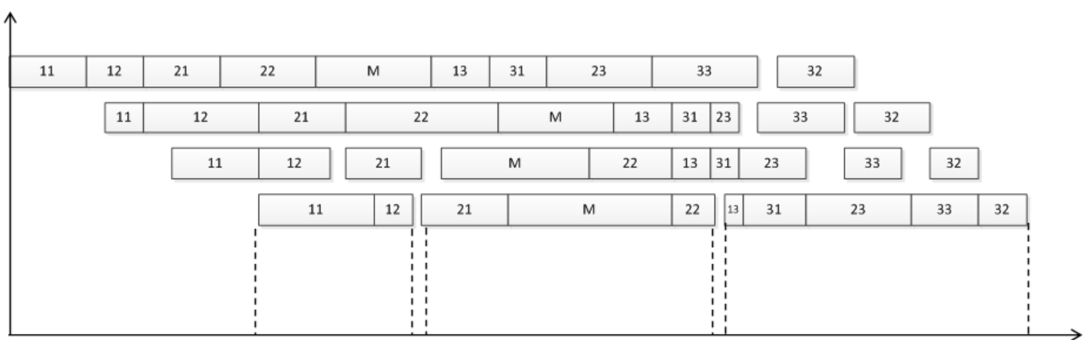

Fig. 2: Optimal solutions of example with intermingling integer consistent sublots where all machines need preventive maintenance

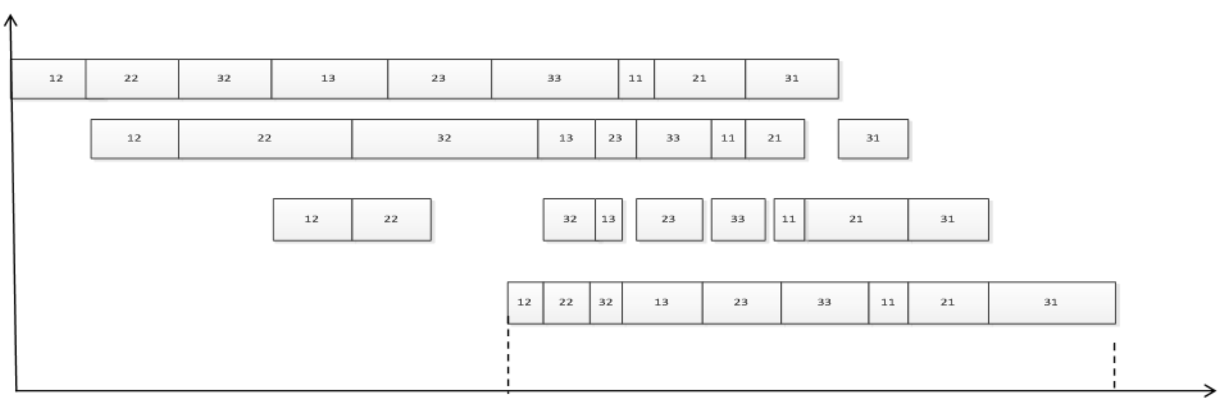

Fig. 3: Optimal solutions of example without intermingling integer consistent sublots

$p_{11} = p_{12} = p_{13} = p_{14} = p_{21} = p_{22} = p_{23} = p_{24} = 20$ and $p_{31} = p_{32} = p_{33} = p_{34} = 15$, with all inventory level or $S_{jk} = 0$

Completion times of maintenance tasks on machines 1 to 4 are 76, 119, 117 and 125 min, respectively. Figure 2 demonstrated the Gantt chart of this problem.

Table 5 summarized the results of lot streaming problems with and without maintenance tasks. Columns 5 and 6 of Table 5 are achieved with the following formulations. Z_{best} and best makespan are 3570 and 170, which belong to consistent sublots with intermingling case when machines do not need preventive maintenance. Comparison of total cost $Z = (z - z_{best}) / z_{best} \times 100$ and comparison of $c_{max} = c_{max} - c_{max_{best}} / c_{max_{best}} \times 100$ For instance, the makespan of consistent sublots with intermingling case is 11% better than the makespan of consistent sublots without intermingling case when all machines are available.

Figure 1 to 3 are Gantt charts of these problems. Optimal sequence for schedule without lot streaming is 1-2-3 and the makespan is 245 (achieved through SPT rule (Johnson, 1954)). The percentage of makespan decrease due to lot streaming in permutation flowshop is 30% as compared to the best makespan.

CONCLUSION

In this research, the first mathematical model for the integration of lot sizing and flow shop scheduling with lot streaming was developed. A mixed-integer linear model for multiple products lot sizing and lot streaming problem was proposed. Mixed-integer programming formulation was proposed which enabled user to find optimal production quantities, inventory levels, sublot sizes and sequences simultaneously. Two situations were considered:

- All machines were available.
- All machines needed preventive maintenance tasks.

For both situations a mixed-integer formulation was developed. Anumerical example was used to demonstrate the practicality of the proposed model. It was shown that, the best makespan can be achieved through the case of consistent sublots with intermingling. Since increase in the number of binary and integer variables generally make lot streaming problems difficult to solve, the use of meta-heuristic methods to handle large-scale problems deserves further research. The proposed model is adapted to consistent sublot type. Extension of this model for variable sublot types could be a topic for further study.

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