

Research Article

Novel Sub-carriers Iterative Restructuring Method in OFDM: Toward Extra Equalization Omitted

Zhao Wang Xing and Zhang Zhong Pei

National Key Laboratory of Science and Technology on Communication, University of Electronics Science and Technology of China, Chengdu, China

Abstract: This study considered a receiver equalization problem in wireless communication area. Receiver equalizations in OFDM proceeding through sub-carriers variedness in frequency guarantee transmit system bit error rate thus are essential. E.g., Zero-Forcing (ZF) equalization, Minimal Mean Square Equalization (MMSE), which can also be considered as channel Estimation (EE) and detection schemes. However, some methods like adaptive equalizations must count priority transceiver information thus may be not practical although they raise performances for receiver. This study considers equalization and detection problems through a sub-carriers restructuring method to omit any complicated extra equalization in receiver except ZF equalization following the elemental LS channel estimation. Optimal constraints model is utilized which leads to an optimal sub-carriers derivation, moreover, we iteratively set up constraints model and give derivation, then under our new sub-carriers no extra equalization is needed. Simulations verify the novel method can approach MMSE equalization although it may bring transceiver complexity.

Keywords: Channel estimation, extra equalization, OFDM, optimal sub-carriers, restructuring

INTRODUCTION

In OFDM system, receiver equalization technical has guaranteed the compensation of bit error (Baskaran *et al.*, 2007; Lee *et al.*, 2012; Eghbali *et al.*, 2011; Bui and Hatzinakos, 2005). The common investigated equalizers are mainly based on channel estimation, such as Zero Forcing (ZF), MMSE channel equalizer. With respect to transform domain, equalizers can be divided into time domain or frequency domain, among them, time domain equalizers usually utilize windowing methods to mitigate Inter-Symbol Interference (ISI) (Dukhyun and Stuber, 1998). But due to the fact that OFDM adopts multi-points IFFT to modulate and may encounter multi-paths fading channel in time domain, investigations of OFDM equalization are mainly proceeded in frequency domain (Song *et al.*, 2013; Feng *et al.*, 2013; Lun *et al.*, 2009). As is this study studied that is based on frequency sub-carriers equalizers (). By contrast to only once channel estimation equalization, if it is easy to access transceiver priority information, we can do equalization twice for receiver, which compensate the bit error a second time in receiver. As shown in Fig. 1 (Zhao, 2012), equalization adopts CP-based LS channel estimation for first equalization and extra equalization is made by the help of the transceiver priority information observed.

Zhao (2012) has proposed three matrix-based extra equalization schemes, named as LS matrix equalization, modified magnitude equalization and MMSE matrix equalization and compared the bit error performance to those without extra equalization. However, in the real world communication system, priority from transceiver must increase extra cost and ill the transmit efficiency thus must be emphasized on.

In the work of paper Zhao (2012), the author has systematically studied the LS channel estimation and deduced that its Minimal Square Error (MSE) of estimation was positively related to the energy of pilots selected and negatively related to the power of an OFDM transmit symbol with Cyclic-Prefix (CP). Therefore based on this mechanism, paper (Zhao, 2012) has generalized the pilot selections for channel estimation and proposed CP pilots, together with which the author had deeply studied the optimized sub-carrier comb-pilots in channel estimation. But in actual circumstance, the CP in receiver is often given up, as the time domain information of CP must be added transmit noise, therefore CP-based LS channel estimation can not accurately extract the channel information except under the condition when the connected two or more time OFDM symbols are mutually studied for estimation (Roque *et al.*, 2012), thus this method can only stay in theoretical design.

Corresponding Author: Zhao Wang Xing, National Key Laboratory of Science and Technology on Communication, University of Electronics Science and Technology of China, Chengdu, China

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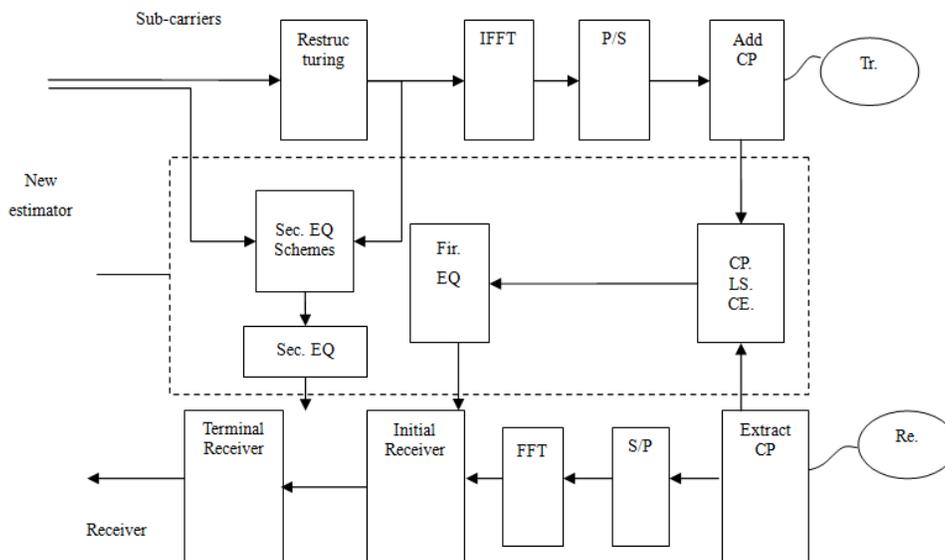


Fig. 1: Twice receiver equalization contained OFDM system model
 CP.LS.CE.: Cyclic-prefix least square channel estimation; Fir. EQ: The first equalization for receiver; Sec. EQ: The twice equalization for receiver

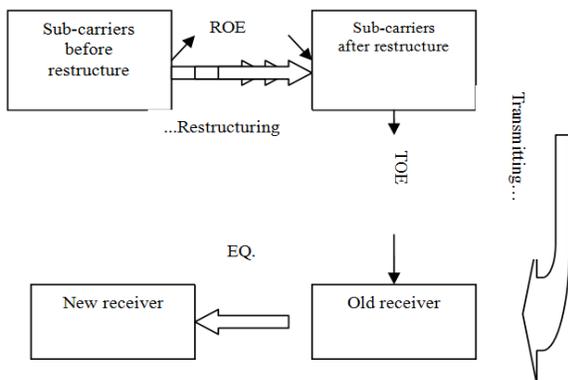


Fig. 2: Simplified OFDM system

Although it is novel work, but the estimator in paper (Zhao, 2012) is also a theoretical study.

Under LS sub-carriers pilot channel estimation, paper (Zhao *et al.*, 2013) has entirely studied optimized pilots, which are subject to equally-spaced discrete distribution, which are known as comb pilots. Under slow fading or frequency selective channel (Jingxian *et al.*, 2005; Gacanin *et al.*, 2013) this pilot type has the largest power in an OFDM symbol. When the channel is dynamic, according to power alteration, the sub-carriers changed, pilots may be staggered distribution (Zhao *et al.*, 2013).

While in this study, we designed an iterative restructuring method of transceiver sub-carriers after constraints model, under different restructuring, comb pilots are also variedly selected. We found for comb-pilots the position of first pilot selection may impact the group comb-pilots energy. In words, under comb-pilots the initial position of a group comb-pilot is of vital importance. Thus in this study, specifically in the first

time of restructuring, we selected the comb pilots whose power were the largest by ensuring the first pilot, the remainder were non-pilots data sub-carriers. While in the following restructurings, under new sub-carriers we searched the first pilot position of comb pilots, the principle was subject to the similarity that kept the non-pilots data sub-carriers closest to the former ones, which in the next part of our article we analyzed it was under our constraints model therefore guaranteed mean square error of channel estimation became lower and lower while sub-carriers remain stationed. As is shown in Fig. 2, which simplifies the OFDM model.

Be noticed that this simplified model divides OFDM system into four sub-parts. The sub-carriers before restructuring X , after restructuring to modulation X' and the older receiver after demodulation and channel estimation and ZF equalization Y' , the new receiver after extra equalization Y . The extra equalization mentioned above may contain variety of adaptive methods and others which account for priority transceiver information. This model can also be recognized into three actions: restructuring, transmitting and extra equalization, these make the existence of Restructuring Offset Error (ROE) presented as $\Delta = X' - X$ and Transmitting Offset Error (TOE) presented as $\Delta' = Y' - X'$. Our aim is to set a constraint optimization model which makes ROE and TOE as small as possible, then from Fig. 2 we know the extra equalization will not be necessary and can be neglected.

MATERIALS: COMB PILOTS AND LS CHANNEL ESTIMATION

Paper Zhao (2012) deduced the linear relation between comb-pilots and total sub-carriers, presented as:

$$\mathbf{X}_{comb} = \mathbf{F}\mathbf{X}_N = \mathbf{F}_p \mathbf{W} [\mathbf{I}_p, \phi, \dots, \phi^{Q-1}] \mathbf{F}_N^{-1} \mathbf{X}_N = \mathbf{F}_p \mathbf{W} \phi \mathbf{F}_N^{-1} \mathbf{X}_N \quad (1)$$

In (1), \mathbf{X}_{comb} means comb-pilots, \mathbf{X}_N is total sub-carriers, \mathbf{F}_p and \mathbf{F}_N^{-1} are Fourier matrix. Comb-pilots are those who are equally-spaced extracted from sub-carriers in frequency:

$$\mathbf{W} = \begin{pmatrix} e^{-\frac{j2\pi^*m*0}{M}} & 0 & 0 & 0 \\ 0 & e^{-\frac{j2\pi^*m*1}{M}} & 0 & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & e^{-\frac{j2\pi^*m*(P-1)}{M}} \end{pmatrix},$$

$$\phi = \begin{pmatrix} e^{-\frac{j2\pi^*m*P}{M}} & 0 & 0 & 0 \\ 0 & e^{-\frac{j2\pi^*m*P}{M}} & 0 & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & e^{-\frac{j2\pi^*m*P}{M}} \end{pmatrix},$$

where $m = 0 \sim P-1$.

Moreover, paper Zhao (2012) has generalized LS mechanism. Due to the arbitral selection of pilots, after deduction of paper (Zhao, 2012), the smallest mechanism of LS estimation MSE is as following:

$$MSE_{HLS} \propto \frac{N_{OFDM}}{N_{pilots}} \quad (2)$$

N_{OFDM} Represents the power of OFDM symbol with cyclic-prefix and N_{pilots} means the power of pilots selected. "∞" means to related to.

METHODS: CONSTRAINT MODEL

Set N , p are number of sub-carriers and comb pilots, then the equal space is $q = \frac{N}{p}$. Selected the largest power pilots can be presented as:

$$\mathbf{X}_{comb}^{(0)} = \arg \max_{j \in \{1, 2, \dots, q\}} \left(\sum_{i=1}^p \|\mathbf{X}_{j+(i-1)p}\|^2 \right), q = \frac{N}{p} \quad (3)$$

$\mathbf{X}_{comb}^{(0)}$ Means the position of first pilot among comb pilots group, as energy of comb-pilots is the largest in the same size pilots among sub-carriers thus $\mathbf{X}_{comb}^{(0)}$ again selects the largest one among comb pilots group. According to (1), power of comb-pilots is:

$$E_{old} = \|\mathbf{F}_p \mathbf{W} \phi \mathbf{F}_N^{-1} \mathbf{X}_N\|^2 \quad (4)$$

While power of non-pilots data sub-carriers is:

$$E_p = \|\mathbf{X}_N\|^2 - \|\mathbf{F}_p \mathbf{W} \phi \mathbf{F}_N^{-1} \mathbf{X}_N\|^2 = \|\mathbf{X}_N\|^2 - E_{old} \quad (5)$$

Then no matter what the new sub-carriers are, we should keep the data sub-carriers unchanged, then, the power of new pilots is:

$$E_{new} = \|\mathbf{X}\|^2 - E_p = \|\mathbf{X}\|^2 - \|\mathbf{X}_N\|^2 + E_{old} \quad (6)$$

According to minimal mechanism of LS channel estimation, we should do:

$$\frac{E_{new}}{\|\mathbf{a}\mathbf{F}_N^{-1}\mathbf{X}\|^2} \geq \frac{E_{old}}{\|\mathbf{a}\mathbf{F}_N^{-1}\mathbf{X}_N\|^2} \Leftrightarrow \|\mathbf{X}\|^2 - \frac{\|\mathbf{F}_p \mathbf{W} \phi \mathbf{F}_N^{-1} \mathbf{X}_N\|^2}{\|\mathbf{a}\mathbf{F}_N^{-1} \mathbf{X}_N\|^2} \|\mathbf{a}\mathbf{F}_N^{-1} \mathbf{X}\|^2 \geq \|\mathbf{X}_N\|^2 - \|\mathbf{F}_p \mathbf{W} \phi \mathbf{F}_N^{-1} \mathbf{X}_N\|^2 \quad (7)$$

where, a make the OFDM symbol combined with cyclic-prefix which is the transceiver symbol, when signal-to-noise ratio is fixed, the power of transceiver symbol is positive related to noise variance. If (7) is established, it guarantees that channel estimation is more precise, which means TOE is smaller, moreover, if we restrict ROE be small, we can build the constraint model as:

$$\min_{\mathbf{x}} f(\mathbf{x}) = \|\mathbf{X} - \mathbf{x}_N\|^2 \quad st. \|\mathbf{X}\|^2 \geq \delta^2 \|\mathbf{N}\mathbf{X}\|^2 + \varepsilon^2 \quad (8)$$

From the constraint model, it is obvious that we restrict ROE subject to smaller TOE, therefore if we iteratively use this model, TOE will be smaller and smaller, while ROE in the other part of our article we will prove that it remains.

According to Pars Val theorem, we transform (8) into the time domain as:

$$\begin{cases} \min_{\mathbf{x}} f(\mathbf{x}) = \|\mathbf{x} - \mathbf{x}_N\|^2, \\ st. \|\mathbf{x}\|^2 \geq \frac{\delta^2}{N} \|\mathbf{a}\mathbf{x}\|^2 + \frac{\varepsilon^2}{N} \end{cases} \quad (9)$$

where,

$$\mathbf{x} = \mathbf{F}_N^{-1} \mathbf{X}, \mathbf{x}_N = \mathbf{F}_N^{-1} \mathbf{X}_N, \delta = \frac{\|\mathbf{F}_p \mathbf{W} \phi \mathbf{F}_N^{-1} \mathbf{X}_N\|}{\|\mathbf{a}\mathbf{F}_N^{-1} \mathbf{X}_N\|}$$

$$\frac{\varepsilon^2}{N} = \|\mathbf{x}_N\|^2 - \frac{\|\mathbf{F}_p \mathbf{W} \phi \mathbf{F}_N^{-1} \mathbf{X}_N\|^2}{N}$$

Methods: model resolution and iterative algorithm: First, let us resolute the constraint model then we will introduce iterative method. By taking Lagrange factor into consideration, the Lagrange function is:

$$g(\mathbf{x}) = \|\mathbf{x} - \mathbf{x}_N\|^2 + \lambda \left(\|\mathbf{x}\|^2 - \frac{\delta^2}{N} \|\mathbf{a}\mathbf{x}\|^2 - \varepsilon^2 \right), \varepsilon^2 > \frac{\varepsilon^2}{N} \quad (10)$$

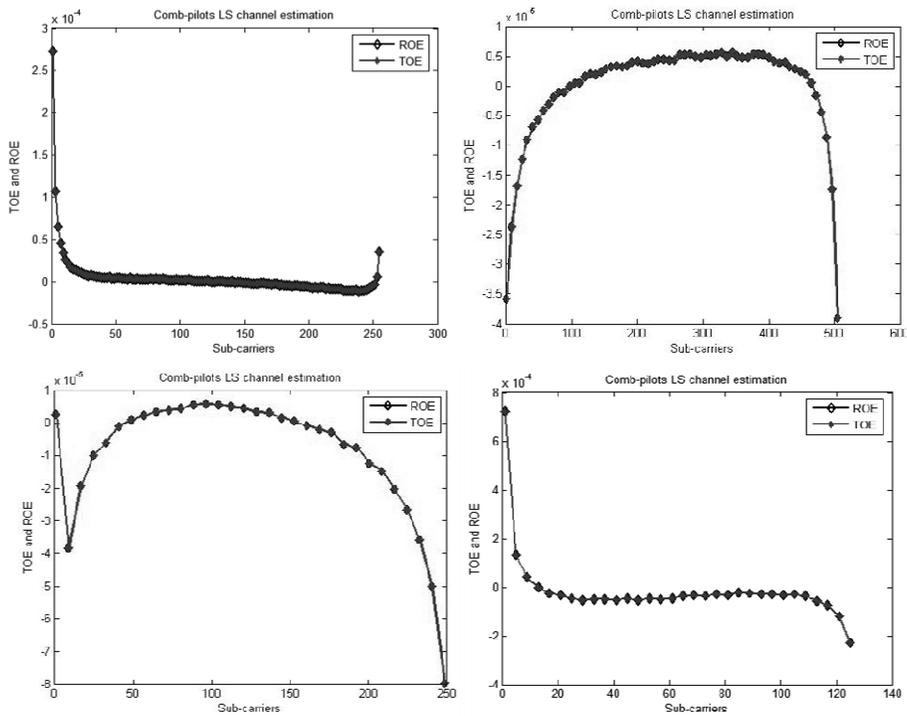


Fig. 3: Real part of TOE and ROE compare under multi-iterations and varied sub-carriers

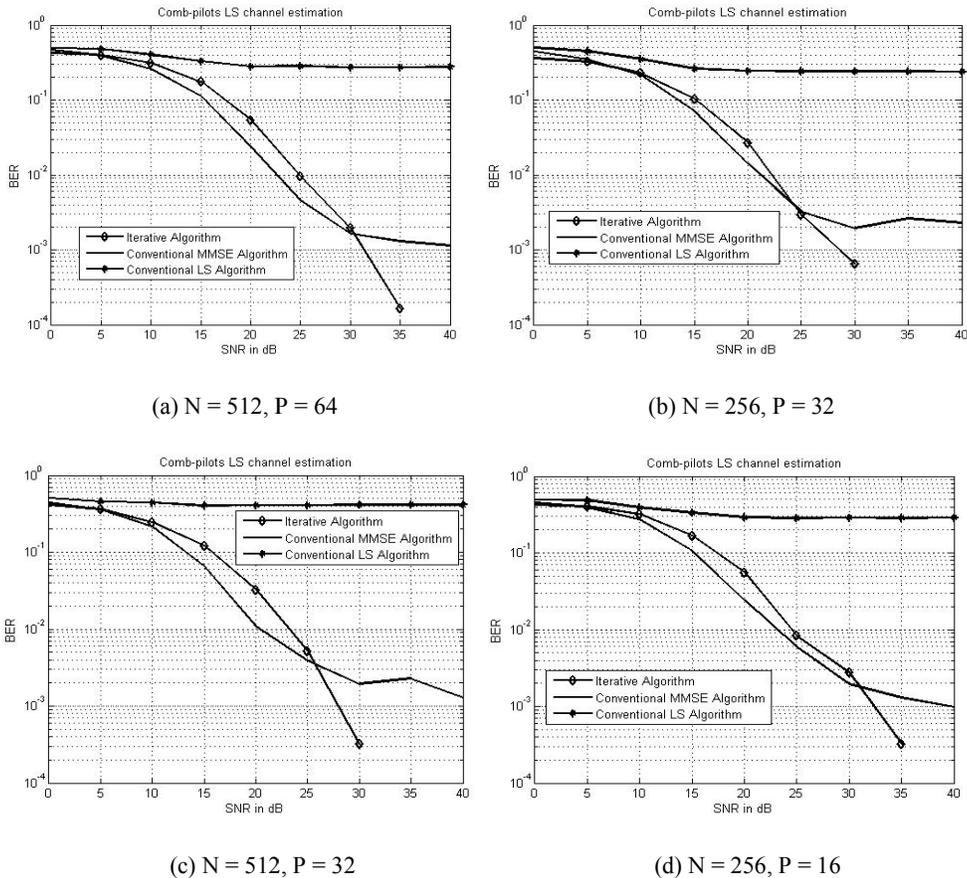


Fig. 4: BER performances of iterative algorithm and conventional algorithm when varied number of sub-carriers

Table 1: Simplified simulation conditions

Conditions	Parameters
Front-end modulation	16-QAM
Number of carriers	$N = 256, 512$
Length of CP	$CP = 32$
Channel type	Rayleigh multi-paths fade channel
Channel paths	8, 16, 32
Number of comb-pilots	$P = 16, 32, 64$
Channel estimation interpolation for receiver	DFT interpolation
ROE	$\Delta = \mathbf{x}' - \mathbf{x}$
TOE	$\Delta' = \mathbf{Y}' - \mathbf{X}'$
Transmission noise type	Addictive gauss white noise
Signal to noise ratio	From 0~40 dB
SNR space	5 dB
$\Delta \epsilon$	$\ \mathbf{x}_{op}^{(i-1)}\ ^2$

number is 64. And our simulations precede under the basic conditions which contain parameters:

Compare of ROE and TOE: First here in order to give reference to the theory, we verified here that both TOE and ROE are very small, see Fig. 3. We only give real part of TOE and ROE although both of them are complex number. Simulations show that under varied number of sub-carriers TOE and ROE are equal, when suddenly our algorithm convergence, however, this is not essential (Table 1).

BER performance: See Fig. 4 (when $ROE = TOE \rightarrow 0$), we find that our iterative algorithm make BER performance approach the performance of MMSE channel estimation in point to point communication and greatly surpass the performance of LS channel estimation in convention, the latter is influenced by energy release caused by DFT interpolation channel estimation. Moreover, our algorithm makes BER curve smooth and in high SNR it surpasses MMSE channel estimation. Simulations verified our theoretical algorithm design.

CONCLUSION

We proposed an iterative method to directly restructure the total sub-carriers at transceiver, the method is based on comb-pilots LS channel estimation and is constrained by minimizing the Transmitting Offset Error (TOE) with restricting Restructure Offset Error (ROE) as objective function. We proved its convergence, simulation verified its gains. Although this may lead to some complexity of calculation because of the iteration, but it avoids the multi-equalization for receiver that must require transceiver priority information, thus save the cost. As a novel proposal, this may bring about some significant to implementation.

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