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Research Article

Fuzzy MADM with Triangular Numbers for Project Investment Model based on Left and Right Scores

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Abstract: Because of the complexity of limitation and realistic decision of decision knowledge, multiple attribute decision making problems with the attribute value often need to characterize fuzzy number. The triangular fuzzy number to describe this kind of ambiguity is very effective and for the project investment selection problem with triangular fuzzy numbers, this study puts forward a new multiple attribute decision making method. The concrete steps are: we first define the left and right scores based the normalized triangular fuzzy numbers and then the decision matrix is transformed into the interval number decision-making matrix. Finally the distance based on alternative and ideal solution as the principle of optimal alternatives for sorting and merit. At the end of the study, an example is given to show that the method proposed in this study is effective and practical.

Keywords: Ideal solution, left and right scores, multiple attribute decision making, triangular fuzzy number

INTRODUCTION

Under the impetus of the economic globalization, competition between enterprises is increasingly fierce, many decision problems become more and more complex waited for decision maker's to deal with. However, due to the limitations of the decision maker's knowledge and incomplete understanding of the world, evaluation or attribute value is usually difficult to express with crisp numbers (Xu, 2002). Even if we force the attribute description value demonstrated by accurate number, it will lose some information, thus it will affect the effectiveness of the decision result. According to the problems above, a lot of fuzzy Multiple Attribute Decision Making (MADM) method is put forward and has become the hot spot of management decision making, its application has been applied to such as investment partner selection, teacher performance evaluation, the program of travel choice and military operations scheme selection and so on all aspects of human life. The triangular fuzzy number has its superiority than fuzzy number and interval fuzzy number in depiction the attribute values in the MADM problems, then the MADM problems with triangular fuzzy numbers have been concerned by many scholars. For MADM problems, in which attribute values and the preference of decision makers are triangular fuzzy numbers, Xu (2002) proposed a MADM method based on the similarity of triangular fuzzy numbers; Wan (2009) analyzed uncertain multi-attribute decision-making problem whose elements of decision-making matrix are triangular fuzzy numbers when the preference information is given by the form of preference ordered pair. Given the attribute evaluation information and attribute of the project, Pan (2012) suggested multiple attribute group decision-making problems given by the form of fuzzy language. Through introducing dominance among projects and dominance comparison matrix, transform language information into triangular fuzzy number and then present multiple attribute group decision-making method based on the ideal weight. With regard to the attribute evaluation information of the project and attribute weight are multiple attribute group decision-making problem formed by fuzzy language, Chen and Yang (2008) converted the language information into triangular fuzzy number, raising a fuzzy multiple attribute group decision algorithm through constructing combined consistency index which concentrates decision-maker's authority and consensus of opinion. It also analyses and proves the feasibility and effectiveness of the whole algorithm through the examples of firms' credit assessment and sensitivity of combined consistency index. Regarding fuzzy of attribute information in the project assessment, Wang *et al*. (2006) presented an evaluation method for the entropy weight multi-attribute project based on fuzzy information only when there is fuzzy judgment matrix but no experts' weights. Through the entropy of fuzzy evaluating matrix and the distance and approach degree of triangular fuzzy number from the assessed to ideal point, the method makes optimization selection evaluation from several

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reasonable projects and gets the optimization project with certain reliability. Lin and Qiu (2009) came up with a decision-making method based on linear programming and fuzzy vector projection, in view of the multiple attribute decision-making problem with completely unknown attribute weight and whose attribute values are triangular fuzzy numbers. The method builds up a linear programming model based on weighted attribute values maximizing deviations, getting the attribute weight through solving the model, calculating the projection of weighted attribute values of projects on the fuzzy positive and negative ideal point and then calculating the relative closeness degree, accordingly, sorting the projects. Wan (2011) studied the type recognition problem in which the characteristic values of object types and observations of sensors are in the form of triangular fuzzy numbers. By solving the maximization optimization model, the vector of characteristic weights is derived. A new fusion method based on triangular fuzzy is proposed, which is also proved that the proposed method can improve the objectivity and accuracy of object recognition. Chen (1985) proposed max-min method of defuzzification method, Chen and Hwang (1989) through the definition of triangular fuzzy number row order about scoring, then put forward to the triangular fuzzy number ranking method based on sort utility function about the left and right scores.

Motivation by the basic idea of Chen (1985), in this study, based on the left and right score of triangular fuzzy number, we will develop a new triangular fuzzy MADM method.

DEFINITIONS AND CONCEPTS OF FUZZY NUMBER

Definition 1: If $\tilde{A}=[a,b,c]$, $0\le a\le b\le c$, then $\tilde{A}$ named triangular fuzzy number. The membership function of $\tilde{A}$ is given as:

$$\mu_{\tilde{A}}(x)=\begin{cases}\dfrac{x-a}{b-a}, a\le x\le b\\ \dfrac{c-x}{c-b}, b\le x\le c\\ 0,\ otherwise\end{cases}$$

Definition 2: Let $\tilde{A}_1=[a_1,b_1]$ and $\tilde{A}_2=[a_2,b_2]$ are two any interval fuzzy number, the distance of $\tilde{A}_1$ and $\tilde{A}_2$ is defined as:

$$d(\tilde{A}_1,\tilde{A}_2)=\frac{\sqrt{2}}{2}\sqrt{(a_1-a_2)^2+(b_1-b_2)^2}$$

Definition 3: An important concept regarding the applications of fuzzy numbers is defuzzification task which transforms a fuzzy number into a crisp value Ebrahimnejad *et al*. (2012). The most commonly used defuzzification method is the centroid defuzzification method given as follows (Chen and Hwang, 1989):

$$\overline{x}_0(\tilde{A})=\frac{\int_a^c x\mu_{\tilde{A}}(x)dx}{\int_a^c \mu_{\tilde{A}}(x)dx}=\frac{a+b+c}{3}$$

For a MADM problem, supposed that $X=\{x_1,x_2,\cdots,x_m\}$ is m alternative set, $O=\{o_1,o_2,\cdots,o_n\}$ is attribute set and $\tilde{a}_{ij}=[a_{ij}^L,a_{ij}^M,a_{ij}^U]$ denotes the attribute value of alternative $X_i$on attribute o_j, the decision-making matrix is given as:

$$A=(\tilde{a}_{ij})_{m\times n}=\begin{array}{c} \\ x_1\\ x_2\\ \vdots\\ x_m\end{array}\begin{array}{c} \begin{array}{cccc} o_1 & o_2 & \cdots & o_n\end{array}\\ \begin{pmatrix}\tilde{a}_{11} & \tilde{a}_{12} & \cdots & \tilde{a}_{1n}\\ \tilde{a}_{21} & \tilde{a}_{22} & \cdots & \tilde{a}_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ \tilde{a}_{m1} & \tilde{a}_{m2} & \cdots & \tilde{a}_{mn}\end{pmatrix}\end{array}$$

Benefit and cost type are most common attribute types in multi-attribute decision making problems. The subscript set of benefit and cost type are respectively denoted by I_1 and I_2 and M = {1, 2, …, m}, N = {1, 2,…, n}. To eliminate the impact of different physical dimension on decision-making result, using the standardized approach from Xu (2002) to deal with, standardized matrix $R=(r_{ij})_{m\times n}$ can be obtained, where $r_{ij}=[r_{ij}^L,r_{ij}^M,r_{ij}^U]$ are calculated by the following equations:

$$\begin{cases} r_{ij}^L = a_{ij}^L \Big/ \sqrt{\sum_{i=1}^m (a_{ij}^U)^2}\\ r_{ij}^M = a_{ij}^M \Big/ \sqrt{\sum_{i=1}^m (a_{ij}^M)^2},\\ r_{ij}^U = a_{ij}^U \Big/ \sqrt{\sum_{i=1}^m (a_{ij}^L)^2}\\ i\in M,\ j\in I_1\end{cases}$$

and

$$\begin{cases} r_{ij}^L = (1/a_{ij}^U) \Big/ \sqrt{\sum_{i=1}^m (1/a_{ij}^L)^2}\\ r_{ij}^M = (1/a_{ij}^M) \Big/ \sqrt{\sum_{i=1}^m (1/a_{ij}^M)^2}\\ r_{ij}^U = (1/a_{ij}^U) \Big/ \sqrt{\sum_{i=1}^m (1/a_{ij}^U)^2}\\ i\in M,\ j\in I_2\end{cases}$$

After get the solution of attribute values, because the attribute value for the triangular Fuzzy number can't

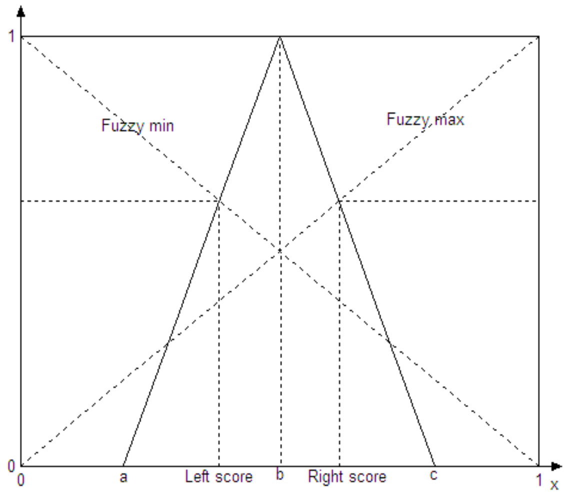

Fig. 1: The left and rignt scores for fuzzy number

direct sequencing, this study will be based on the proposed set of maximum and minimum legal thoughts (Chen, 1985), application of Fuzzy and Fuzzy Max Min intersection point definition about scoring with Fuzzy Numbers, then the triangle Fuzzy Numbers is transformed into interval number. Figure 1 illustrates the mentioned notion (using fuzzy max and fuzzy min that defined by Chen and Hwang (1989) graphically.
About score values are defined as follows:

Definition 4: For given normalized triangular fuzzy number $\tilde{A}$, = [a, b, c] the maximizing set and minimizing set are defined as Mokhtarian (2011) and Mokhtarian and Hadi-Vencheh (2012):

$$\mu_{\max}(x) = \begin{cases} x, & 0 \le x \le 1 \\ 0, & otherwise \end{cases}$$

$$\mu_{\min}(x) = \begin{cases} 1-x, & 0 \le x \le 1 \\ 0, & otherwise \end{cases}$$

Then we call L_s and R_s are the left and right score of $\tilde{A}$ ZZ and respectively defined as:

$$L_S = \sup_x [\mu_{\tilde{r}}(x) \wedge \mu_{\min}(x)]$$

and

$$R_S = \sup_x [\mu_{\tilde{r}}(x) \wedge \mu_{\max}(x)]$$

Through simple and easy calculation, the concrete expression equations of the scores are given as bellow:

$$L_S = \frac{b}{1+b-a}$$

and

$$R_S = \frac{c}{1+c-b}$$

VARIATION COEFFICIENT WEIGHT METHOD

If an attribute value r_{ij}, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$ in each alternatives are the same, then the attribute index has no effect in the ranking scheme, on the contrary, if the attribute value of the index difference is large, then the indicators identified in the medium-term target larger role can therefore be based on the measure of the difference between the attribute values, using the coefficient of variation to determine the weight of each characteristic index. Men and Liang (2005) proposed the coefficient of variation:

$$w_j = \frac{\delta_j}{\sum_{j=1}^{n} \delta_j}, j = 1, 2, \ldots, n$$

As the attribute index weights, where $\delta_j = \frac{s_j}{\bar{x}_j}$,

$$\bar{x}_j = \frac{1}{m} \sum_{i=1}^{m} x_{ij}$$

and

$$s_j = \sqrt{\frac{1}{m} \sum_{i=1}^{m} (x_{ij} - \bar{x}_j)^2}$$

Obviously:

$$w_j \ge 0, \sum_{j=1}^{n} w_j = 1, j = 1, 2, \ldots, n$$

MADM method based on left and right scores: In this section, we will give the calculation steps of the proposed MADM method based on left and right scores as follows:

Step 1: For a MADM problem, supposed that $X = \{x_1, x_2, \cdots, x_m\}$ is m alternative set, $O = \{o_1, o_2, \cdots, o_n\}$ is attribute set and $\tilde{a}_{ij} = [a_{ij}^L, a_{ij}^M, a_{ij}^U]$ denotes the attribute value of alternative x_i on attribute o_j, the decision-making matrix is given as:

$$A = (\tilde{a}_{ij})_{m \times n} = \begin{array}{c} \\ x_1 \\ x_2 \\ \vdots \\ x_m \end{array} \begin{array}{c} \begin{array}{cccc} o_1 & o_2 & \cdots & o_n \end{array} \\ \begin{pmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \cdots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{m1} & \tilde{a}_{m2} & \cdots & \tilde{a}_{mn} \end{pmatrix} \end{array}$$

Step 2: Change the decision matrix A into the normal decision matrix $R=(r_{ij})_{m\times n}$

Step 3: Use centroid defuzzification method by definition 3, the decision matrix $A=(\tilde{a}_{ij})_{m\times n}$ can be converted into crisp decision matrix $X_C=(x_{ij})_{m\times n}$, where:

$$x_{ij}=(a_{ij}^L+a_{ij}^M+a_{ij}^U)/3$$

Step 4: Calculating the attribute weights according to the coefficient of variation method

Step 5: Using definition 4, the fuzzy normal decision matrix $R=(r_{ij})_{m\times n}$ is transformed into interval decision matrix $Q=(\tilde{Q}_{ij})_{m\times n}$, where $\tilde{Q}_{ij}=[(L_s)_{ij},(R_s)_{ij}]$, $(L_s)_{ij}$ and $(R_s)_{ij}$ are the left and right scores of r_{ij}

Step 6: Calculate the relative closeness index of the ith alternative $\tilde{A}_i$ corresponding to the ideal solution $x*=([1,1],[1,1],[1,1],[1,1])$ and the index is defined as follows:

$$d(x_i,x^*)=\sum_{j=1}^{n}\frac{\sqrt{2}}{2}w_j\sqrt{(1-(L_S)_{ij})^2+(1-(R_S)_{ij})^2}$$

Step 7: According to the value of $d(x_i, x^*)$, the smaller $d(x_i, x^*)$ demonstrates the better alternative. Then we can sort and merit the alternatives and select the best one.

PROJECTION INVESTMENT SELECTION EXAMPLE

To illustrate the effectiveness and practicability of the proposed method in this study, we use the project risk selection problem to illustrate it. Suppose that a risk investment company want to select the best enterprise to invest. There are five alternative enterprises $x_j (j = 1, 2, \dots, 5)$ the investment evaluation should consider the following four attributes $O=\{o_1,o_2,o_3,o_4\}$ respectively, risk factors o_1, growth factors o_2, social and political factors o_3 and environmental factors o_4. The attribute o_1 is cost type, other attributes are benefit type and the attributes values of each alternative are given in the form of triangular fuzzy number and shown in Table 1. Try to sort these five alternatives and to determine the best investment enterprise.

To sort the five alternatives using this paper's method, the steps are given as follows:

Step 1: Transform fuzzy decision-making matrix $A=(a_{ij})_{m\times n}$ into standardized decision-making matrix:

$$R=\begin{bmatrix} [0.3899,0.4339,0.4877] & [0.3557,0.3879,0.4280] \\ [0.3482,0.3704,0.4013] & [0.4680,0.4978,0.5350] \\ [0.4431,0.4746,0.5032] & [0.3869,0.4137,0.4414] \\ [0.5042,0.5523,0.5981] & [0.3370,0.3685,0.4013] \\ [0.3566,0.3797,0.4064] & [0.5117,0.5430,0.5685] \end{bmatrix}$$
$$\rightarrow\begin{bmatrix} [0.4345,0.4651,0.5031] & [0.3226,0.3804,0.4371] \\ [0.3988,0.4341,0.4709] & [0.4751,0.5093,0.5464] \\ [0.4465,0.4838,0.5289] & [0.2992,0.3375,0.3857] \\ [0.3750,0.4155,0.4515] & [0.4517,0.4909,0.5271] \\ [0.4048,0.4341,0.4644] & [0.4575,0.4909,0.5335] \end{bmatrix}$$

Step 2: Calculate the score vector decision matrix:

$$Q=\begin{bmatrix} [0.4157,0.4628] & [0.3758,0.4115] \\ [0.3624,0.3893] & [0.4834,0.5158] \\ [0.4601,0.4892] & [0.4029,0.4295] \\ [0.5270,0.5719] & [0.3572,0.3885] \\ [0.3711,0.3958] & [0.5265,0.5544] \end{bmatrix}$$
$$\rightarrow\begin{bmatrix} [0.4513,0.4847] & [0.3597,0.4137] \\ [0.4193,0.4542] & [0.4925,0.5269] \\ [0.4664,0.5061] & [0.3250,0.3680] \\ [0.3994,0.4358] & [0.4724,0.5087] \\ [0.4218,0.4508] & [0.4751,0.5117] \end{bmatrix}$$

Step 3: Calculate the attribute weight vector decision matrix:

$$w=(0.2773,0.2974,0.1152,0.3101)$$

Step 4: Calculate the relative closeness index of the ith alternative A_i corresponding to the ideal solution $x*=([1,1],[1,1],[1,1],[1,1])$, is given as:

$$d(x_1,x^*)=0.5877, d(x_2,x^*)=0.5391, d(x_3,x^*)=0.5814,$$
$$d(x_4,x^*)=0.5369, d(x_5,x^*)=0.5299$$

Table 1: Attribute evaluation value of alternatives

	O_1	O_2	O_3	O_4
x_1	(0.65, 0.70, 0.75)	(0.57, 0.60, 0.64)	(0.73, 0.75, 0.78)	(0.55, 0.62, 0.68)
x_2	(0.79, 0.82, 0.84)	(0.75.0.77, 0.80)	(0.67, 0.70, 0.73)	(0.81, 0.83, 0.85)
x_3	(0.63, 0.64, 0.66)	(0.62, 0.64, 0.66)	(0.75, 0.78, 0.82)	(0.51, 0.55, 0.60)
x_4	(0.53, 0.55, 0.58)	(0.54, 0.57, 0.60)	(0.63, 0.67, 0.70)	(0.77, 0.80, 0.82)
x_5	(0.78, 0.80, 0.82)	(0.82, 0.84, 0.85)	(0.68.0.70, 0.72)	(0.78, 0.80, 0.83)

Step 5: According to the smaller $d(x_i, x^*)$ the better alternative, the ranking order of the alternatives is $x_5 > x_4 > x_2 > x_3 > x_1$ and the best investment enterprise is x_5.

CONCLUSION

Multi-attribute decision making problems have been widely used in the solution of practical decision making problems. A new MADM method based on left and right scores are proposed. The attribute weight are often artificially designated, thus often have the subjective arbitrariness and uncertainty. To fully consider the degree of importance of the attributes, we use the coefficient of variation method, based on the difference of each attribute index to determine index weight, so that the weight data with the outside world and change, not only to better reflect the objective reality, but determine the weight to avoid the subjective arbitrariness, reducing the interference of artificial subjective factors, thus improving the results of human's objectivity, while the proposed method defines the relative closeness of alternatives with the positive ideal, which is the result of better reliability and rationality of this algorithm is simple and easy to use Matlab and other software programmable computing for solving the problem, thus provides a new MADM approach.

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