

Research Article

The Prediction for Shanghai Business Climate Index by Grey Model

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Abstract: This study predicts Shanghai Business Climate Index from the third quarter of 2013 to the first quarter of 2014 by Grey Model. Business climate index (enterprise integrated production business index) is the judgments and expectations index to the development of an enterprise in the future. By comparing the forecasting accuracy of three grey models, i.e., GM (1, 1) model, Verhulst model and the DGM (2, 1) model, the results show that the forecasting accuracy of GM (1, 1) model is higher than Verhulst model and DGM (2, 1) model. And GM (1, 1) model is used to predict Shanghai business climate index in the next three quarters.

Keywords: Business climate index, GM (1, 1) model, grey model

INTRODUCTION

Business climate index (enterprise integrated production business index) is the judgments and expectations index which is based on integrated production and management of the enterprise and comprehensively reflects the production and operation of enterprises. Climate index is an indicator that reflects the state or trend for a particular social group or an economic phenomenon. Climate index values are between 0 and 200, its threshold is 100. When the climate index is greater than 100, it indicates that the situation tends to rise or improve and the closer to 200, the better; when climate index is less than 100, it indicates that the situation tends to decline or deteriorate and the closer to 0, the worse.

Business sentiment survey which is conducted by periodically investigating the responsible persons through periodic sample surveys. According to the company's business situation and the persons responsible for the judgment of the macroeconomic environment and expectations to compile climate index, it can reflect the macroeconomic and business conditions accurate and timely, the forecast movements in economic development as a survey method. It is to adapt to China's socialist market economic development in the new situation, learn from the experience of Western countries and set up a statistical survey system in advance, it is to enhance the timeliness of statistical services and expand the scope of statistical services, improve the quality of statistical services of new investigations.

The form of the business sentiment survey, mainly qualitative, quantitative, supplemented by a combination of qualitative and quantitative indicators of

economic system in order to determine the macroeconomic environment of the enterprise and micro business conditions determine the intention of combining survey content. High ahead of its information, objectivity, reliability and continuity, both in terms of time or in the index are set up for the shortcomings of traditional statistical methods.

Grey system theory is composed by a professor of Huazhong University of Science named Deng (1982, 1989) in the early 1980s and has already have a considerable development in about three decades. Grey system theory has been widely used in several areas, such as agricultural, industrial and environmental system. Grey system theory is an important part of predictive models and has been popularized in the time series prediction due to its simplicity and ability and high accuracy to characterize an unknown system, using as little as four data points.

In recent years, the grey system theory has been widely used to forecast in various fields and demonstrated satisfactory results. For instance, Che and Chen (2003), had used improved GM (1, 1) model for power demand forecasting. Pin *et al.* (2003), had applied grey model to analyze human and vehicle factors for Taiwan freeway traffic accidents. Wu and Chen (2005), had used the Grey Model GM (1, n) to conduct a case study on internet access population forecast. Zhou *et al.* (2006), had used a trigonometric grey prediction approach to forecasting electricity demand. Lu (2007), had used grey model to analyze and forecast the Road traffic safety improvement in Netherlands. Huang and Wang (2007), had used Grey Model-GM (1, 1) to predict the urban traffic accidents. Lin *et al.* (2009), had presented the adaptive and high precision grey forecasting model to predict the stock

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index in Taiwan. Kayacan *et al.* (2010), had used Grey System Theory-Based models in time Series prediction. Zhu (2010), had used the composite grey BP neural network forecasting model to motor vehicle fatality risk in China. Mao and Sun (2011), had applied the Grey-Markov model in forecasting fire accidents in China.

Because the factors that affect the business climate index are complex and uncertain, therefore, to forecast the business climate index, There are a lot of known information, there are also a lot of unknown information, which can be regarded as a grey system, so it can be analyzed by using grey system theory.

In this study, we first introduce the concept of business climate index and then we introduce the GM (1, 1) model, Verhulst model and the DGM (2, 1) model of the basic concepts and compare these three models predict accuracy of the results. Finally, predicts Shanghai Business Climate Index from the third quarter of 2013 to the first quarter of 2014 by the GM (1, 1) model.

METHODOLOGY

Grey models: The most common grey prediction model is GM (1, 1), Then we can use the raw sequence data to make the Accumulation Generating Operation (AGO). The AGO reveal the hidden regular pattern in the system development.

Before the algorithm of GM (1, 1) is described, the raw data series is assumed to be:

$$X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\} \tag{1}$$

where n is the total number of modeling data. The AGO formation of $X^{(1)}$ is defined as:

$$X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n)\} \tag{2}$$

where,

$$x^{(1)}(k) = \sum_{m=1}^k x^{(0)}(m) \quad (k = 1, 2, \dots, n) \tag{3}$$

The GM (1, 1) model can be constructed by establishing a first order differential equation for $x^{(1)}(k)$ as:

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b \tag{4}$$

where, parameters a and b are called the developing coefficient and grey input, respectively.

In practice, parameters a and b are not calculated directly from Eq. (4). Therefore, the solution of (4) can be obtained by using the least square method. That is:

$$\hat{x}^{(1)}(k+1) = [x^{(0)}(1) - \frac{\hat{b}}{a}]e^{-\hat{a}k} + \frac{\hat{b}}{a} \quad (k = 2, 3, \dots, n) \tag{5}$$

where,

$$\hat{P} = (\hat{a}, \hat{b})^T = (B^T B)^{-1} B^T Y$$

And

$$B = \begin{bmatrix} -\frac{1}{2}(x^{(1)}(1) + x^{(1)}(2)) & 1 \\ -\frac{1}{2}(x^{(1)}(2) + x^{(1)}(3)) & 1 \\ \vdots & \vdots \\ -\frac{1}{2}(x^{(1)}(n-1) + x^{(1)}(n)) & 1 \end{bmatrix} \tag{6}$$

$$Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} \tag{7}$$

Applying the Inverse Accumulated Generation Operation (IAGO). Namely:

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) = (1 - e^{-\hat{a}})[x^{(0)}(1) - \frac{\hat{b}}{\hat{a}}]e^{-\hat{a}k} + \frac{\hat{b}}{\hat{a}} \tag{8}$$

where,

$$\hat{x}^{(1)}(1) = \hat{x}^{(0)}(1) \quad (k = 2, 3, \dots, n).$$

The Verhulst model was first introduced by a German biologist Pierre Franois Verhulst. The main purpose of Verhulst model is to limit the whole development for a real system.

For an initial time sequence:

$$X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\}$$

The initial sequence $X^{(0)}$ is used to directly construct the Verhulst model:

$$\frac{dx^{(0)}}{dt} + ax^{(0)} = b(x^{(0)})^2 \tag{9}$$

where a is development coefficient and b is grey action quantity. The solution of the parameter vector $\hat{P} = (\hat{a}, \hat{b})^T$ can be obtained by utilizing the least square method.

where,

$$\hat{P} = (\hat{a}, \hat{b})^T = [(A : B)^T (A : B)]^{-1} (A : B)^T Y$$

And

$$A = \begin{bmatrix} -\frac{1}{2}(x^{(0)}(1) + x^{(0)}(2)) \\ -\frac{1}{2}(x^{(0)}(2) + x^{(0)}(3)) \\ \vdots \\ -\frac{1}{2}(x^{(0)}(n-1) + x^{(0)}(n)) \end{bmatrix} \quad (10)$$

$$B = \begin{bmatrix} [\frac{1}{2}(x^{(0)}(1) + x^{(0)}(2))]^2 \\ [\frac{1}{2}(x^{(0)}(2) + x^{(0)}(3))]^2 \\ \vdots \\ [\frac{1}{2}(x^{(0)}(n-1) + x^{(0)}(n))]^2 \end{bmatrix} \quad (11)$$

$$Y = \begin{bmatrix} (x^{(0)}(2) - x^{(0)}(1)) \\ (x^{(0)}(3) - x^{(0)}(2)) \\ \vdots \\ (x^{(0)}(n) - x^{(0)}(n-1)) \end{bmatrix} \quad (12)$$

The resolution of (9) is:

$$\hat{x}^{(0)}(k+1) = \frac{\hat{a}x^{(0)}(1)}{\hat{b}x^{(0)}(1) + [\hat{a} - \hat{b}x^{(0)}(1)]e^{\hat{a}k}} \quad (k=0,1,\dots,n) \quad (13)$$

The DGM (2, 1) model is a single sequence second-order linear dynamic model and is fitted by differential equations.

Assume an original series to be $X^{(0)}$:

$$X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\}$$

A new sequence $X^{(1)}$ is generated by the Accumulated Generating Operation (AGO):

$$X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n)\}$$

where,

$$x^{(1)}(k) = \sum_{m=1}^k x^{(0)}(m) \quad (k=1,2,\dots,n)$$

Setting up a second-order differential equation:

$$\frac{d^2 x^{(1)}}{dt^2} + a \frac{dx^{(1)}}{dt} = b \quad (14)$$

where,

$$\hat{P} = (\hat{a}, \hat{b})^T = (B^T B)^{-1} B^T Y$$

$$B = \begin{bmatrix} -x^{(0)}(2) & 1 \\ -x^{(0)}(3) & 1 \\ \vdots & \vdots \\ -x^{(0)}(n) & 1 \end{bmatrix} \quad (15)$$

$$Y = \begin{bmatrix} (x^{(0)}(2) - x^{(0)}(1)) \\ (x^{(0)}(3) - x^{(0)}(2)) \\ \vdots \\ (x^{(0)}(n) - x^{(0)}(n-1)) \end{bmatrix} \quad (16)$$

According to (14), we have:

$$\hat{x}^{(0)}(k+1) = \left(\frac{\hat{b}}{a} - \frac{x^{(0)}(1)}{a}\right)e^{-\frac{\hat{a}}{a}k} + \frac{\hat{b}}{a}(k+1) + (x^{(0)}(1) - \frac{\hat{b}}{a})\frac{1+a}{a} \quad (17)$$

The prediction values of original sequence can be obtained by applying inverse AGO to $X^{(1)}$. Namely:

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) \quad (k=0,1,\dots,n)$$

Case study: In this section, the GM (1, 1), the Verhulst model and the DGM (2, 1) are used for comparison. The business climate index data in Shanghai from the second quarter of 2011 to the second quarter of 2013 is adopted to demonstrate the effectiveness and practicability of these models. The business climate index data in the second quarter of 2011 to the third quarter of 2012 is employed to set up the three grey prediction models and the business climate index data from the fourth quarter of 2012 to the second quarter of 2013 is used as data set to compare the three models accuracy. The evaluation criterion is the Mean Relative Percentage Error (MRPE), which measures the percent of prediction accuracy:

$$MRPE = \frac{1}{n} \sum_{k=1}^n \left[\frac{|x^{(0)}(k) - \hat{x}^{(0)}(k)|}{x^{(0)}(k)} \right]$$

The real and predictive values are shown in Table 1. So we can compare the forecasting accuracy and relative error of the three models. The corresponding calculated results (the mean error in the different stage) are shown in Table 2.

Table 1 demonstrates that the relative error of the GM (1, 1) prediction model is smaller than the others.

From Table 2, it can be seen that the MRPE of the GM (1, 1) model, the Verhulst model and the DGM (2, 1) model from the fourth quarter of 2012 to the second quarter of 2013, respectively are 6.23, 11.36 and 7.81%.

Table 1: Model values and prediction error of business climate index in Shanghai

Shanghai business climate index (the second quarter of 2011-the second quarter of 2013)								
Year	Real value	GM (1, 1)		Verhulst		DGM (2, 1)		
		Model value	Error R (%)	Model value	Error R (%)	Model value	Error R (%)	
Model building stage	The second quarter of 2011	133.1	128.8	3.251	147.7	-10.936	144.6	-8.670
	The third quarter of 2011	128.9	120.5	6.543	119.1	7.634	125.9	2.290
	The fourth quarter of 2011	125.6	128.0	-1.912	100.7	19.798	113.2	9.899
	The first quarter of 2012	120.9	134.0	-10.811	147.0	-21.622	131.4	-8.696
	The second quarter of 2012	122.4	115.7	5.466	111.3	9.068	109.6	10.435
	The third quarter of 2012	117.5	112.5	4.258	129.2	-9.935	96.6	17.806
Ex-post building stage	The fourth quarter of 2012	121.9	118.0	3.164	114.2	6.327	117.7	3.439
	The first quarter of 2013	131.1	133.5	-1.826	120.2	8.287	134.0	-2.247
	The second quarter of 2013	123.5	134.5	-8.900	141.1	-14.239	138.7	-12.298

Table 2: MRPE for the three prediction models

Stage	GM (1, 1) MRPE (%)	Verhulst MRPE (%)	DGM (2, 1) MRPE (%)
The second quarter of 2011 to the third quarter of 2012	5.31	12.74	6.37
The fourth quarter of 2012 to the second quarter of 2013	6.23	11.36	7.81

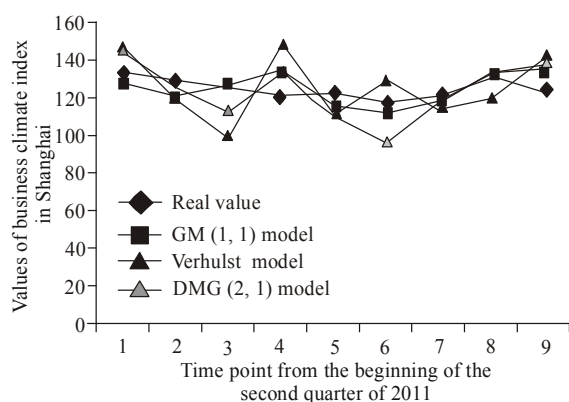


Fig. 1: Real values and models values

Table 3: Forecasting values for business climate index

Year	Model values	Real values
The fourth quarter of 2012	118.0	121.9
The first quarter of 2013	133.5	131.1
The second quarter of 2013	134.5	123.5
The third quarter of 2013	129.7	
The fourth quarter of 2013	128.6	
The first quarter of 2014	131.2	

The effectiveness and accuracy of GM (1, 1) model is higher than the Verhulst model and the DGM (2, 1) model.

Figure 1 shows that the GM (1, 1) model and the DGM (2, 1) model have the better forecasting precision from the second quarter of 2011 to the second quarter of 2013, but the GM (1, 1) prediction model seems to obtain the lowest mean relative percentage error, besides the GM (1, 1) model is more suitable to make a short-term prediction. So the GM (1, 1) model is used to predict business climate index in Shanghai from the third quarter of 2013 to the first quarter of 2014 (Table 3).

CONCLUSION

In this study, we compare the accuracy of three grey forecasting models by predicting business climate

index in Shanghai. The grey system theory could deal with the problems with incomplete or unknown information or problems with only few samples, so it is suitable to use it in this study.

The result shows that the forecast value of GM (1, 1) model from the fourth quarter of 2012 to the second quarter of 2013 is more accurate than the Verhulst grey model and the DGM (2, 1) model. Based on the above analysis, the GM (1, 1) model appeals to be the best model, because it is simply to apply and accurate to forecast, thus we use the GM (1, 1) model to forecast business climate index in Shanghai from the third quarter of 2013 to the first quarter of 2014.

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