

## Research Article

### Similar Rock Specimen's Creep Constitutive Test and Realization by FLAC<sup>3D</sup>

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**Abstract:** In order to reveal rocks' deformation law under long-time load, similar rock specimens' creep constitutive was studied with a comprehensive method of theoretical analysis, test and numerical simulation. Samples' conventional tests were done with WAW-32000 testing machine and indoor long-time creep tests were done with RLJW-2000 testing machine. Results show that, similar rock specimens' uniaxial compressive strength is 2.65 MPa; when stress is lower than its failure threshold 4.792 KN, specimens appear only two stages of initial creep and steady-state creep; when stress is higher than the threshold, specimens appear accelerated creep stage. According to the test and equivalent coordination deformation principle, nonlinear CYJ creep constitutive is derived and nested into FLAC<sup>3D</sup>, model's solution agrees well with the test results.

**Keywords:** Creep test, CYJ model, nonlinear constitutive, numerical simulation, similar rock specimen

## INTRODUCTION

Underground's stress and strain fields have time effect, project damage is often controlled by the rock creep. Currently, the scale and depth of underground engineering tends to increase, especially in the coal industry, mining depth increases year by year and deep pressure becomes more and more prominent, which brings tremendous challenges to the core work. Therefore, for convenience, intuitiveness and safeness, a considerable number of similar specimens' creep tests and corresponding constitutive models are necessary to reflect deep rock masses' creep characteristics.

Yang *et al.* (2007) did creep tests with different water content shales for studying elastoplastic deformation. Chen *et al.* (2007) proposed a new hyperbolic viscoplastic model for red sandstone coarse-grained soil's creep characteristic, which stringed the hyperbolic viscoplastic model and Burgers model together, a new six components nonlinear constitutive was established. Song *et al.* (2005) introduced a damage creep model based on the creep characteristics of gypsum breccia. Yang (2008) proposed an improved Burgers creep damage model considering the injury degradation effects with parameters varying as time, which could describe the instable creep. Xi *et al.* (2009) used Generalized Xi-Yuan model to study the creep behavior of granite under thermal coupling effects, based on which stress threshold and temperature threshold existed. Although in recent years great

progresses had been made for rock creep and models (Enrico and Tsutomu, 2001; Cazacu *et al.*, 1997; Maranini and Brignoli, 1999; Lade, 1994; Tai-Tien and Tsan-Hwei, 2009; Okubo *et al.*, 1991; Cristescu, 1993; Dahou *et al.*, 1995; Shao and Henry, 1991), similar rock specimens' creep test itself is a very special test style, self-defined creep constitutive model and FLAC<sup>3D</sup> secondary development by creep tests is even rare. In view of this, similar rock specimens' creep test was done, based on which nonlinear creep constitutive model was built and the model was nested into FLAC<sup>3D</sup>, numerical simulation results were in line with the test results.

## METHODOLOGY

**Specimen preparation and conventional test:** The prototype specimen is siltstone of Shandong Xinwen Mining -1100 m level, some mechanical parameters are as follows: the average uniaxial compressive strength is 52.87 MPa, the average diameter is 52.93 mm and the average height is 99.75 mm. The similar scale is 20:1. Similar rock materials are gypsum powder, iron powder barite powder, quartz sand, rosin and alcohol, the ratio is shown in Table 1.

During the production process, molds should be wiped by alcohol to prevent the adhesion of specimen to mold walls and the top surface of specimen should be scratched while filling to void the fault phenomenon. Specific operation procedures are shown in Fig. 1.

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Table 1: The ratio of similar rock materials

| Similar rock materials | Iron powder: barite powder: quartz sand | Gypsum powder's proportion of the total weight | Rosin alcohol solution's molar concentration | Rosin alcohol solution's proportion of the total weight |
|------------------------|---|--|--|---|
| Ratio                  | 1:1.5:0.28                              | 2.5%   | 30.0%  | 5.0%  |

Table 2: Similar rock specimen's average mechanical parameters

| Parameters    | Quality/ (g) | Height/ (mm) | Density/ (kg/m <sup>3</sup> ) | Ultimate load/ (kN) | Compressive strength/ (MPa) |
|---------------|--------------|--------------|-------------------------------|---------------------|-----------------------------|
| Average value | 489.19       | 100          | 2491.4                        | 5.20                | 2.65                        |

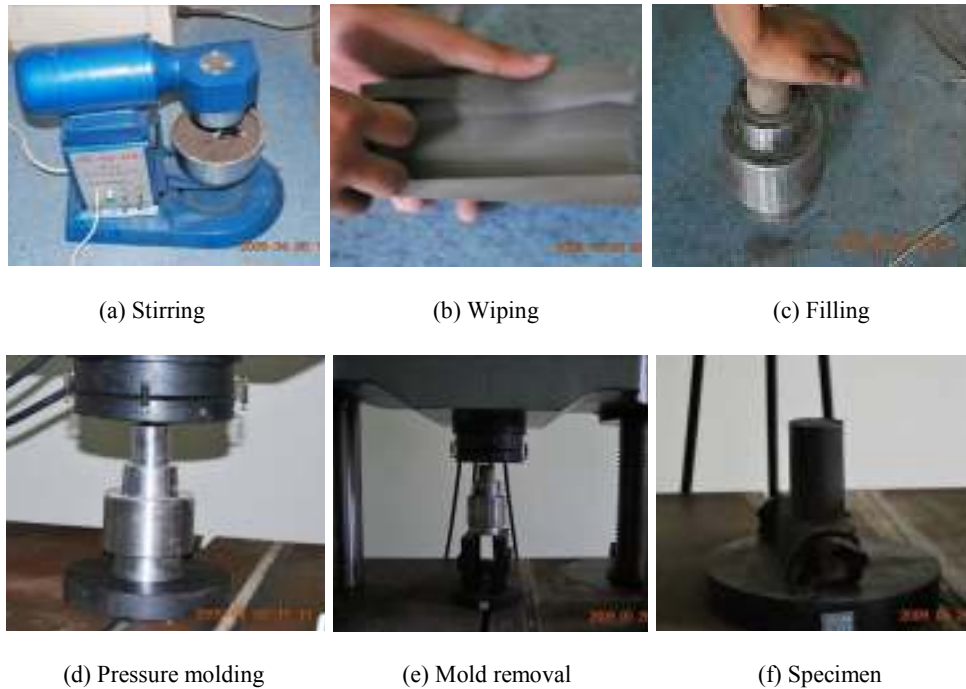


Fig. 1: Specimen production process



Fig. 2: Suppressed specimens



Fig. 3: (a) WAW-32000 test system, (b) specimens' failure modes

The successfully made similar rock specimens are shown in Fig. 2.

Specimens' conventional uniaxial compressive strength is tested with WAW-32000 test system, shown as Fig. 3.

According to the test, specimens appear cleavage fracture, the same with sites' rock mass. The average compressive strength of specimens is 2.65 MPa, meeting the similar scale. Samples' average mechanical parameters are shown in Table 2.

**Similar rock specimens' creep test:** The multi-stage loading uniaxial creep test is simple and can meet the study needs. The initial load is 1.07 kN (21% of ultimate strength), firstly displacement control and then stress control. When the displacement increment is less than 0.001 mm/h, the next load level can be exerted. The duration of load level is determined according to specimen's strain rate or stress rate, here it is about 2 h. During the process, environmental factors within the laboratory are controlled strictly, the room temperature is constant at 25°C, without exceeding  $\pm 1^\circ\text{C}$ .

Similar rock specimens' creep test is done with RLJW-2000 testing machine, the system is shown in Fig. 4 and 5.

Table 3: Uniaxial creep statistics of specimen

| Specimen     | Creep value (mm) |                |                |                |                |                |                |
|--------------|------------------|----------------|----------------|----------------|----------------|----------------|----------------|
|              | 21% $\sigma_c$   | 33% $\sigma_c$ | 45% $\sigma_c$ | 58% $\sigma_c$ | 70% $\sigma_c$ | 82% $\sigma_c$ | 94% $\sigma_c$ |
| Specimen (a) | 0.0020           | 0.0030         | 0.0020         | 0.0040         | 0.0100         | 0.0120         | -              |
| Specimen (b) | 0.0018           | 0.0025         | 0.0030         | 0.0039         | 0.0090         | 0.0110         | -              |
| Specimen (c) | 0.0023           | 0.0028         | 0.0032         | 0.0041         | 0.0118         | 0.0125         | -              |
| Specimen (d) | 0.0021           | 0.0028         | 0.0025         | 0.0036         | 0.0098         | 0.0119         | -              |

Table 4: Sample damage threshold

| Threshold                     | Specimen     |              |              |              |
|-------------------------------|--------------|--------------|--------------|--------------|
|                               | Specimen (a) | Specimen (b) | Specimen (c) | Specimen (d) |
| Damage threshold (kN)         | 4.792        | 4.69         | 4.802        | 4.876        |
| Average damage threshold (kN) |              | 4.79         |              |              |



Fig. 4: RLJW-2000 test system



Fig. 5: Windows interface system

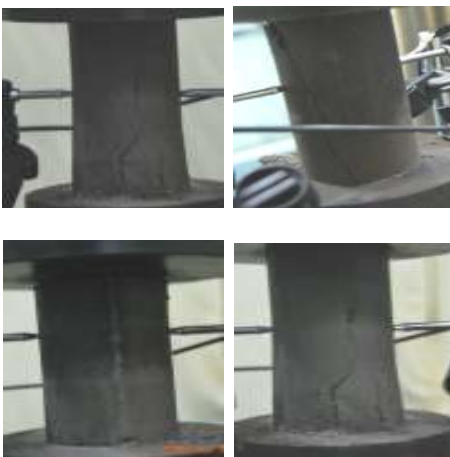


Fig. 6: Creep failure modes of specimen

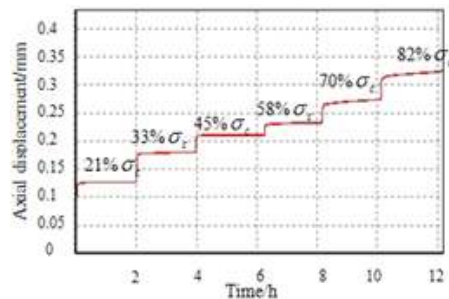


Fig. 7: Axial deformation of specimen (a) vs. time

Four samples are selected for tests at random, the load levels are 21, 33, 45, 58, 70, 82 and 94%  $\sigma_c$  ( $\sigma_c$  is the ultimate strength), respectively. Results are shown in Table 3 and 4.

Samples' failure modes are shown in Fig. 6.

Samples' creep failure modes show that: similar rock specimen appears interface splitting destruction and the degree is serious, which is similar to rock masses' creep damage.

Take sample (a) for an example, creep deformation of the specimen under all levels' load is described in detailed below.

Figure 7 shows that: from 21 to 82%  $\sigma_c$ , it appears only initial creep and steady-state creep and quickly accesses to steady-state after short-time initial phase. When the stress level is low, the creep increment is very small, especially the steady-state creep (nearly zero), the increment increases with increasing stress and when the stress level is closer to the failure threshold (4.792 kN), the increment increases obviously. This creep characteristic is similar to Burgers model, therefore, Burgers model is recommended to simulate the similar rock specimens' creep properties at this linear stage. Once the stress level reaches the damage threshold, creep will rapidly goes into the nonlinear accelerated destruction stage in a short period. Creep properties of this stage should be described by a nonlinear creep model.

**Similar rock specimens' nonlinear creep constitutive:** According to creep tests, CYJ nonlinear creep constitutive is established, which strings Burgers model and a nonlinear part CY. When stress is lower

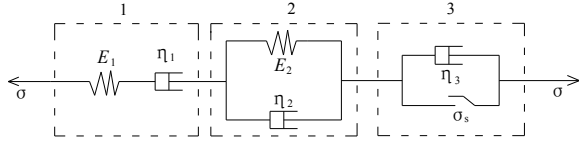


Fig. 8: Non-linear model CYJ

than the failure threshold, CYJ model can be transformed into Burgers model, as shown in Fig. 8.

Where,  $\sigma$  is the stress,  $E$  is the modulus of elasticity,  $\eta$  is the viscosity and  $\sigma_s$  is the stress threshold. Subscript 1 means Maxwell body, subscript 2 means Kelvin body and subscript 3 means the nonlinear element CY.

The first part of CYJ model is Maxwell body, one dimensional constitutive equation:

$$\dot{\varepsilon}_1 = \frac{\dot{\sigma}}{E_1} + \frac{\sigma}{\eta_1} \quad (1)$$

where,  $\dot{\sigma}$ ,  $\dot{\varepsilon}_1$  represent stress rate and strain rate. The three-dimensional differential form of Maxwell:

$$S_{ij}^N = \frac{1}{A} \Delta e_{ij,1} + \frac{B}{A} S_{ij}^O \quad (2)$$

$$A = \frac{1}{2G_1} + \frac{\Delta t}{4\eta_1} \quad (3)$$

$$B = \frac{1}{2G_1} - \frac{\Delta t}{4\eta_1} \quad (4)$$

where,

- $\Delta t$  = The time step
- $\Delta e_{ij,1}$  = The deviator strain increment
- $S_{ij}^N$  = The new deviator stress
- $S_{ij}^O$  = The old deviator stress
- $G_1$  = The shear modulus

The second part is Kelvin body, one dimensional constitutive equation:

$$\sigma = E_2 \varepsilon_2 + \eta_2 \dot{\varepsilon}_2 \quad (5)$$

Through differential derivation and the simplified form of Kelvin:

$$\Delta e_{ij,2} = \frac{\Delta t}{4D\eta_2} (S_{ij}^N + S_{ij}^O) - \frac{2G_2\Delta t}{2\eta_2 + G_2\Delta t} e_{ij,2}^O \quad (6)$$

$$C = 1 - \frac{G_2\Delta t}{2\eta_2} \quad (7)$$

$$D = 1 + \frac{G_2\Delta t}{2\eta_2} \quad (8)$$

where,  $e_{ij,2}^O$  is the old deviator strain.

Burgers model is composed of Maxwell model and Kelvin model in series. So, the three-dimensional differential form of Burgers:

$$S_{ij}^N = \frac{1}{A} (\Delta e_{ij} - \Delta e_{ij,2}) + \frac{B}{A} S_{ij}^O \quad (9)$$

The third part is the nonlinear element CY (Xu *et al.*, 2005). One-dimensional constitutive equation:

$$\varepsilon_3 = \frac{\sigma - \sigma_s}{\eta_3} t^2 \quad (10)$$

Three-dimensional form (Xu *et al.*, 2006):

$$\dot{\varepsilon}_{ij,3} = \frac{tH(F)}{\eta_3} \times \frac{\partial g}{\partial \sigma_{ij}} \quad (11)$$

and

$$F = \sigma_{\max} - \sigma_{\min} N_\phi + 2C\sqrt{N_\phi} \quad (12)$$

$$H(F) = \begin{cases} F, & F > 0 \\ 0, & F \leq 0 \end{cases} \quad (13)$$

$$N_\phi = \frac{1 + \sin \phi}{1 - \sin \phi} \quad (14)$$

$$g = \sigma_{\max} - \sigma_{\min} N_\psi \quad (15)$$

$$N_\psi = \frac{1 + \sin \psi}{1 - \sin \psi} \quad (16)$$

where,

- $\sigma_{\max}$  = The maximum principal stress
- $\sigma_{\min}$  = The minimum principal stress
- $C$  = The cohesion
- $\phi$  = The friction angle
- $\psi$  = The dilation angle
- $F$  = The yield function
- $g$  = The plastic potential function

The three-dimensional differential form:

$$\Delta e_{ij,3}^N = \Delta e_{ij,3}^O + \left[ \frac{H(F)}{\eta_3} \times \frac{\partial g}{\partial \sigma_{ij}} - \frac{H(F)}{\eta_3} \times \frac{\delta_{ij} \times (1 - N_\psi)}{3} \right] \Delta t^2 \quad (17)$$

Therefore, the constitutive equation of CYJ model:

$$S_{ij}^N = \frac{1}{A} (\Delta e_{ij} - \Delta e_{ij,2} - \Delta e_{ij,3}) + \frac{B}{A} S_{ij}^O \quad (18)$$

and

$$\sigma_m^N = \sigma_m^O + 3K\Delta\varepsilon_m \quad (19)$$

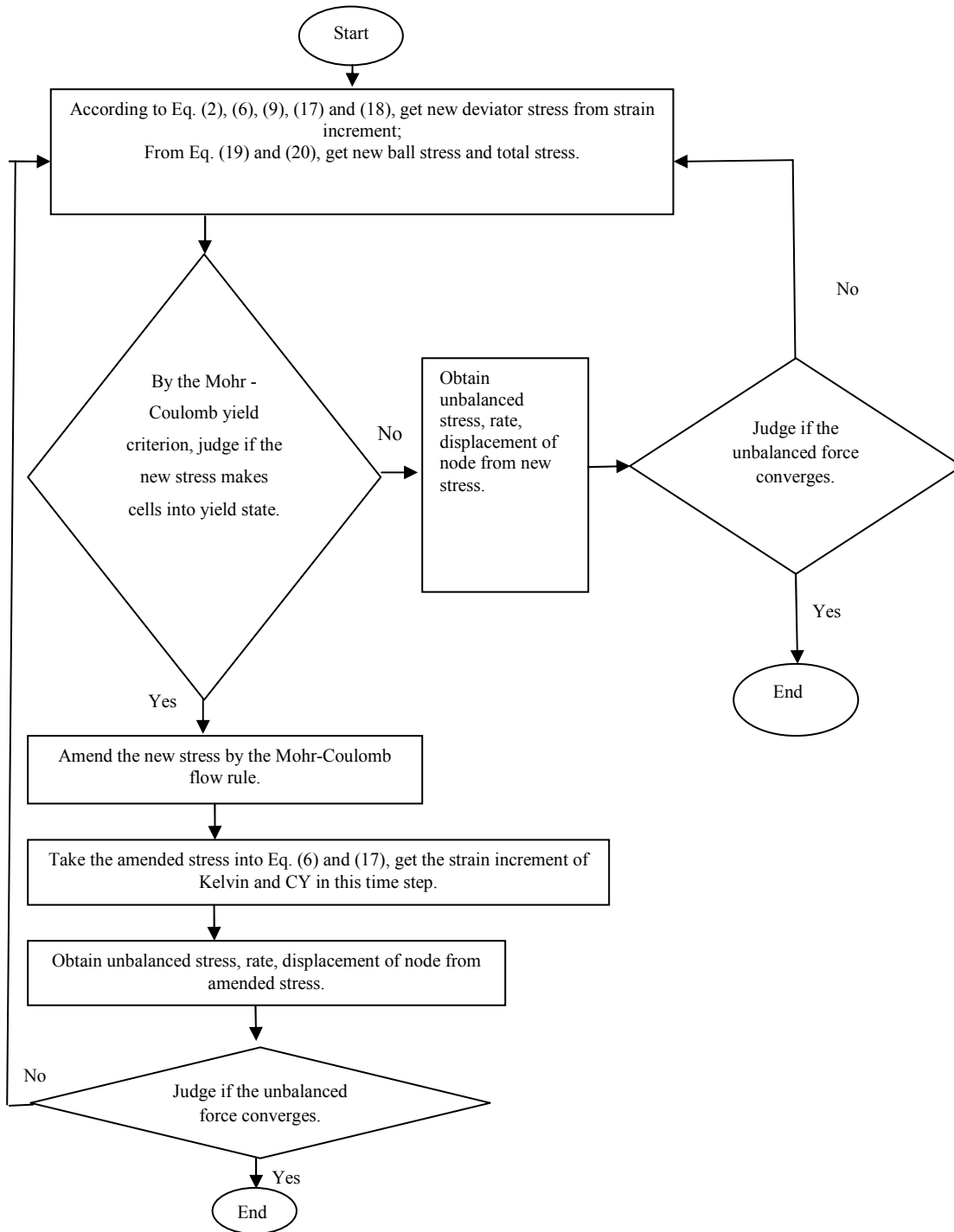


Fig. 9: Programming flow chart of CYJ model

where,

- $\sigma_m^N$  = The new ball stress
- $\sigma_m^o$  = The old ball stress
- $\Delta\varepsilon_m$  = The ball strain increment
- $K$  = The bulk modulus

The total stress is the ball stress add the deviator stress:

$$\sigma_{ij} = \sigma_m \delta_{ij} + S_{ij} \quad (20)$$

From Eq. (3), (4), (6), (7), (8), (9), (17) to (20), the defined nonlinear creep constitutive model YJ can be compiled into dynamic link library (.dll) which can be called by FLAC<sup>3D</sup>.

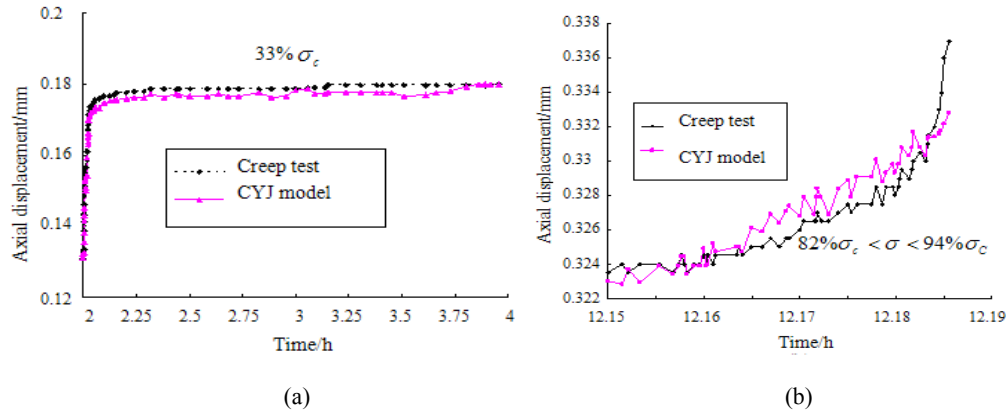


Fig. 10: CYJ model solution vs. test results

The programming flow chart of defined model is shown in Fig. 9.

**Case studies:** Based on the creep test results, the average stress threshold of CYJ model is 4.79 kN ( $2.44 \times 10^6$  Pa), other parameters can be obtained by regression analysis. Compare the CYJ model simulation solution with the test results, as shown in Fig. 10.

Figure 10 shows that nonlinear CYJ model has been nested into FLAC<sup>3D</sup> successfully, numerical simulation results fit test curve well, it can describe this type of similar rock's creep characteristics, which has important safety value for geotechnical engineering.

### CONCLUSION

- The similar rock specimen is made and studied with creep test. Similar rock is as same as real rock: when stress level is lower than the failure threshold, the specimen appears only the initial creep and the steady-state creep; when higher, test records the whole creep process including the nonlinear accelerated phase.
- Based on the similar rock specimen's creep test, nonlinear CYJ model which can be degenerated into Burgers is defined and its constitutive equations are derived, it can describe the three creep stages under certain conditions.
- With FLAC<sup>3D</sup>'s secondary development, CYJ model is nested into FLAC<sup>3D</sup> and has simulated the whole creep behavior, which fits the test results well, CYJ model can be used in FLAC<sup>3D</sup> and has some engineering significance for rock masses' creep.

### ACKNOWLEDGMENT

Project (51139004) supported by the State Key Program of National Natural Science Foundation of China; Project (20110131110030) supported by the Education Ministry Foundation for doctoral program; Project (ZR2009AZ001) supported by the National Natural Science Foundation of Shandong Province;

Project (ZR2011EL049) supported by the National Natural Science Foundation of Shandong Province.

### REFERENCES

- Cazacu, O., J. Jin and N.D. Cristescu, 1997. A new constitutive model for alumina powder compaction [J]. *KONA*, 15: 103-112.
- Chen, X.B., J.S. Zhang and Z.P. Feng, 2007. Experimental study on rheological engineering properties of coarsely granular red sandstone soil [J]. *Chinese J. Rock Mech. Eng.*, 26(3): 601-607, (In Chinese).
- Cristescu, N.D., 1993. A general constitutive equation for transient and stationary creep of rock salt [J]. *Int. J. Rock Mech. Min.*, 30(2): 41-64.
- Dahou, A., J.F. Shao and M. Bederiat, 1995. Experimental and numerical investigations on transient creep of porous chalk [J]. *Mech. Mater.*, 21: 147-158.
- Enrico, M. and Y. Tsutomu, 2001. A non-associated viscoplastic model for the behavior of granite in triaxial compression [J]. *Mech. Mater.*, 33(5): 283-293.
- Lade, P.V., 1994. Creep effects on static and cyclic instability of granular soils [J]. *J. Geotech. Eng.*, 120(2): 404-419.
- Maranini, E. and M. Brignoli, 1999. Creep behavior of a weak rock: Experimental characterization [J]. *Int. J. Rock Mech. Min.*, 36(1): 127-138.
- Okubo, S., Y. Nishimatsu and K. Fukui, 1991. Complete creep curves under uniaxial compression [J]. *Int. J. Rock Mech. Min.*, 28(1): 77-82.
- Shao, J.F. and J.P. Henry, 1991. Development of an elastoplastic model for porous rocks [J]. *Int. J. Plasticity*, 7(1): 1-13.
- Song, F., F.S. Zhao and Y.L. Li, 2005. Testing study on creep properties for gypsum breccias [J]. *Hydrogeol. Eng. Geol.*, 3: 94-96.
- Tai-Tien, W. and H. Tsan-Hwei, 2009. A constitutive model for the deformation of a rock mass containing sets of ubiquitous joints [J]. *Int. J. Rock Mech. Min.*, 46(3): 521-530.

- Xi, B.P., Y.S. Zhao, Z.J. Wan, Z. Jinchang and W. Yi, 2009. Study of constitutive equation of granite rheological model with thermo-mechanical coupling effects [J]. *Chinese J. Rock Mech. Eng.*, 28(5): 956-967, (In Chinese).
- Xu, W., S. Yang and W. Chu, 2006. Nonlinear viscoelasto-plastic rheological model (Hohai Model) of rock and its engineering application. *Chinese J. Rock Mech. Eng.*, 25(3): 433-447, (In Chinese).
- Xu, W.Y., S.Q. Yang, S.Y. Xie and S. Jian-Fu, 2005. Investigation on triaxial rheological mechanical properties of greenschist specimen (II): Model analysis. *Rock Soil Mech.*, 26(5): 693-698.
- Yang, W.D., 2008. Dam soft rock's nonlinear creep damage constitutive model and its application [J]. Shandong University, (In Chinese).
- Yang, C.H., Y. Wang, J.G. Li *et al.*, 2007. Test studies on influence of water content on rock creep law [J]. *J. China Coal Soc.*, 32(7): 695-699, (In Chinese).