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Research Article

Stochastic Analysis of a Parallel System under Maintenance and Repair Subject to Random Shocks

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Abstract: This study presents the stochastic analysis of a two unit parallel system subject to random shocks. There is a single server who visits immediately to do maintenance and repair of the system. The operative units in the system undergo for maintenance if they are affected by the impact of shocks with some probabilities. However, repair of the units is done when they fail due to some other reasons. The unit works as new after maintenance and repair. The distributions of random shocks and failure times of the unit follow negative exponential while that of maintenance and repair times are taken as arbitrary with different probability density functions. Using regenerative point technique and semi-Markov process, several measures of system effectiveness are obtained in steady state. The graphical behavior of MTSF, availability and profit function has been analyzed for particular values of various parameters and costs. Profit of the present model has also been made with the profit of the system model proposed by Malik and Chhillar (2012).

Keywords: Maintenance and repair, parallel system, random shocks

INTRODUCTION

The method of cold standby redundancy has widely been adopted by the researchers including Murari and Goyal (1984) and Singh (1989) while analyzing systems of two or more units not only to attain better reliability but also to reduce the down time of the system. But sometimes cold standby redundancy is not suggestive when shocks occur during operation of the system. In such a situation, the units in the system may be may be operated in parallel mode to share the impact of shocks. It is a known fact that cold standby redundancy is better than parallel redundancy so far as reliability is concerned. But, no research study has been written by the researchers on these types of systems so far in the subject of reliability. However, a little work has been carried out by the authors including Murari and Al-Ali (1988) and Wu and Wu (2011) on the reliability modeling of shock models with the concept of maintenance and repairs. Shocks are the external environmental conditions which cause perturbation to the system, leading to its deterioration and consequent failure. The shocks may be caused by external factors such as fluctuation of unstable electric power, power failure, change in climate conditions, change of operator, etc., or due to internal factors such as stress and strain. Many systems like power generation and automotive industries are vulnerable to damage caused by shock attacking that may occur over the service life. Sometimes, a system may or may not be affected by the impact of shocks and the system may fail due to operation and/or due to random shocks.

Keeping the above observations and practical situations in mind, here stochastic analysis of a two unit parallel system subject to random shocks. There is a single server who visits immediately to do maintenance and repair of the system. The operative units in the system undergo for maintenance if they are affected by the impact of shocks with some probabilities. However, repair of the units is done when they fail due to some other reasons. The unit works as new after maintenance and repair. The distributions of random shocks and failure times of the unit follow negative exponential while that of maintenance and repair times are taken as arbitrary with different probability density functions. Various reliability indices such as transition probabilities, mean sojourn times, Mean Time to System Failure (MTSF), availability, busy period of the server due to repair and maintenance, expected number of maintenance and repair and profit function are evaluated in steady state using semi-Markov process and regenerative point technique. The numerical results giving particular values to various costs and parameters are obtained for MTSF, availability and profit to depict their graphical behavior with respect to shock rate.

MATERIALS AND METHODS

The following are the possible transition states of the system:

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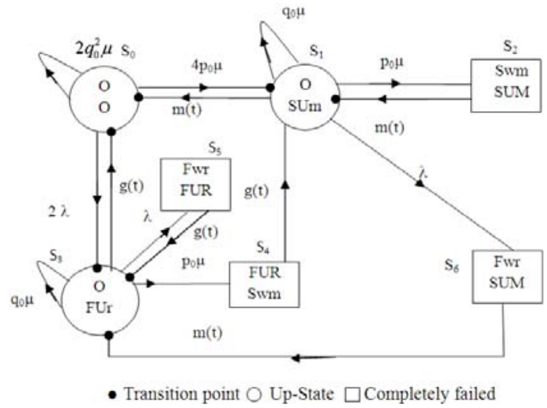

Fig. 1: State transition diagram

S_0 = (O, O), S_1 = (SUm, O), S_2 = (SUM, SWm), S_3 = (O, FUr), S_4 = (FUR, SWm), S_5 = (FUR, FWr), S_6 = (SUM, FWr)

The transition states S_0, S_1, S_3, are regenerative and states S_2, S_4, S_5, S_6 are non-regenerative as shown in Fig. 1.

Transition probabilities and mean sojourn times: Simple probabilistic considerations yield the following expressions for the non-zero elements:

$$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt$$

as:

$$p_{00} = \frac{2q_0^2\mu}{2q_0^2\mu + 2\lambda + 4p_0\mu},\ p_{01} = \frac{4p_0\mu}{2q_0^2\mu + 2\lambda + 4p_0\mu},$$
$$p_{03} = \frac{2\lambda}{2q_0^2\mu + 2\lambda + 4p_0\mu},\ p_{10} = \frac{\theta}{\lambda+\mu+\theta},\ p_{11} = \frac{q_0\mu}{\lambda+\mu+\theta},$$
$$p_{12} = \frac{p_0\mu}{\lambda+\mu+\theta},\quad p_{16} = \frac{\lambda}{\lambda+\mu+\theta},$$
$$p_{30} = \frac{\gamma}{\lambda+\mu+\gamma},\ p_{33} = \frac{q_0\mu}{\lambda+\mu+\gamma},$$
$$p_{34} = \frac{p_0\mu}{\lambda+\mu+\gamma},\ p_{35} = \frac{\lambda}{\lambda+\mu+\gamma}$$

$$p_{21} = m^*(0),\ p_{41} = g^*(0),$$
$$p_{53} = g^*(0),\ p_{63} = m^*(0) \tag{1}$$

For m (t) = $\theta e^{-\theta t}$ and g (t) = $\alpha e^{-\alpha t}$ we have:

$$p_{11.2} = \frac{p_0\mu}{\lambda+\mu+\theta},\ p_{13.6} = \frac{\lambda}{\lambda+\mu+\theta},$$
$$p_{33.5} = \frac{\lambda}{\lambda+\mu+\gamma},\ p_{31.4} = \frac{p_0\mu}{\lambda+\mu+\gamma} \tag{2}$$

It can be easily verified that:

$$p_{00} + p_{01} + p_{03} = p_{10} + p_{11} + p_{12} + p_{16} = p_{21} = p_{30} + p_{33} + p_{34} + p_{35} = p_{30} + p_{33} + p_{33.5} + p_{31.4} = p_{41} = p_{53} = p_{63} = p_{10} + p_{11.2} + p_{11} + p_{13.6} = 1 \tag{3}$$

The mean sojourn times μ_i for m (t) = $\theta e^{-\theta t}$, g (t) = $\alpha e^{-\alpha t}$ are:

$$\mu_0 = \frac{1}{2q_0^2\mu + 2\lambda + 4p_0\mu},\ \mu_1 = \frac{1}{\lambda+\mu+\theta},$$
$$\mu_3 = \frac{1}{\lambda+\mu+\gamma},\ \mu_1' = \frac{\lambda + p_0\mu + \theta}{\theta(\lambda+\mu+\theta)},$$
$$\mu_3' = \frac{\lambda + p_0\mu + \gamma}{\gamma(\lambda+\mu+\gamma)} \tag{4}$$

where,

$$m_{00} + m_{01} + m_{03} = \mu_0,\ m_{10} + m_{11} + m_{12} + m_{16} = \mu_1,$$
$$m_{30} + m_{33} + m_{34} + m_{35} = \mu_3,$$
$$m_{10} + m_{11} + m_{11.2} + m_{13.6} = \mu_1',$$
$$m_{30} + m_{33} + m_{33.5} + m_{31.4} = \mu_3' \tag{5}$$

RESULTS AND DISCUSSION

Reliability and Mean Time to System Failure (MTSF): Let ϕ_i (t) be the cdf of first passage time from regenerative state i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for ϕ_i (t):

ϕ_0 (t) = Q_{00} (t) Ⓢϕ_0 (t) + Q_{01} (t) Ⓢϕ_1 (t) + Q_{03} (t) Ⓢϕ_3 (t)
ϕ_1 (t) = Q_{10} (t) Ⓢϕ_0 (t) + Q_{11} (t) Ⓢϕ_1 (t) + Q_{12} (t) + Q_{16} (t) ϕ_3 (t) = Q_{30} (t) Ⓢϕ_0 (t) + Q_{34} (t) + Q_{35} (t) + Q_{33} (t) Ⓢϕ_3 (t) (6)

Taking LST of above relations (6) and solving for $\tilde{\phi}_0(s)$:
We have:

$$R^*(s) = \frac{1-\tilde{\phi}_0(s)}{s} \tag{7}$$

The reliability of the system model can be obtained by taking Laplace inverse transform of (7).
The Mean Time to System Failure (MTSF) is given by:

$$\text{MTSF} = \lim_{s \to o} \frac{1-\tilde{\phi}_0(s)}{s} = \frac{N_1}{D_1} \tag{8}$$

where,

$N_1 = \mu_0 (1-p_{11}) (1-p_{33}) + \mu_1 p_{01} (1-p_{33}) + \mu_3 p_{03} (1-p_{11})$

and

$D_1 = (1-p_{00}) (1-p_{11}) (1-p_{33}) - p_{01} p_{10} (1-p_{33}) - p_{03} p_{30} (1-p_{11})$

Steady state availability: Let A_i (t) be the probability that the system is in up-state at instant 't' given that the system entered regenerative state i at t = 0.
The recursive relations for A_i (t) are given as:

$A_0(t) = M_0(t) + q_{00}(t) © A_0(t) + q_{01}(t) © A_1(t) + q_{03}(t) © A_3(t)$
$A_1(t) = M_1(t) + q_{10}(t) © A_0(t) + [q_{11}(t) + q_{11.2}(t)] © A_1(t) + q_{13.6}(t) © A_1(t)$
$A_3(t) = M_3(t) + q_{30}(t) © A_0(t) + [q_{33.5}(t) + q_{33}(t)] © A_3(t) + q_{31.4}(t) © A_1(t)$ (9)

where, M_i (t) is the probability that the system is up initially in state $S_i \epsilon E$ is up at time t without visiting to any other regenerative state, we have:

$$M_0(t) = e^{-(\lambda+\mu)t},\ M_1(t) = e^{-(\lambda+\mu)t}\overline{M(t)},$$
$$M_3(t) = e^{-(\lambda+\mu)t}\overline{G(t)} \quad (10)$$

Taking LT of above relations (9) and solving for A_0^* (s), the steady state availability is given by:

$$A_0(\infty) = \lim_{s\to 0} sA_0^*(s) = \frac{N_2}{D_2} \quad (11)$$

where,

$N_2 = [(1 - p_{11.2} - p_{11})(1 - p_{33} - p_{33.5}) - p_{31.4}p_{13.6}]\mu_0 + [p_{01}(1 - p_{33} - p_{33.5}) + p_{31.4}p_{03}]\mu_1 + [p_{01}p_{13.6} + p_{03}(1 - p_{11.2} - p_{11})]\mu_3$

and,

$D_2 = [(1 - p_{11.2} - p_{11})(1 - p_{33} - p_{33.5}) - p_{31.4}p_{13.6}]\mu_0 + [p_{01}(1 - p_{33} - p_{33.5}) + p_{31.4}p_{03}]\mu_1' + [p_{01}p_{13.6} + p_{03}(1 - p_{11.2} - p_{11})]\mu_3'$

Busy period analysis of the server:
Due to repair: Let B^R_i (t) be the probability that the server is busy in repairing of the system at instant t given that the system entered regenerative state i at t = 0. The recursive relation for B^R_i (t) are as follows:

$B^R_0(t) = q_{00}(t) © B^R_0(t) + q_{01}(t) © B^R_1(t) + q_{03}(t) © B^R_3(t)$
$B^R_1(t) = q_{10}(t) © B^R_0(t) + [q_{11}(t) + q_{11.2}(t)] © B^R_1(t) + q_{13.6}(t) © B^R_3(t)$
$B^R_3(t) = W_3(t) + q_{30}(t) © B^R_0(t) + [q_{33.5}(t) + q_{33}(t)] © B^R_3(t) + q_{31.4}(t) © B^R_1(t)$ (12)

where,

$$W_3(t) = e^{-(\lambda+\mu)t}\overline{G(t)} + (\lambda e^{-(\lambda+\mu)t} © 1)\overline{G(t)} + (p_0\mu e^{-(\lambda+\mu)t} © 1)\overline{G(t)} + (q_0\mu e^{-(\lambda+\mu)t} © 1)\overline{G(t)}$$

Now taking L.T. of relations (12) and obtain the value of B^{R*}_0 (s) and by using this, the time for which server is busy in steady state is given by:

$B^R_0 = N_3/D_2$

where,
$N_3 = [p_{10}\ p_{13.6} + p_{03}(1 - p_{11.2} - p_{11})]\, w_3^*(s)$ and D_2 is already defined

Due to maintenance: Let B^M_0 (t) be the probability that the server is busy in maintenance of the system at instant t given that the system entered regenerative state i at t = 0. The recursive relation for B^R_0 (t) are as follows:

$B^M_0(t) = q_{00}(t) © B^M_0(t) + q_{01}(t) © B^M_1(t) + q_{03}(t) © B^M_3(t)$
$B^M_1(t) = W_1(t) + q_{10}(t) © B^M_0(t) + [q_{11}(t) + q_{11.2}(t)] © B^M_1(t) + q_{13.6}(t) © B^M_3(t)$
$B^M_3(t) = q_{30}(t) © B^M_0(t) + [q_{33.5}(t) + q_{33}(t)] © B^M_3(t) + q_{31.4}(t) © B^M_1(t)$ (13)

where,

$$W_1(t) = e^{-(\lambda+\mu)t}\overline{M(t)} + (\lambda e^{-(\lambda+\mu)t} © 1)\overline{M(t)} + (p_0\mu e^{-(\lambda+\mu)t} © 1)\overline{M(t)} + (q_0\mu e^{-(\lambda+\mu)t} © 1)\overline{M(t)}$$

Now taking L.T. of relations (13) and obtain the value of B^{M*}_0 (s) and by using this, the time for which server is busy in steady state is given by:

$B^M_0 = N_4/D_2$

where,
$N_4 = [p_{01}(1 - p_{33} - p_{33.5}) + p_{31.4}p_{03}]\, w_1^*(s)$ and D_2 is already defined

Expected number of maintenance: Let $N_i^M(t)$ the expected number of maintenance of the system by the server in (0, t) given that the system entered the regenerative state i at t = 0. The recursive relations for $N_i^M(t)$ are given as:

$N^M_0(t) = Q_{00}(t) Ⓢ N^M_0(t) + Q_{01}(t) Ⓢ N^M_1(t) + Q_{03}(t) Ⓢ N^M_3(t)$
$N^M_1(t) = Q_{10}(t) Ⓢ (1 + N^M_0(t)) + Q_{11}(t) Ⓢ N^M_1(t) + Q_{11.2}(t) Ⓢ (1 + N^M_1(t)) + Q_{13.6}(t) Ⓢ (1 + N^M_3(t))$
$N^M_3(t) = Q_{30}(t) Ⓢ N^M_0(t) + [Q_{33.5}(t) + Q_{33}(t)] Ⓢ N^M_3(t) + Q_{31.4}(t) Ⓢ N^M_1(t)$ (14)

Now taking L.T. of relations (14) and obtain the value of N^{M*}_0 (s) and by using this, the time for which server is busy in steady state is given by:

$N^M_0 = N_5/D_2$

where,
$N_5 = (p_{10} + p_{11.2} + p_{13.6})\ [p_{01}\ (1 - p_{33} - p_{33.5}) + p_{31.4}p_{03}]$ and D_2 is already defined

Expected number of repairs: Let $N_i^R(t)$ the expected number of repairs of the system by the server in (0, t) given that the system entered the regenerative state i at t = 0. The recursive relations for $N_i^R(t)$ are given as:

$N^R_0(t) = Q_{00}(t) Ⓢ N^R_0(t) + Q_{01}(t) Ⓢ N^R_1(t) + Q_{03}(t) Ⓢ N^R_3(t)$
$N^R_1(t) = Q_{10}(t) Ⓢ N^R_0(t) + Q_{11}(t) Ⓢ N^R_1(t) + Q_{11.2}(t) Ⓢ N^R_1(t) + Q_{13.6}(t) Ⓢ N^R_3(t)$
$N^R_3(t) = Q_{30}(t) Ⓢ [1 + N^R_0(t)] + Q_{33.5}(t) Ⓢ [1 + N^R_3(t)] + Q_{33}(t) Ⓢ N^R_3(t) + Q_{31.4}(t) Ⓢ [1 + N^R_1(t)]$ (15)

Now taking L.T. of relations (15) and obtain the value of $N^{R*}_0(s)$ and by using this, the time for which server is busy in steady state is given by:

$N^R_0 = N_6/D_2$

where,
$N_6 = (p_{30} + p_{33.5} + p_{31.4})\ [p_{03}\ (1 - p_{11} - p_{11.2}) + p_{01}p_{13.6}]$ and D_2 is already defined

Profit analysis: The profit incurred to the system model in steady state can be obtained as:

$$P = K_0A_0 - K_1B_0^M - K_2B_0^R - K_3N_0^M - K_4N_0^R - K_5 \quad (16)$$

where,
K_0 = Revenue per unit up-time of the system
K_1 = Cost per unit time for which server is busy due to maintenance
K_2 = Cost per unit time for which server is busy due to repair
K_3 = Cost per unit maintenance of the shocked unit
K_4 = Cost per unit repair of the failed unit
K_5 = Total cost for the busy of the server and $A_0, B_0^M, B_0^R, N_0^M, N_0^R$ are already defined

Particular cases:

Suppose $g(t) = ge^{-gt}$, $m(t) = \theta e^{-\theta t}$

We can obtain the following results:

MTSF $(T_0) = \frac{N_1}{D_1}$, Availability $(A_0) = \frac{N_2}{D_2}$
Busy period due to repair $(B_0^R) = \frac{N_3}{D_2}$
Busy period due to maintenance $(B_0^M) = \frac{N_4}{D_2}$
Expected number of maintenance $(N_0^M) = \frac{N_5}{D_2}$
Expected number of repairs $(N_0^R) = \frac{N_6}{D_2}$ (17)

where,

$$N_1 = \frac{[(\lambda+\mu+\gamma)(\lambda+\mu+\theta) - q_0\mu(\lambda+\mu+\gamma) - q_0\mu(\lambda+\mu+\theta) + (q_0\mu)^2 + 4p_0\mu(\lambda+\mu+\gamma) - 4q_0p_0\mu^2 + 2\lambda(\lambda+\mu+\theta) - 2\lambda q_0\mu]}{(2q_0^2\mu + 2\lambda + 4p_0\mu)(\lambda+\mu+\gamma)(\lambda+\mu+\theta)}$$

$$D_1 = \frac{\begin{aligned}&[(\lambda+\mu+\gamma)(\lambda+\mu+\theta)(2q_0^2\mu+2\lambda+4p_0\mu) - (\lambda+\mu+\gamma)(\lambda+\mu+\theta)2q_0^2\mu - q_0\mu(\lambda+\mu+\gamma)\\&(2q_0^2\mu+2\lambda+4p_0\mu) + 2q_0^3\mu^2(\lambda+\mu+\gamma) - 2q_0^4\mu^3(\lambda+\mu+\theta) - 4p_0\mu\theta(\lambda+\mu+\gamma) + 4q_0p_0\mu^2\\&-q_0\mu(\lambda+\mu+\theta)(2q_0^2\mu+2\lambda+4p_0\mu) + 2q_0^3\mu^2(\lambda+\mu+\theta) + (2q_0^2\mu+2\lambda+4p_0\mu)(q_0\mu)^2\\&-2\lambda\gamma(\lambda+\mu+\theta) + 2\lambda\mu q_0]\end{aligned}}{(2q_0^2\mu + 2\lambda + 4p_0\mu)(\lambda+\mu+\gamma)(\lambda+\mu+\theta)}$$

$$N_2 = \frac{\begin{aligned}&[(\lambda+\mu+\gamma)(\lambda+\mu+\theta) - \mu(\lambda+\mu+\gamma) - \lambda(\lambda+\mu+\theta) - q_0\mu(\lambda+\mu+\theta)\\&+q_0\mu^2 + \mu\lambda - p_0\mu\lambda + 4p_0\mu(\lambda+\mu+\gamma) - 4q_0p_0\mu^2 + 2\lambda(\lambda+\mu+\theta) - 2\lambda\mu]\end{aligned}}{(2q_0^2\mu + 2\lambda + 4p_0\mu)(\lambda+\mu+\gamma)(\lambda+\mu+\theta)}$$

$$D_2 = \frac{\begin{aligned}&\gamma\theta[(\lambda+\mu+\gamma)(\lambda+\mu+\theta) - \mu(\lambda+\mu+\gamma) - \lambda(\lambda+\mu+\theta) - q_0\mu(\lambda+\mu+\theta)\\&+q_0\mu^2 + \mu\lambda - p_0\mu\lambda] + \gamma(\lambda + p_0\mu + \theta)[4p_0\mu(\lambda+\mu+\gamma) - 4q_0p_0\mu^2 - 2p_0\mu\lambda]\\&+\theta(\lambda + p_0\mu + \gamma)[2\lambda(\lambda+\mu+\theta) - 2\lambda\mu + 2p_0\mu\lambda]\end{aligned}}{\gamma\theta(2q_0^2\mu + 2\lambda + 4p_0\mu)(\lambda+\mu+\gamma)(\lambda+\mu+\theta)}$$

$$N_3 = \frac{[4p_0\mu\gamma + 2\lambda(\lambda+\mu+\theta) - 2\lambda\mu]}{\gamma(2q_0^2\mu + 2\lambda + 4p_0\mu)(\lambda+\mu+\theta)}$$

$$N_4 = \frac{[2q^2{}_0\mu(\lambda+\mu+\gamma) - 2q^3{}_0\mu^2 - 2q^2{}_0\mu\lambda + 2p_0\mu\lambda]}{\theta(2q_0^2\mu + 2\lambda + 4p_0\mu)(\lambda+\mu+\gamma)}$$

$$N_5 = \frac{[2q^2{}_0\mu(\lambda+\mu+\gamma) - 2q^3{}_0\mu^2 - 2q^2{}_0\mu\lambda + 2p_0\mu\lambda]}{(2q_0^2\mu + 2\lambda + 4p_0\mu)(\lambda+\mu+\gamma)} \frac{(\lambda + p_0\mu + \theta)}{(\lambda+\mu+\theta)}$$

$$N_6 = \frac{[4p_0\mu\gamma + 2\lambda(\lambda+\mu+\theta) - 2\lambda\mu]}{(2q_0^2\mu + 2\lambda + 4p_0\mu)(\lambda+\mu+\theta)} \frac{(\lambda + p_0\mu + \gamma)}{(\lambda+\mu+\gamma)}$$

CONCLUSION

In the present study some important reliability measures including MTSF, availability and profit for a two- unit parallel system subject to random shocks have been obtained numerically giving particular values to various costs with $p_0 = 0.6$ and $q_0 = 0.4$. The graphical behavior of these measures as shown in Fig. 2 to 4 go on decline with increase of shock rate (μ) and failure

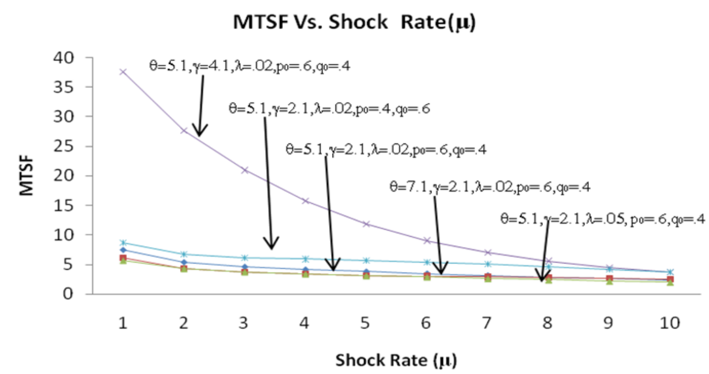

Fig. 2: MTSF vs. shock rate (μ)

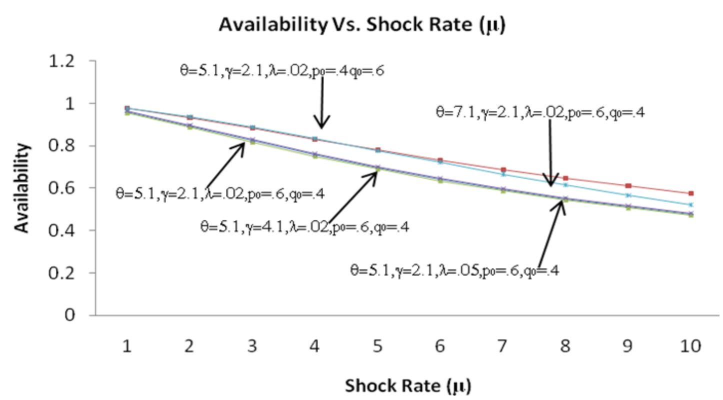

Fig. 3: Availability vs. shock rate (μ)

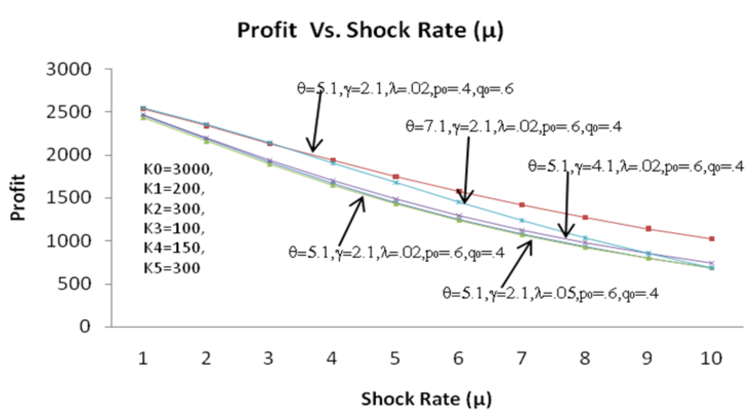

Fig. 4: Profit vs. shock rate (μ)

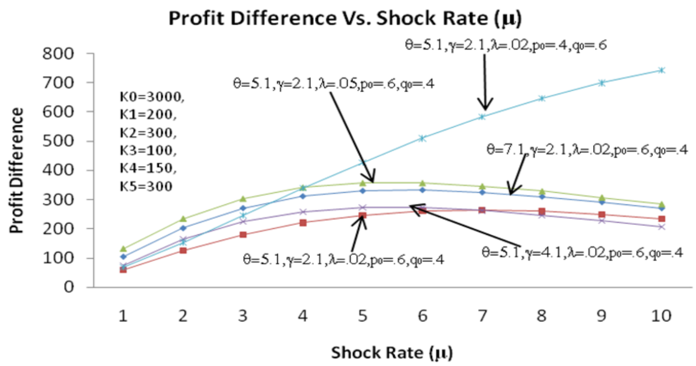

Fig. 5: Profit difference vs. shock rate (μ)

rate (λ). However, there values increase with the increase of repair and maintenance rate. Thus, the study reveals that a parallel system in which operative units subject to random shocks can be made more reliable and profitable to use by doing repair and maintenance of the system with higher rates.

Comparative study: As shown in Fig. 5, the profit of the present model has been compared with the profit of the model proposed by Malik and Chhillar (2012). It is concluded that a two-unit cold standby system subject to random shocks would have more profit as compare to the two unit parallel system.

NOTATIONS

E	: Set of regenerative states
O	: The unit is operative and in normal mode
p_0	: The probability that shock is effective
q_0	: The probability that shock is not effective
μ	: Constant rate of the occurrence of a shock
λ	: Constant failure rate of the unit
m (t) /M (t)	: pdf/cdf of maintenance time of the unit after the effect of a shock
FUr/FWr/FUR	: The Unit is completely failed and under repair/waiting for repair/under Continuous repair from previous state
SUm/SUM	: Shocked unit under maintenance and under maintenance continuously from previous state
SWm	: Shocked unit waiting for maintenance
g (t) /G (t)	: pdf/cdf of repair time of the completely failed unit
q_{ij} (t) /Q_{ij} (t)	: pdf and cdf of direct transition time from a regenerative state i to a regenerative state j without visiting any other regenerative state
$q_{ij.k}$ (t) /$Q_{ij.k}$ (t)	: pdf and cdf of first passage time from a regenerative state i to a regenerative State j or to a failed state j visiting state k once in (0, t)
M_i (t)	: Probability that the system is up initially in state $S_i \in E$ is up at time t without visiting to any other regenerative state
W_i (t)	: Probability that the server is busy in state S_i up to time t without making transition to any other regenerative state or returning to the same via one or more non regenerative states
m_{ij}	: Contribution to mean sojourn time in state S_i when system transits directly to state S_j (S_i, $S_j \in E$) so that $\mu_i = \sum_j m_{ij}$ where $m_{ij} = \int t d\, Q_{ij}(t) = - q_{ij}^{*/}(0)$ and μ_i is the mean sojourn time in state $S_i \in E$
(s) /©	: Symbol for Stieltjes convolution/ Laplace convolution
~/*	: Symbol for Laplace Stieltjes Transform (LST) /Laplace Transform (LT)
/ (desh)	: Symbol for derivative of the function

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