Research Journal of Applied Sciences, Engineering and Technology 7(16): 3395-3404, 2014
DOI:10.19026/rjaset.7.688
ISSN: 2040-7459; e-ISSN: 2040-7467

Submitted: November 04, 2013 Accepted: December 04, 2013 Published: April 25, 2014

Research Article

Thermal Stresses in an Anisotropic Thin Plate Subjected to Moving Plane Heat Sources

[1]Malak Naji, [2]Moh'd Al-Nimr, [2]Eyad Alsyed and [3]Mahir Es-Saheb
[1]Jubail University College, Female Branch, Saudi Arabia
[2]Department of Mechanical Engineering, Jordan University of Science and Technology, Jordan
[3]Department of Mechanical Engineering, King Saud University, Saudi Arabia

Abstract: The aim of this study is to numerically simulate the plane moving heat source through anisotropic mild steal thin plate. Heat conduction problems in anisotropic material, where the thermal conductivity varies with direction and involving a moving heat source have several industrial applications, such like metal cutting, flame or laser hardening of metals, welding and others. The parabolic heat conduction model is used for the prediction of the temperature history. The temperature distribution inside the plate is determined from the solution of heat equation. Thus, the heat equation is solved numerically using finite deference method and the temperature distributions are determined. The thermal stresses in this case are, also, investigated and computed numerically. It is found that the thermal conductivity ratio affect in both temperature and thermal stresses distributions, in additional to the speed and heat source intensity.

Keywords: Anisotropic, heat conduction model, moving plane heat source, parabolic heat conduction model, thermal stresses

INTRODUCTION

Heat conduction problems in anisotropic material have numerous important applications in various branches of science; that is the thermal conductivity varies with direction. Crystals, wood, sedimentary rocks and many others are example of anisotropic materials. Heat conduction problems involving a moving heat source have several applications, such as metal cutting, flame of laser hardening of metals, welding and others.

The early published work on this topic by Al-Huniti *et al*. (2004) investigated the variation of thermal and residual stresses inside a thin mild steel plate during welding process. It was found that welding speed and heat source intensity are the main factors that affect the residual stress formation in the plate. Al-Nimr and Naji (2000) described the thermal behavior of anisotropic material using hyperbolic heat conduction model, which assumed different phase lags between each component of the heat flux vector and the summation of temperature gradient in all direction of the orthogonal coordinate system. Zibdeh and Al Farran (1995) presented a steady-state solution for the thermal stresses of a homogeneous, orthotropic hollow cylinder subjected to asymmetric temperature distribution at the outer surface and heat convection into a medium at zero reference temperature at the inner surface. The results show that the orientation of fibers of each layer affects the distribution of the stresses. Hou and Komanduri (2000) presented general solution for temperature rise at any point due to stationary/moving plane heat source of different shapes and heat intensity distributions (uniform, parabolic, rectangular and normal) using the Jaeger's classical heat source method. The solutions for the stationary heat source were obtained from the moving source solution by simply equating velocity of sliding to zero. The result shows that the temperature rise as well as its distribution around the heat source depends on several factors, including the heat intensity and its distribution, the shape and size of the heat source, the thermal properties and the velocity of sliding. Al-Huniti and Al-Nimr (2001) investigated the dynamic thermal and elastic behavior of a rod due to a moving heat source. The hyperbolic heat conduction model was used to predict the temperature history. Also they presented the effect of different parameters such as moving source speed and the convection heat transfer. Laplace transformer and Riemann-sum approximations were used to determine the temperature, displacement and stresses distribution within the rod. Francis (2002) investigated the simulations of the welding process with moving heat sources for butt and tee joints using finite element analyses. From the transient heat transfer equation he obtained the thermal analysis, followed by a separate mechanical analysis based on the thermal history. Also presented the residual stresses for both butt and tee joints. The results shows for the butt joint that the maximum residual longitudinal normal stress was within 3.6% of published data and for a fully

Corresponding Author: Malak Naji, Jubail University College, Female Branch, Saudi Arabia

transient analysis this maximum stress was within 13% of the published result. And also shows for the tee, the maximum residual stresses were found to be 90-100% of the room-temperature yield strength. Araya (2004) presented a numerical simulation of the temperature field and the removed material resulting from the impingement of a moving laser beam on a ceramic surface. Finite volume was used to predict the temperature field including phase changes generated during the process. The derivation of the energy equation in anisotropic material with its boundary condition is reported by Ozisisik (1993). The same reference offers different analytical technique to solve energy equation. Also, this reference classifies different types of anisotropic materials.

In present work, the plane moving heat source through anisotropic thin plate is simulated numerically. The temperature distribution inside the plate is determined from the solution of heat equation. The heat equation is solved numerically using finite deference method definition implicit scheme. The calculation of thermal stresses based on numerically method.

METHODOLOGY

Analysis:

Formulation of heat equation: Figure 1 shows a schematic diagram of the physical domain for a square thin anisotropic metal plate with moving plane heat source. The plate has a length *L* and thickness *th*. The plane moving heat source has constant speed *u* in x-direction.

Assumptions: The assumption can be summarized as follows:

- Plate is thin ($\frac{th}{L}$ <<1)
- The heat source is a plane heat source
- The speed of moving heat source is constant
- The thermal properties of material under consideration are constant with temperature. Without this assumption, the heat conduction equation nonlinear, Al-Huniti *et al*. (2004)
- Convection and radiation losses are neglected

The plane heat source is moving at a constant speed *u* along *x* direction and is releasing its energy continuously while moving. Therefore, the form of the heat source is given by Ozisisik (1993) as:

$$g(t,x,y) = g_0\delta(x-ut)$$

where,

$g(t, x, y)$ = The volumetric source W/m^3
g_0 = The plane source W/m^2
δ = Dirac delta function $1/m$
t = The time variable

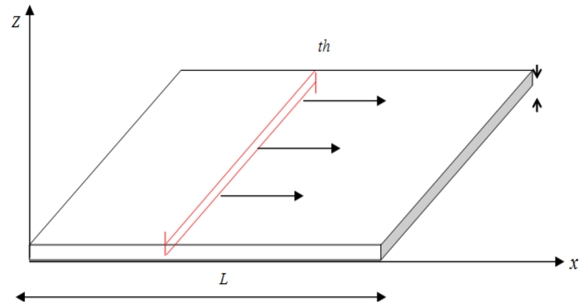

Fig. 1: Thin plate undergoing moving plane heat source

Cartesian coordinate system (x y) is chosen fixed to the work piece ($\frac{th}{L}$ <<1) so the z-axis can be neglected. The transient heat conduction equation with heat generation in Cartesian coordinates for a thin plate can be expressed as follow (Ozisisik, 1993):

$$\rho C\frac{\partial T}{\partial t} = k_{11}\frac{\partial^2 T}{\partial x^2} + k_{22}\frac{\partial^2 T}{\partial y^2} + 2k_{12}\frac{\partial^2 T}{\partial xy} - \frac{hu}{w}(T - T_\infty) - \frac{h1}{w}(T - T_\infty) + g_0\delta(x-ut) \quad (1)$$

where,

ρ = The mass density (kg/m^3)
C = The specific heat (J/kg.k)
k = The thermal conductivity (W/m.k)
T = Temperature at any location (K)
$h1$ = Lower convection heat transfer coefficient (W/m^2.K)
hu = The upper convection heat transfer coefficient (W/m^2. K)
T_∞ = The ambient temperature (K)
u = The velocity of moving heat source (m/s)

Let $h1 = hu = h$ then the equation becomes:

$$\rho C\frac{\partial T}{\partial t} = k_{11}\frac{\partial^2 T}{\partial x^2} + k_{22}\frac{\partial^2 T}{\partial y^2} + 2k_{12}\frac{\partial^2 T}{\partial xy} - 2\frac{h}{w}(T - T_\infty) + g_0\delta(x-ut) \quad (2)$$

Initial and boundary condition: Initially, the plate temperature is assumed to be uniform and equal to ambient temperature. Hence, the initial and boundary condition are:

$$\begin{aligned} T(x,y,0) &= T_\infty \\ T(0,y,t) &= T_\infty \\ T(L,y,t) &= T_\infty \\ T(x,0,t) &= T_\infty \\ T(x,L,t) &= T_\infty \end{aligned} \quad (3)$$

The heat conduction equation for anisotropic equation with moving line heat source (2) will be transformed into dimensionless parameter form. For this, the following dimensionless parameters are introduced:

$$\theta = \frac{T - T_\infty}{T_\infty},\ \varsigma = \frac{x}{L}, \eta = \frac{y}{L},\ \tau = \frac{t}{t_0}$$
$$\gamma = \frac{L^2}{wk_{11}}(2h),\ G_0 = \frac{g_0 L}{T_\infty k_{11}},\ U = \frac{ut_0}{L},\ t_0 = \frac{L^2 \rho C}{k_{11}}$$
$$\varepsilon_{22} = \frac{k_{22}}{k_{11}},\ \varepsilon_{12} = \frac{k_{12}}{k_{11}}$$

Substituting these parameters in Eq. (2) gives:

$$\frac{\partial\theta}{\partial\tau} = \frac{\partial^2\theta}{\partial\varsigma^2} + \varepsilon_{22}\frac{\partial^2\theta}{\partial\eta^2} + 2\varepsilon_{12}\frac{\partial^2\theta}{\partial\varsigma\eta} - \gamma\theta + G_0\delta(\varsigma - U\tau) \qquad (4)$$

The dimensionless initial and boundary conditions are:

$$\theta(\varsigma,\eta,0) = 0$$
$$\theta(0,\eta,\tau) = 0$$
$$\theta(L,\eta,\tau) = 0$$
$$\theta(\varsigma,0,\tau) = 0$$
$$\theta(\varsigma,L,\tau) = 0 \qquad (5)$$

The Following a procedure to solve the governing equations by implicit method, which transformed to algebraic equations as shown below:

$$\frac{\theta_{i,j}^{n+1} - \theta_{i,j}^{n}}{\Delta\tau} = \frac{\theta_{i+1,j}^{n+1} - 2\theta_{i,j}^{n+1} + \theta_{i-1,j}^{n+1}}{\Delta\varsigma^2} + \varepsilon_{22}\frac{\theta_{i,j+1}^{n+1} - 2\theta_{i,j}^{n+1} - \theta_{i,j-1}^{n+1}}{\Delta\eta^2} + \qquad (6)$$
$$2\varepsilon_{12}\frac{\theta_{i+1,j+1}^{n+1} - \theta_{i+1,j-1}^{n+1} - \theta_{i-1,j+1}^{n+1} + \theta_{i-1,j-1}^{n+1}}{4\Delta\varsigma\eta} - \gamma\theta_{i,j}^{n+1} + G_0\delta(\varsigma - U\tau)$$

Let $\Delta\varsigma = \Delta\eta$ arranging the above Eq. (6) gives the final solutions for parabolic heat equation:

$$(2A + 2A\varepsilon_{22} + \gamma\Delta\tau + 1)\theta_{i,j}^{n+1}$$
$$- A(\theta_{i+1,j}^{n+1} + \theta_{i-1,j}^{n+1}) - A\varepsilon_{22}(\theta_{i,j+1}^{n+1} + \theta_{i,j-1}^{n+1}) \qquad (7)$$
$$- A\frac{\varepsilon_{12}}{2}(\theta_{i+1,j+1}^{n+1} - \theta_{i+1,j-1}^{n+1} - \theta_{i-1,j+1}^{n+1} + \theta_{i-1,j-1}^{n+1})$$
$$= \theta_{i,j}^{n} + G_0\delta(\varsigma - U\tau)\Delta\tau$$

where,

$$A = \frac{\Delta\tau}{\Delta\varsigma^2}$$

Thermal stresses: The thermal stresses present in many branches of engineering, considered an important factors affect on the life of material, so that become a very significant in application involving large temperature difference and they important to determine the life of material, when the temperature rise in a homogeneous body, different element of body tend to expand by different amount, an amount proportional to the local temperature raise. The thermo-elastic formulation of the deflection and stress in thin plate under the effect temperature field requires the reformulation of the classical stress-strain relations. Thin plate subjected to the moving plane heat source. As a result, the transient temperature is varying in thin plate.

The plate is simply supported at the four edges, homogeneous, of uniform thickness, isotropic and behaves elastically at all times and the plane stress condition exits in the plate. The stress-strain-temperature relations are (Zibdeh and Al Farran, 1995):

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma xy \end{Bmatrix} = -z\frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 1-\nu \end{bmatrix}\begin{Bmatrix} \frac{\partial^2 w}{\partial x^2} \\ \frac{\partial^2 w}{\partial y^2} \\ \frac{\partial^2 w}{\partial xy} \end{Bmatrix} - \frac{E\alpha}{1-\nu}\begin{bmatrix} T - T_\infty \\ T - T_\infty \\ 0 \end{bmatrix} \qquad (8)$$

where,
w = The transverse deflection of mid-plane of plate
E = Young's modulus
ν = Poisson's ratio
α = The coefficient of thermal expansion

The equation of motion of plate as result of a time dependent temperature filed (Zibdeh and Al Farran, 1995) is given by:

$$D\nabla^4 w + \rho th\frac{\partial^2 w}{\partial t^2} = -\frac{\nabla^2 M_T}{1-\nu} \qquad (9)$$

where, D is the bending rigidity of the plate, given by:

$$D = \frac{Eth^3}{12(1-\nu^2)} \qquad (10)$$

M_T is the thermal moment define as (Zibdeh and Al Farran, 1995):

$$M_T = \alpha E\int_{-th/2}^{th/2}(T - T_\infty)zdz \qquad (11)$$

The boundary condition for simply-supported plate along the four edges are given by Al-Nimr and Al-Huniti (2000) as:

$$w(0, y, t) = w(a, y, t) = 0$$
$$w(x, 0, t) = w(x, a, t) = 0$$
$$D\frac{\partial^2 w(0,y,t)}{\partial x^2} + \frac{M_T}{(1-\nu)} = D\frac{\partial^2 w(L,y,t)}{\partial x^2} + \frac{M_T}{1-\nu} = 0$$
$$D\frac{\partial^2 w(x,0,t)}{\partial y^2} + \frac{M_T}{1-\nu} = D\frac{\partial^2 w(x,L,t)}{\partial y^2} + \frac{M_T}{1-\nu} = 0 \qquad (12)$$

It is assumed that the plate is initially at rest i.e., in the reference position, so the initial conditions as given by Al-Nimr and Al-Huniti (2000) are:

$$w(x, y, 0) = \frac{\partial w(x, y, 0)}{\partial t} = 0 \tag{13}$$

When the thermal load is applied to the thin plate during moving plane heat source, it generates thermal moments along the thickness of thin plate, but the thermal load is uniform in z-direction, then (T-T_∞) is independent of z. Therefore, the thermal moment integral at Eq. (11) becomes zero. The thermal moment zero this means no deflection occurs in the plate. It can be concluded that the temperature gradient in the plate is the main contribution to thermal stresses, from Eq. (8) the stresses generations in the plate becomes:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = -\frac{E\alpha}{1-\nu}\begin{bmatrix} T - T_\infty \\ T - T_\infty \\ 0 \end{bmatrix} \tag{14}$$

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma xv \end{Bmatrix} = -\frac{E\gamma}{1-\nu}\begin{bmatrix} T - T_{in} \\ T - T_{in} \\ 0 \end{bmatrix} \tag{15}$$

Solutions: After applying finite difference we have a system of linear algebraic Eq. (7). To solve algebraic equation we should create grids (steps) for the time and space (ς, η), let us divided the length of the plate in to equal space $\Delta\zeta = \Delta\eta = 0.1$ and for dimensionless time $\Delta\tau = 1.3888888889e-3$, then linear algebraic Eq. (7) can be written as matrix equation:

$$\Psi\theta = b \tag{16}$$

So that, in each time step we have a matrix to be solved. The matrix inverse method is used to determine the dimensionless temperature distribution in each time step. Hence, when the matrix inverse is known ψ^{-1}, the solution θ is simply the product of the matrix inverse ψ^{-1} and the right hand side vector b as shown in equation below:

$$\theta = \Psi^{-1} b \tag{17}$$

Evaluate the thermal stresses in the plate can be computed from Eq. (14) after the history of temperature are determined.

RESULTS AND DISCUSSION

Equation (7) represented the dimensionless solution of the parabolic heat equations of moving line; afford the transient dimensionless temperature variation with space and time. The property of material will be used (mild steal), reported by Al-Huniti *et al*. (2004), are given in Table 1.

The ratio of thermal conductivity (ε_{22}, ε_{12}) will be change during the solution to understand the effect of

Table 1: Properties of mild steal

Mass density (ρ)	7800 kg/m^3
Specific heat (C)	450 J/kg°C
Thermal conductivity (K_{11})	65 W/m°k
Plate length (L)	1 m
Plate thickness (th)	0.005 m
Velocity of moving heat source (u)	0.001333 m/s
Plate initial and ambient temperature (T_∞)	300 K
Convection heat transfer coefficient (from steal to air) (h)	15 W/m^{2}K
Modula's of elasticity	1.99e11 pa
Coefficient thermal expansion α	1.06355e-5 K^{-1}
Heat source intensity (G)	1500

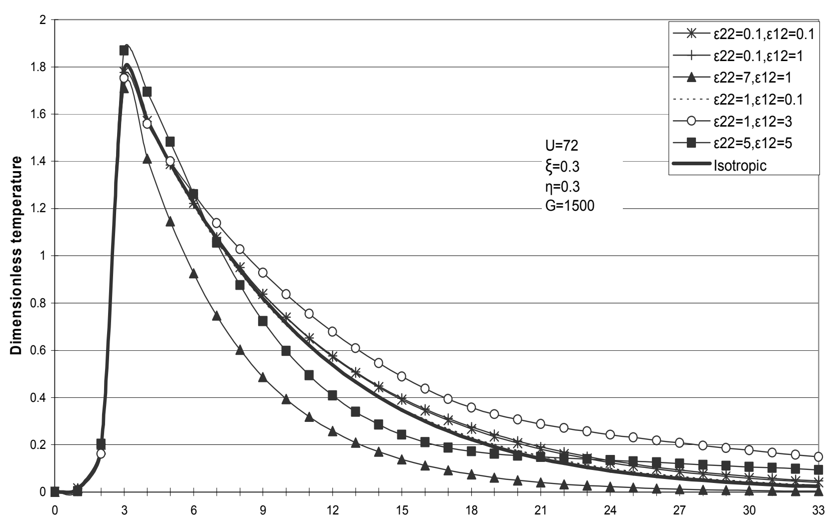

Fig. 2: Temperature distributions for anisotropic material (different cases)

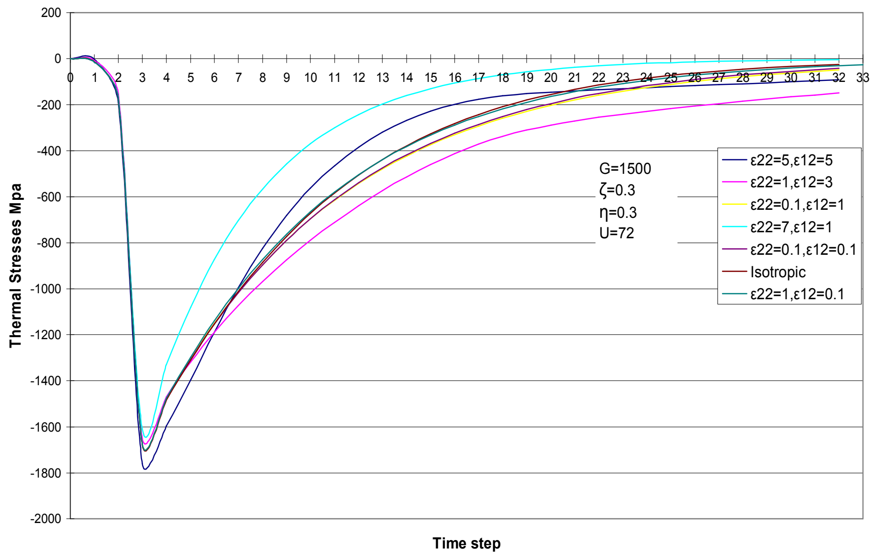

Fig. 3: Thermal stresses for anisotropic material (different case)

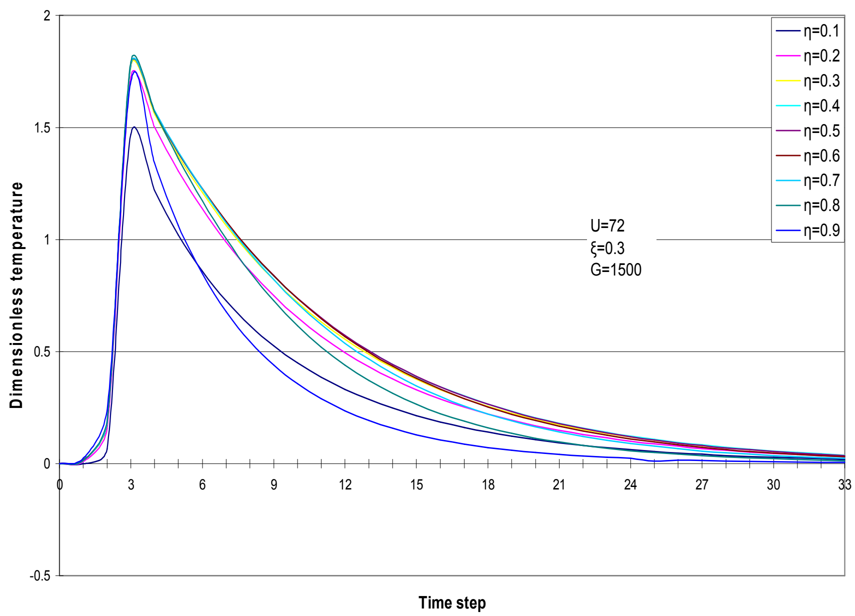

Fig. 4: Temperature distributions along direction for isotropic

anisotropic on the temperature and thermal stresses distribution at moving plane heat source.

Figure 2 shows the dimensionless temperature variation with time for specific point ($\varsigma = 0.3$, $\eta = 0.3$) for different ratio of thermal conductivity (ε_{22} ε_{12}), during moving plane heat source. It is clear from figure that the ratios of thermal conductivity have the main effect on the behavior temperature distribution. Therefore, increase the ratio of thermal conductivity (ε_{22} ε_{12}) let the cooling processes within the plate occurred slowly. In the other hand decrease the ratio of thermal conductivity (ε_{22} ε_{12}) let the cooling process within the plate occurred rapidly. Where Fig. 3 show the thermal stresses at this point. It's clear from figure the behavior of thermal stresses follow the behavior of temperature distribution.

Figure 4 to 6 shows the dimensionless temperature variation with time along the longitudinal line along $\varsigma = 0.3$ at different ratio of thermal conductivity (ε_{22} ε_{12}), during moving plane heat source. It is a clear from Fig. 5 the temperature distribution became more uniform along longitudinal line when the ratio of thermal conductivity decrease while in other hand when the ratio of thermal conductivity increase the temperature distribution near the edges less than other's there are due increase the ratio of conductivity and the conductivity for the plate increase and became

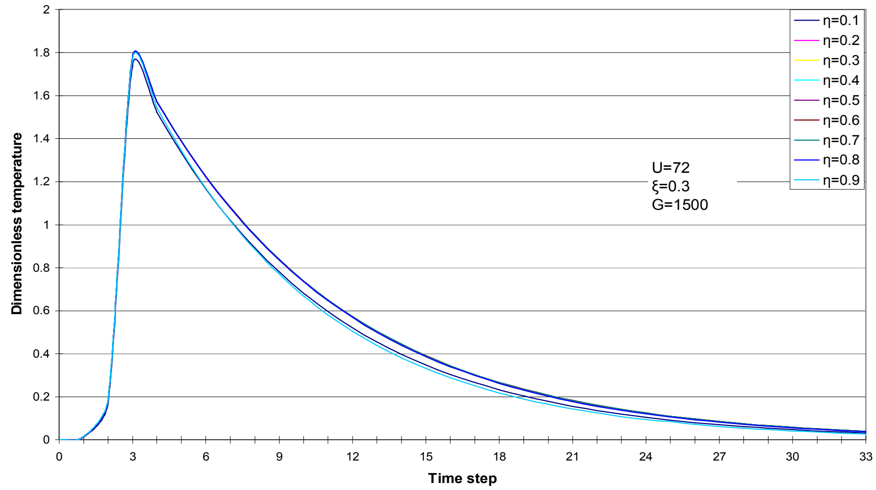

Fig. 5: Temperature distributions along direction for anisotropic ($\varepsilon_{22} = 0.1$, $\varepsilon_{12} = 0.1$)

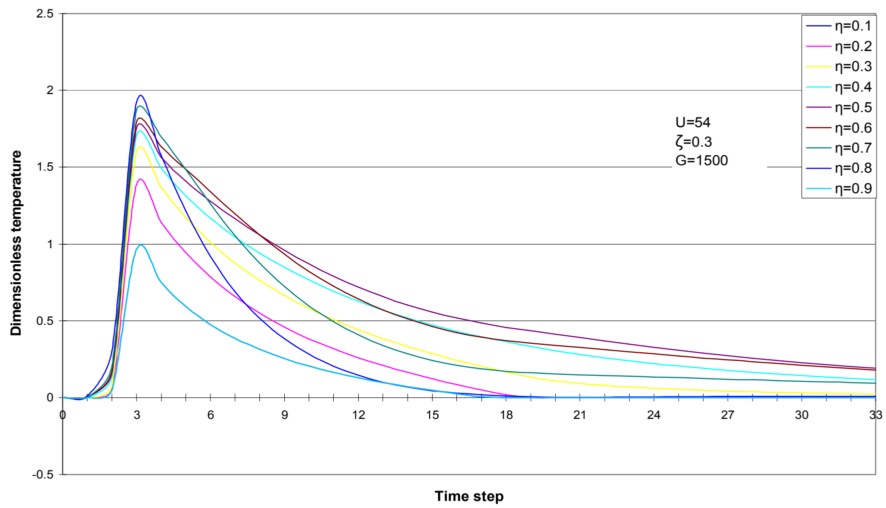

Fig. 6: Temperature distributions along direction ($\varepsilon_{22} = 5$, $\varepsilon_{12} = 5$)

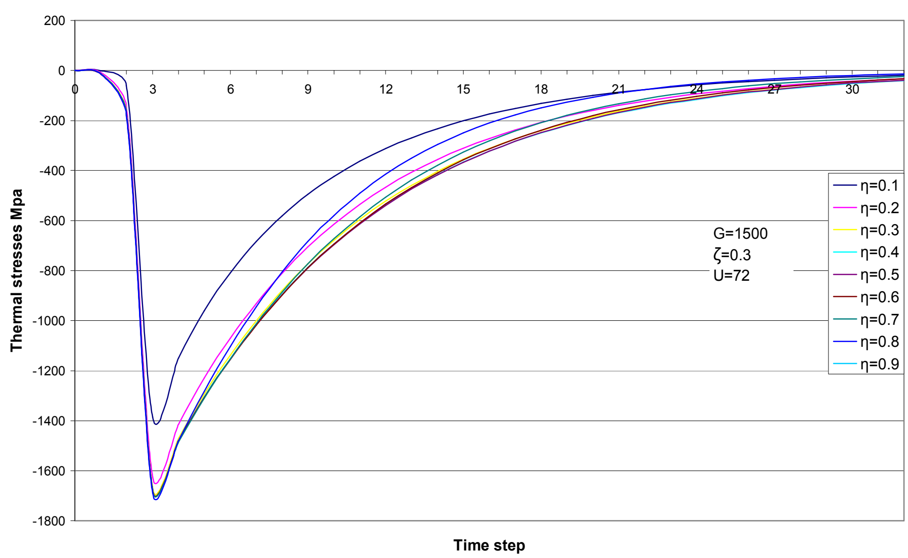

Fig. 7: Thermal stresses along direction isotropic

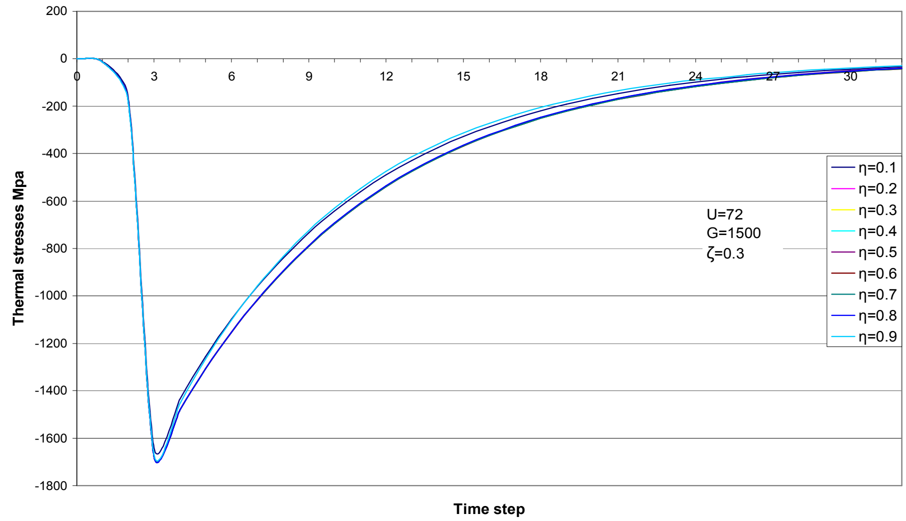

Fig. 8: Thermal stresses along direction ($\varepsilon_{22} = 0.1$, $\varepsilon_{12} = 0.1$)

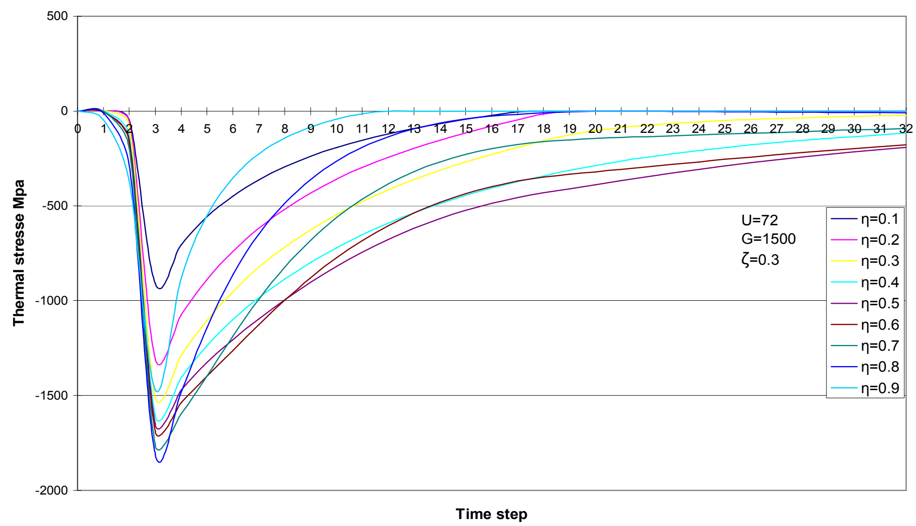

Fig. 9: Thermal stresses along direction ($\varepsilon_{22} = 5$, $\varepsilon_{12} = 5$)

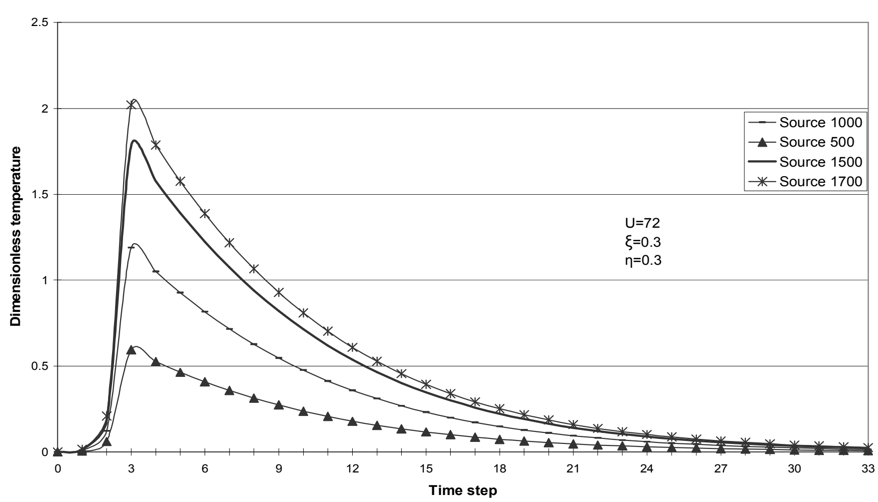

Fig. 10: Temperature various with different heat source for isotropic

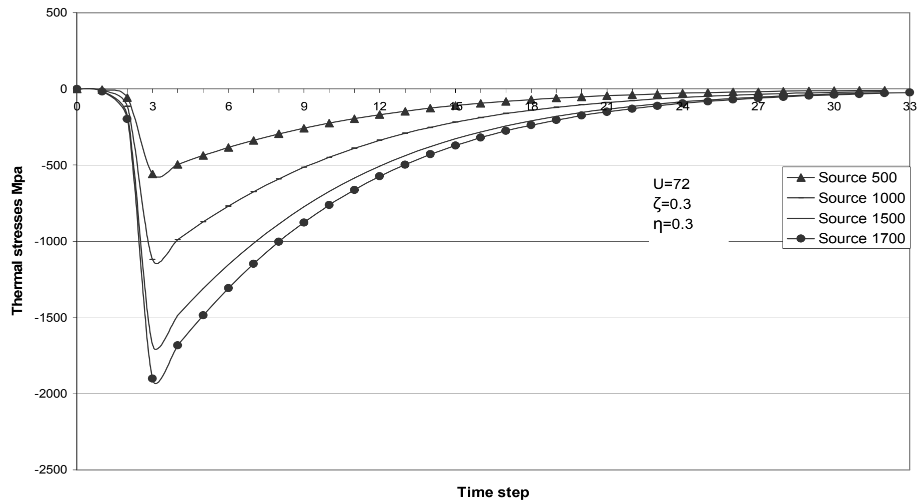

Fig. 11: Thermal stress for isotropic with different heat source

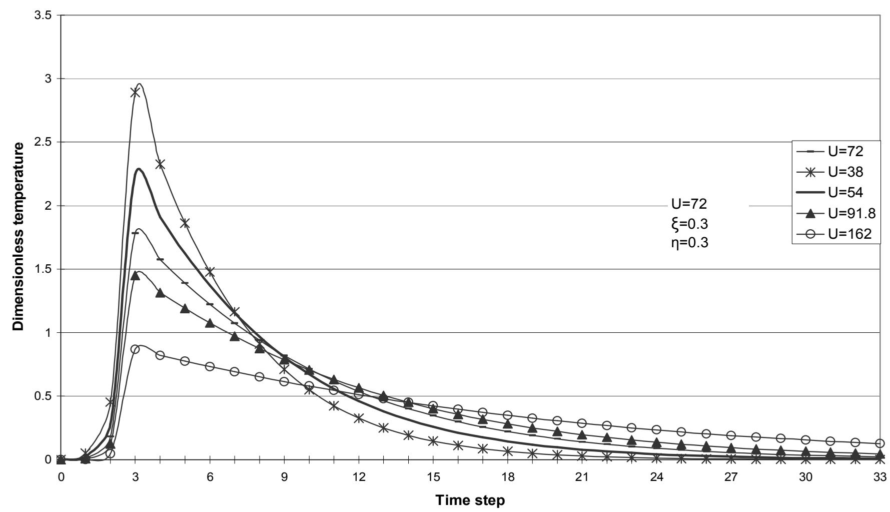

Fig. 12: Temperature various with different heat source velocity for isotropic

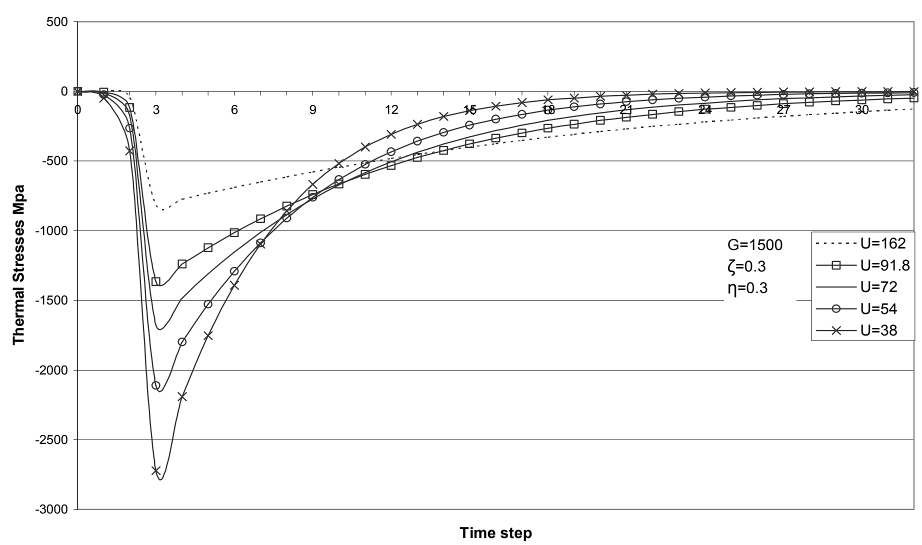

Fig. 13: Thermal stresses with different velocity for isotropic

more abler to passing the heat through the plate. Where Fig. 7 to 9 illustrate the behavior of thermal stresses distribution for these cases. It's clear from figures the behaviors of thermal stresses are compressive and follow the behaviors of temperature distribution.

Figure 10 shows the transient thermal behavior of specific point: (ς = 0.5, η = 0.7) at different values of heat source input, namely for G = 500, 1000, 1500 and 2100. It is illustrated the heat intensity increase, higher temperature distributions will be given, due to the more heat reached the plate, this situation for all cases for anisotropic. Where Fig. 11 show the transient thermal stresses of this point at different values of heat sources.

It is illustrated the thermal stresses intensity increase, due higher temperature distributions that given due to the more heat reached the plate. This situation for all cases for anisotropic.

Figure 12 shows the effect of changing the moving heat source speed on thermal cycle. Five moving heat source dimensionless speed are taken into consideration: U = 36, 54, 72, 91.8 and 160. From figure illustrate that is the moving heat source speed decrease, higher temperature value will be reached inside the plate, this mean more amount of power delivered to the plate per unit time. This situation for all cases for anisotropic. Where Fig. 13 show the effect of changing the moving heat source speed on the thermal stresses. From figure illustrate that when moving heat source decrease, higher thermal stresses value will be reached inside the plate, due to increase the temperature this mean more amount of power delivered to the plate per unit time. This situation for all cases for anisotropic.

CONCLUSION

The thermal stresses in thin anisotropic plate involving a moving plane heat source are presented. The parabolic heat conduction model is used to evaluate the thermal behavior of thin anisotropic plate. The governing equation is derived and solved using finite difference method by implicit scheme. The effect of dimensionless ratio of thermal conductivity and moving heat source value or speed were studied. The ratios of thermal conductivity are found have the main effect on the behavior of temperature and thermal stresses distribution in the plate. Therefore, increase the ratios of thermal conductivity are found the cooling processes within the plate occurred slowly. And also, decreases the ratio of thermal conductivity are found the cooling process within the plate occurred rapidly. The thermal stresses in the plate are found compressive and follow the behavior of temperature.

The temperature and thermal stresses of the plate are found to increase at small moving heat source speed. This due to fact that decrease the speed means the source release more amount of energy. Decrease of the value of moving heat source are found to decrease the temperature and thermal stresses. This is due to the fact decrease the value means the mount of energy reached to the plate will be reduce.

NOMENCLATURE

L	:	Plate side length, m
C	:	Specific heat, J/m^3 k
g, g_0	:	Heat source W/m^2
E	:	Young's modulus, N/m^2
th	:	Plate thickness, m
k	:	Thermal conductivity, W/m k
ε_{12}, ε_{22}	:	Ratio of thermal conductivity in xy and y direction
q	:	Heat flux vector, W/m^2
T_∞	:	Plate initial and ambient temperature, K
T	:	Temperature, k
t	:	Times
u	:	Moving plane heat source speed
w	:	Transverse deflection of plate
G	:	Dimensional heat source
h_l, h_u	:	Lower and upper surface heat convection coefficients

Greek symbols

α	:	Coefficient of thermal expansion
τ	:	Dimensionless time
ς	:	Dimensionless coordinate in longitudinal direction
η	:	Dimensionless coordinate transverse directions
Θ	:	Dimensionless temperature
σ	:	Thermal stresses
ρ	:	Mass density, kg/m^3
δ	:	Unit step function

ACKNOWLEDGMENT

The Authors extend their appreciation to the Deanship of Scientific Research at King Saud University for funding the work through the research group project No. RGP-VPP-036.

REFERENCES

Al-Huniti, N.S. and M.A. Al-Nimr, 2001. Dynamic response of a rod due to a moving heat source under hyperbolic heat conduction model. J. Sound Vib., 242(4): 629-640.

Al-Huniti, N.S., M.A. Al-Nimr and M.A. Da'as, 2004. Transiant variations of thermal stresses within a thin plate during welding process. J. Therm. Stresses, 27: 671-689.

Al-Nimr, M.A. and M. Naji, 2000. The hyperbolic heat condition equation in anisotropic material. Int. J. Thermophys., 21(1).

Al-Nimr, M.A. and N.S. Al-Huniti, 2000. Transient thermal stresses in thin elastic plate due to a rapid dual-phase-lag heating. J. Therm. Stresses, 23: 731-746.

Araya, J.G., 2004. Transient, Three-dimension numerical model of laser cutting process in ceramics with phase change consideration. M.Sc Thesis, Department of Mechanical Eng., College of Eng., University of Puerto Rico.

Francis, J.D., 2002. Welding simulations of aluminum alloy joints by finite element analysis. M.Sc. Thesis, Department of Aerospace, College of Eng., University of the Virginia Polytechnic Institute and State University.

Hou, Z.B. and R. Komanduri, 2000. General solution for stationary/moving plane heat source problem in manufacturing and tribology. Int. J. Heat Mass Tran., 43: 1679-1698.

Ozisisik, M., 1993. Heat Conduction. 2nd Edn., Wiley, New York.

Zibdeh, H.S. and J.M. Al Farran, 1995. Stresses analysis in composite hollow cylinders due to an asymmetric temperature distribution. J. Press. Vess-T. ASME, 117(1): 59-65.