Research Article
A Novel Scheme for Reduction of Torque and Speed Ripple in Rotor Field Oriented Control of Single Phase Induction Motor Based on Rotational Transformations

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Abstract: This study discusses about an exact and novel scheme for Rotor Field Oriented Control (RFOC) method in single phase Induction Motors (IMs) with two different main and auxiliary stator windings. In the presented technique, rotational transformations are introduced and applied to the single phase IM equations. It is proved by using these proposed rotational transformations, asymmetrical equations of single phase IM changed into symmetrical equations. As a result, with some modifications in the conventional field oriented controller, vector control of single phase IM can be done. Results are presented to show the good performance of the presented technique.

Keywords: Exact and novel scheme, RFOC, rotational transformations, single phase IM

INTRODUCTION

Single phase Induction Motors (IMs) are broadly used in the low power domestic and industrial applications where a three phase AC supply is not available. They employ in washers, fans, vacuum cleaners, compressors and many other applications. Generally, single phase IMs are categorized to: shaded-pole motors, split phase motors, capacitor-start/run motors and two capacitor motors. In this study, single phase IM without startup and running capacitors is investigated. Actually single phase IM is considered as an asymmetric two phase IM. In the recent years, adjustable speed control of single phase IMs has been aroused among researchers in literature (Azza *et al.*, 2011; Blaabjerg *et al.*, 2004; Chomat and Lipo, 2003; De Rossiter Corrêa *et al.*, 2000, 2004; Jang, 2013; Jemli *et al.*, 2009a; Rajaei *et al.*, 2010; Reicy and Vaez-Zadeh, 2005; Vaez-Zadeh and Harooni, 2005; Zahedi and Vaez-Zadeh, 2009).

Field Oriented Control (FOC) is one of the accepted techniques for controlling single phase IMs. One of the drawbacks in the presented FOC method by researchers for single phase IMs is that, with supposition of $L_{qs}/L_{ds} = (M_q/M_d)^2 = a^2$, coupling between stator main and auxiliary inductance in single phase IMs is not considered (Azza *et al.*, 2011; de Rossiter Corrêa *et al.*, 2000, 2004; Jang, 2013; Jemli *et al.*, 2009b; Reicy and Vaez-Zadeh, 2005; Vaez-Zadeh and Harooni, 2005). Using of this assumption is reflected in the speed and hence the torque responses of the drive system.

This study shows the single phase IM can be modeled as a balanced circuit. Based on this simplifying, the transformation matrixes are achieved. In this note, by using these proposed transformation matrixes and without assumption of $L_{qs}/L_{ds} = (M_q/M_d)^2 = a^2$, we obtain the balanced equations for single phase IM. By using this similarity, a new and exact RFOC method for single phase IM control is presented. By applying a few modifications on the conventional FOC method for balanced IM, vector control of single phase or unbalanced two phase IMs can be done. The presented method is analyzed and verified by using computer simulations based on the experimental values of the inputs.

The single phase IM model: The equations of single phase IM with unequal stator windings in a stator reference frame (superscript "*s*") can be shown as following equations:

$$v_{ds}^s = r_{ds}i_{ds}^s + \left(d\lambda_{ds}^s / dt\right), \; v_{qs}^s = r_{qs}i_{qs}^s + \left(d\lambda_{qs}^s / dt\right)$$

$$0 = r_r i_{dr}^s + \left(d\lambda_{dr}^s / dt\right) + \omega_r \lambda_{qr}^s, 0 = r_r i_{qr}^s + \left(d\lambda_{qr}^s / dt\right) - \omega_r \lambda_{dr}^s$$

$$\lambda_{ds}^s = L_{ds}i_{ds}^s + M_d i_{dr}^s, \; \lambda_{qs}^s = L_{qs}i_{qs}^s + M_q i_{qr}^s$$

$$\lambda_{dr}^s = M_d i_{ds}^s + L_r i_{dr}^s, \; \lambda_{qr}^s = M_q i_{qs}^s + L_r i_{qr}^s$$

$$\tau_e = \left(Pole/2\right)(M_q i_{qs}^s i_{dr}^s - M_d i_{ds}^s i_{qr}^s)$$

$$\left(Pole/2\right)(\tau_e - \tau_l) = J\left(d\omega_r/dt\right) + F\omega_r \qquad (1)$$

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All the parameters in Eq. (1) are defined in (De Rossiter Corrêa *et al.*, 2000, 2004). In the RFOC method, we need to transfer equations of machine to the RFO reference frame. The conventional rotational transformation that should be applied to the equations of IM is as follows (this matrix transfer equations of IM from stationary reference frame to rotating reference frame):

$$[T_s^e] = \begin{bmatrix} \cos\theta_e & \sin\theta_e \\ -\sin\theta_e & \cos\theta_e \end{bmatrix} \qquad (2)$$

In this equation, θ_e is the angle between the stationary reference frame and rotating reference frame. It can be shown, the equations of single-phase IM Eq. (1), by applying conventional rotational transformation Eq. (2), are divided into forward terms and backward terms (it is because of unequal inductances in the equations of single phase IM: $M_d \neq M_q$ and $L_{ds} \neq L_{qs}$). Each term has the same structure with equations of balanced motor. One of them is rotated in the forward direction and one of them is rotated in the backward direction. By controlling of forward and backward components, controlling of single phase IM can be done but the controlling structure for single phase IM will be complex.

PROPOSED METHOD FOR VECTOR CONTROL OF SINGLE PHASE IM

As it was mentioned before by using of conventional rotational transformation, the backward terms is generated in the RFOC equations of single phase IM. In this study, new rotational transformations are introduced and applied to the single phase IM equations. By using of proposed matrices, the unbalanced equations of single phase IM change into balanced equations. The main idea of using these transformations is obtained from the steady state equivalent circuit of the single phase IM. Figure 1 shows the equivalent circuit of single phase IM. All the parameters in Fig. 1 are defined in (Zahedi and Vaez-Zadeh, 2009).

Based on the Fig. 1, the motor main and auxiliary voltage is obtained as:

$$V_m = Z_{lm} I_m + E_{fm} - \frac{j}{\alpha} E_{fa} + E_{bm} + \frac{j}{\alpha} E_{ba} \qquad (3)$$

$$V_a = Z_{la} I_a + E_{fa} + j\alpha E_{fm} + E_{ba} - j\alpha E_{bm}$$

where,

$$E_{fm} = Z_f I_m \ , \ E_{bm} = Z_b I_m \ , \ E_{fa} = \alpha^2 Z_f I_a$$
$$E_{ba} = \alpha^2 Z_b I_a \ , \ Z_f = R_f + jX_f \ , \ Z_b = R_b + jX_b \qquad (4)$$
$$Z_{lm} = R_{lm} + jX_{lm} \quad , \quad Z_{la} = R_{la} + jX_{la}$$

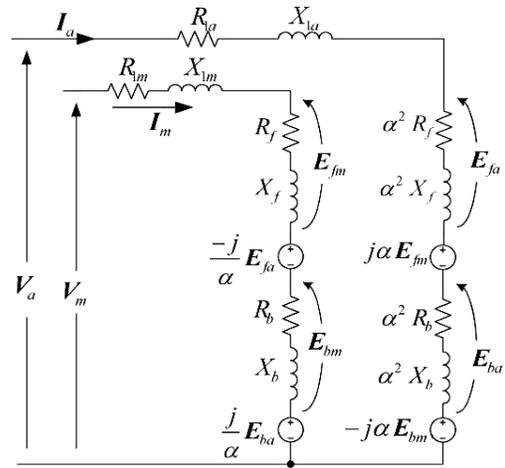By applying of following change of variables:

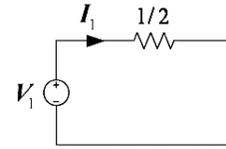Fig. 1: Steady state equivalent circuit of the single phase IM

Fig. 2: Simplified equivalent circuit of single phase IM

$$I_m = \frac{1}{2}(I_1 + I_2), \ I_a = \frac{j}{2\alpha}(I_1 - I_2), \ V_1 = Z_3 V_m + jZ_4 V_a \qquad (5)$$

where,

$$Z_3 = \left(Z_1(Z_{lm} + 2Z_f) + Z_2(Z_{lm} + 2Z_b)\right)/2Z_1$$
$$Z_4 = -j\left(Z_1(Z_{lm} + 2Z_f) + Z_2(Z_{lm} + 2Z_b)\right)/2Z_1 \qquad (6)$$
$$Z_1 = \alpha Z_{lm} + jZ_{la} + 2\alpha Z_b(\alpha j + 1)$$
$$Z_2 = -\alpha Z_{lm} + jZ_{la} + 2\alpha Z_f(\alpha j - 1)$$

Figure 1 can be simplified as Fig. 2.

As it was shown, with presented change of variables Eq. (5), the equivalent circuit of single phase IM changed into a balanced circuit. Equation (5) can be written as following equations:

$$\begin{cases} jV_1 = -Z_4 V_a + jZ_3 V_m \\ V_1 = jZ_4 V_a + Z_3 V_m \end{cases}, \quad \begin{cases} jI_1 = (N_a / N_m)I_a + jI_m \\ I_1 = -j(N_a / N_m)I_a + I_m \end{cases} \qquad (7)$$

With following substitutions:

$$N_a / N_m \rightarrow N_q / N_d = M_q / M_d$$
$$j \rightarrow \sin\theta_e, jV_1 \rightarrow v_{ds}^e, V_1 \rightarrow v_{qs}^e, V_a \rightarrow v_{ds}^s, V_m \rightarrow v_{qs}^s \qquad (8)$$
$$1 \rightarrow \cos\theta_e, jI_1 \rightarrow i_{ds}^e, I_1 \rightarrow i_{qs}^e, I_a \rightarrow i_{ds}^s, I_m \rightarrow i_{qs}^s$$

The proposed rotational transformations for stator voltage and current variables are obtained as follows:

$$\begin{bmatrix} v_{ds}^e \\ v_{qs}^e \end{bmatrix} = [T_{vs}^e]\begin{bmatrix} v_{ds}^s \\ v_{qs}^s \end{bmatrix} = \begin{bmatrix} -Z_4\cos\theta_e & Z_3\sin\theta_e \\ Z_4\sin\theta_e & Z_3\cos\theta_e \end{bmatrix}\begin{bmatrix} v_{ds}^s \\ v_{qs}^s \end{bmatrix}$$

$$\begin{bmatrix} i_{ds}^e \\ i_{qs}^e \end{bmatrix} = [T_{is}^e]\begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix} = \begin{bmatrix} (M_d/M_q)\cos\theta_e & \sin\theta_e \\ -(M_d/M_q)\sin\theta_e & \cos\theta_e \end{bmatrix}\begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix} \quad (9)$$

Equation (9) are transformation matrixes for transformation of variables from unbalanced mode (e.g., Fig. 1) to the balanced mode (Fig. 2). It is expected by using of these transformation matrixes the equations of unbalanced single phase IM become similar to the equations of balanced motor (in this study, $Z_3 = 1$ and $Z_4 = -M_q/M_d$ is considered). By applying (9) to the equations of single phase IM, stator voltage equations are obtained as follows:

$$\begin{bmatrix} v_{ds}^e \\ v_{qs}^e \end{bmatrix} = \begin{bmatrix} r_{qs}+L_{qs}\frac{d}{dt} & -\omega_e L_{qs} \\ \omega_e L_{qs} & r_{qs}+L_{qs}\frac{d}{dt} \end{bmatrix}\begin{bmatrix} i_{ds}^e \\ i_{qs}^e \end{bmatrix} + \begin{bmatrix} M_q\frac{d}{dt} & -\omega_e M_q \\ \omega_e M_q & M_q\frac{d}{dt} \end{bmatrix}\begin{bmatrix} i_{dr}^e \\ i_{qr}^e \end{bmatrix}$$

$$+ \begin{bmatrix} \left(\begin{array}{c}(\frac{M_q^2}{M_d^2}r_{ds}-r_{qs})+ \\ (\frac{M_q^2}{M_d^2}L_{ds}-L_{qs})\frac{d}{dt}\end{array}\right) & -\omega_e(\frac{M_q^2}{M_d^2}L_{ds}-L_{qs}) \\ \omega_e(\frac{M_q^2}{M_d^2}L_{ds}-L_{qs}) & \left(\begin{array}{c}(\frac{M_q^2}{M_d^2}r_{ds}-r_{qs})+ \\ (\frac{M_q^2}{M_d^2}L_{ds}-L_{qs})\frac{d}{dt}\end{array}\right) \end{bmatrix}\begin{bmatrix} i_{ds}^{-e} \\ i_{qs}^{-e} \end{bmatrix} \quad (10)$$

where,

$$\begin{bmatrix} i_{ds}^{-e} \\ i_{qs}^{-e} \end{bmatrix} = \begin{bmatrix} \cos^2\theta_e & -\sin\theta_e.\cos\theta_e \\ -\sin\theta_e.\cos\theta_e & \sin^2\theta_e \end{bmatrix}\begin{bmatrix} i_{ds}^e \\ i_{qs}^e \end{bmatrix} \quad (11)$$

Moreover, rotor and torque equations are obtained as following equations:

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} M_q\frac{d}{dt} & \omega_r M_q \\ -\omega_r M_q & M_q\frac{d}{dt} \end{bmatrix}\begin{bmatrix} i_{ds}^s \\ i_{qs}^s \end{bmatrix} + \begin{bmatrix} r_r+L_r\frac{d}{dt} & \omega_r L_r \\ -\omega_r L_r & r_r+L_r\frac{d}{dt} \end{bmatrix}\begin{bmatrix} i_{dr}^s \\ i_{qr}^s \end{bmatrix} \quad (12)$$

$$\tau_e = (Pole/2)M_q(i_{qs}^e i_{dr}^e - i_{ds}^e i_{qr}^e) \quad (13)$$

As it was shown by using of these rotational transformations, equations single phase IM became like balanced equations. So, RFOC equations of single phase IM are obtained as follows (in RFOC method, the rotor flux vector is aligned with d-axis; $\lambda_{dr}^e = |\lambda_r|$, $\lambda_{qr}^e = 0$):

$$|\lambda_r| = (M_q i_{ds}^e)/(1+T_r(d/dt))$$

$$T_r(\omega_e - \omega_r)|\lambda_r| = M_q i_{qs}^e \quad (14)$$

$$\tau_e = (Pole/2)(\frac{M_q}{L_r})(|\lambda_r|i_{qs}^e)$$

In (14), T_r and ω_e are the rotor time constant and the angular velocity of the rotor field oriented reference frame respectively. Based on (10), equations of RFOC for stator voltages can be written as follows:

$$v_{ds}^e = v_{ds}^d + v_{ds}^{ref} + v_{ds}^{-e} \quad , \quad v_{qs}^e = v_{qs}^d + v_{qs}^{ref} + v_{qs}^{-e} \quad (15)$$

where,

$$v_{ds}^d = -\omega_e i_{qs}^e(L_{qs}-\frac{M_q^2}{L_r}) + (\frac{M_q}{L_r})(\frac{M_q i_{ds}^e - |\lambda_r|}{T_r})$$

$$+ (r_{qs} - \frac{(-\frac{M_q^2}{M_d^2}r_{ds}+r_{qs})(L_{qs}-\frac{M_q^2}{L_r})}{-\frac{M_q^2}{M_d^2}L_{ds}+L_{qs}})i_{ds}^e \quad (16)$$

$$v_{qs}^d = \omega_e i_{ds}^e(L_{qs}-\frac{M_q^2}{L_r}) + \omega_e M_q\frac{|\lambda_r|}{L_r}$$

$$+ (\frac{M_q^2}{M_d^2}r_{ds} - \frac{(-\frac{M_q^2}{M_d^2}r_{ds}+r_{qs})(\frac{M_q^2}{M_d^2}L_{qs}-\frac{M_q^2}{L_r})}{-\frac{M_q^2}{M_d^2}L_{ds}+L_{qs}})i_{qs}^e \quad (17)$$

$$\begin{bmatrix} v_{ds}^{-e} \\ v_{qs}^{-e} \end{bmatrix} = \omega_e\left(-(M_q^2/M_d^2)L_{ds}+L_{qs}\right)$$

$$\times \begin{bmatrix} \sin\theta_e.\cos\theta_e & -\cos^2\theta_e \\ \sin^2\theta_e & -\sin\theta_e.\cos\theta_e \end{bmatrix}\begin{bmatrix} i_{ds}^e \\ i_{qs}^e \end{bmatrix} \quad (18)$$

$$\begin{bmatrix} v_{ds}^{ref} \\ v_{qs}^{ref} \end{bmatrix} =$$

$$\underbrace{\begin{bmatrix} -\cos^2\theta_e + \frac{L_{qs}-\frac{M_q^2}{L_r}}{-\frac{M_q^2}{M_d^2}L_{ds}+L_{qs}} & \sin\theta_e.\cos\theta_e \\ \sin\theta_e.\cos\theta_e & \cos^2\theta_e + \frac{\frac{M_q^2}{M_d^2}L_{qs}-\frac{M_q^2}{L_r}}{-\frac{M_q^2}{M_d^2}L_{ds}+L_{qs}} \end{bmatrix}}_{[T]}$$

$$\times \begin{bmatrix} (-\frac{M_q^2}{M_d^2}r_{ds}+r_{qs})i_{ds}^e + (-\frac{M_q^2}{M_d^2}L_{ds}+L_{qs})\frac{di_{ds}^e}{dt} \\ (-\frac{M_q^2}{M_d^2}r_{ds}+r_{qs})i_{qs}^e + (-\frac{M_q^2}{M_d^2}L_{ds}+L_{qs})\frac{di_{qs}^e}{dt} \end{bmatrix} \quad (19)$$

Based on Eq. (14)-(19), Fig. 3 can be presented for RFOC of single phase IM.

PERFORMANCE EVALUATION

In this part, based on Fig. 3, MATLAB/SIMULINK simulation results have been presented. A 0.5 hp single

Fig. 3: Block diagram of proposed RFOC for single phase IM

(a)

(b)

(a)

(b)

Fig. 4: Simulation results of comparison between speed response in single-phase IM (a) not assuming $(M_q/M_d)^2 = L_{qs}/L_{ds}$ (b) assuming $(M_q/M_d)^2 = L_{qs}/L_{ds}$

phase IM is fed by a PWM, Four Switch Inverter (FSI) as was proposed in De Rossiter Corrêa *et al.* (2000, 2004).

Based on Eq. (10), the difference in the d and q stator voltages between the conditions in which the supposition of $(M_q/M_d)^2 = L_{qs}/L_{ds}$ is considered (Azza *et al.*, 2011; De Rossiter Corrêa *et al.*, 2000, 2004; Vaez-Zadeh and Harooni, 2005) and otherwise is as following equation:

$$
\begin{bmatrix} \Delta v_{ds}^e \\ \Delta v_{qs}^e \end{bmatrix} = \begin{bmatrix} (\dfrac{M_q^2}{M_d^2} L_{ds} - L_{qs})\dfrac{d}{dt} & -\omega_e(\dfrac{M_q^2}{M_d^2} L_{ds} - L_{qs}) \\ \omega_e(\dfrac{M_q^2}{M_d^2} L_{ds} - L_{qs}) & (\dfrac{M_q^2}{M_d^2} L_{ds} - L_{qs})\dfrac{d}{dt} \end{bmatrix} \quad (20)
$$
$$
\times \begin{bmatrix} \sin\theta_e.\cos\theta_e & -\cos^2\theta_e \\ \sin^2\theta_e & -\sin\theta_e.\cos\theta_e \end{bmatrix} \begin{bmatrix} i_{ds}^e \\ i_{qs}^e \end{bmatrix}
$$

A comparison between the steady state speed response of the RFOC with considering of $(M_q/M_d)^2 = L_{qs}/L_{ds}$ and without considering of $(M_q/M_d)^2 = L_{qs}/L_{ds}$ is shown in Fig. 4. Small magnitude of oscillations at the nominal reference speed can be seen in the torque and speed responses when the assumption $(M_q/M_d)^2 = L_{qs}/L_{ds}$ is used. It is concluded in comparison with the previous schemes for FOC of single phase IM, the proposed controller in this study is more accurate. The waveform in Fig. 4 is obtained based on the experimental values of the current and speed.

CONCLUSION

This study presents a new methodology for control of single phase or unbalanced two phase IMs based on RFOC. The theory and analysis of the proposed vector control is based on using rotational transformations which is obtained from steady state equivalent circuit of single phase IM. The results are shown the ability of the proposed drive system in the speed control of single phase IMs.

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