

Research Article

Reduction of Train-induced Vibrations by using Barriers

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Abstract: The problem of the ground-borne vibration caused by high speed trains has received considerable attention in recent years, due to the effects of vibration on buildings, in terms of physical damage and on population, in terms of discomfort. The problem has become more significant with the increase of speed and weight of trains, which results in heavier loads on the tracks. Therefore, there is the necessity to find a method, which allows investigating the propagation of vibration waves in the soil. This study aims to study the train-induced ground vibration and the mitigation effects of barriers using a Finite Element Method (FEM) model. Two different types of barriers were evaluated considering their stiffness and a benchmark model without mitigation measures was also analyzed to evaluate the effectiveness of the considered barriers. The results of the proposed elaborations have been finalized to the assessment of the incidence of the barrier on the vibration state induced from the passage of a high speed trains and the following conclusions can be made: concrete seems to provide a significative reduction of the vibration. The proposed method can be successfully applied to a preliminary analysis of the influence of different types of barriers on the dynamic properties of vibration waves in the soil.

Keywords: Damping coefficient, FEM, finite element analysis, ground vibration, railway, Rayleigh wave, trains, wave barriers, wave propagation

INTRODUCTION

Trains traffic related noise and vibration is a source of pollution which can cause severe damage to communities in terms of health and social welfare (Cirianni and Leonardi, 2012, 2011).

Large forces between wheels and rails cause railway vibrations during the passage of the train. The main source of excitation of the track is represented by vertical force determined by the wheel-rail interaction during the passage of trains (Ferrara *et al.*, 2012, 2013). Such vibrations are transmitted through the track structure, including rails, sleepers, ballast and sub-layers and propagate through the ground by means of two types of non-dispersive waves, which can freely propagate and are called the body waves (Fig. 1). The first is a dilatational wave, or a pressure wave (known as the P-wave). This is a longitudinal wave, where the wave-front and the medium particles move in the same direction. The second is the shear wave (known as the S-wave). This is a transverse wave, where the wave-front and the medium particles move perpendicular to each other.

Both types of body waves decay at the surface and only Rayleigh wave propagates freely at the surface. The pressure wave is the fastest and arrives first, the

shear wave comes next and finally Rayleigh wave arrives and it is associated with large amplitudes.

Rayleigh wave (known as the R-wave or the surface wave) is non-dispersive and it is confined near to the surface at a depth approximately equal to the wavelength. The particles move in elliptical shape in the same plane as the wave-front. The vertical component of particle motion is greater than the horizontal component; both components decay exponentially with depth. Away from the surface where the Rayleigh wave disappears, motion of particles is attributed to body waves.

For all type of waves, the energy decreases as the distance from the source increases. This is due to the geometric dispersion and energy absorption in the ground. The lowest frequencies are the least damped.

Among the mitigation measures, the trenches (open and in-filled) and barriers between the source and the structure or building to be protected have exhibited a good performance.

The mitigation effects of different types of wave barriers, varying from very stiff concrete walls to very flexible elements, are discussed in different studies (Chouw *et al.*, 1991; Massarsch, 1994; Kattis *et al.*, 1999a, b; Buonsanti *et al.*, 2009b).

These researches reveal that efforts have been directed mainly towards analytical studies and some

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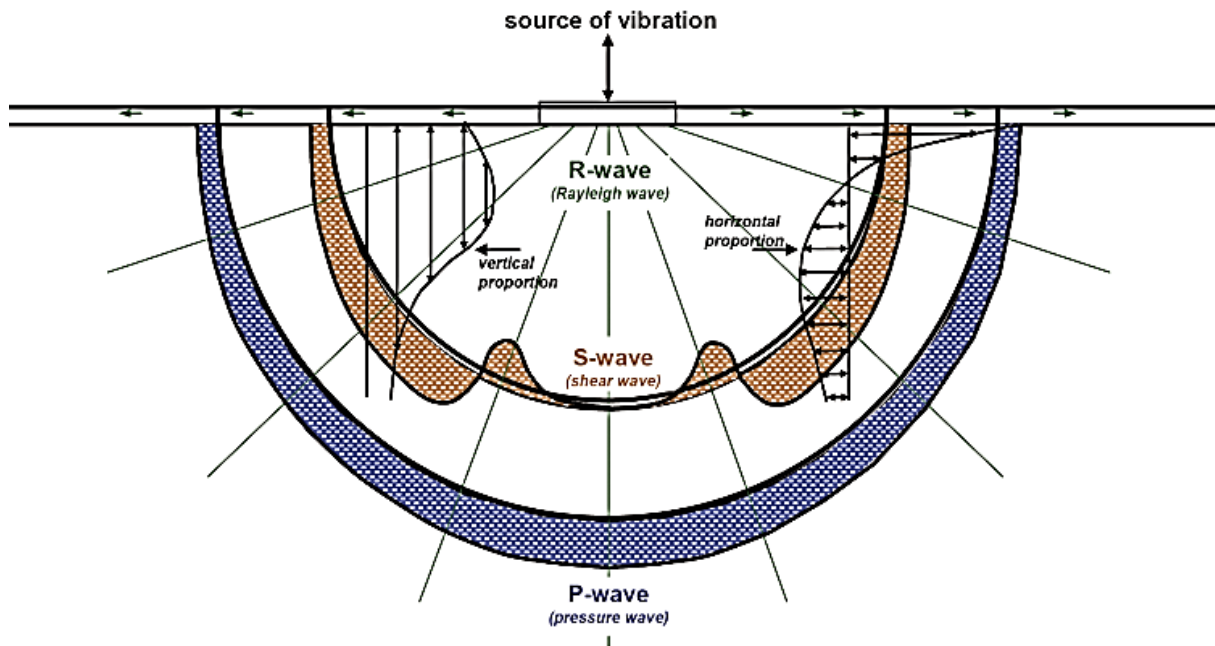


Fig. 1: Modes of propagation of ground vibration

experimental works to investigate the problem of isolation by means of trench barriers. Only a few experimental studies present some practical design guidelines that can be applied in the process of buildings and infrastructures project (Adam and Von Estorff, 2005). We can remember the studies of Barkan (1962), Woods (1968) and Richart *et al.* (1970), they conducted a series of field experiments and analytical studies in order to study the effectiveness of open trenches.

Nevertheless, open trenches pose stability problems from the engineering point of view. This difficulty can be overcome by using in-filled trenches or barriers.

In the last decades, various researches, both experimental and numerical, have been carried out to investigate the propagation of vibration waves across different types of barriers or in-filled trenches (Shrivastava and Kameswara Rao, 2002; Kim *et al.*, 2000; Ahmad and Al-Hussaini, 1991; Al-Hussaini and Ahmad, 1991; Ahmad *et al.*, 1996; Al-Hussaini *et al.*, 2000).

While full-scale model test results are often difficult to extrapolate, numerical methods, such as Finite Difference Method (FDM), Finite Element Method (FEM) and Boundary Element Method (BEM), can be a valid alternative to study the efficiency of barriers and trenches.

Aboudi (1973) and Fuyuki and Matsumoto (1980) applied the Finite Difference Method (FDM) to model the ground vibration propagation and response in different conditions.

Several studies used Finite Element Method (FEM) to study the influence of barriers and trenches on

vibration, such as Buonsanti *et al.* (2009a), Hall (2003), Yang *et al.* (2010), Yang and Hung (2009) and Gao *et al.* (2008).

The Boundary Element Method (BEM) can be one of the ideal methods to study the vibration isolation problem since it requires only surface discretization and because the radiation condition at the boundary is automatically accounted in the formulation. Many authors adopted the BEM for the analysis of isolation effects of open and in-filled trenches in different types of soils. Banerjee *et al.* (1988), Emad and Manolis (1985), Dasgupta *et al.* (1986) and Ahmad and Al-Hussaini (1991) adopted the BEM for the analysis of isolation effects of open and in-filled trenches, investigating different types of soils.

In this study, after an introduction on some general concepts of theory of the vibration, the focus is set on vibrations in a homogenous elastic and isotropic semi-space, where a two-dimensional constitutional discontinuity is localized, representing the artificial barrier. Also THE ground-borne vibrations produced by a high-speed moving load on an embankment are investigated, studying in particular the effects due to the presence of concrete and compacted soil barriers (De Azevedo and Patricio, 2010). A benchmark model was also analyzed without any mitigation measure and the results of the models were compared to evaluate the barriers effectiveness.

MATERIALS AND METHODS

Theoretical characterization: wave's theory in semi-infinite media: We analyze a more particular case of waves generated from a surface source. Under general

conditions of normal and shear loads applied to the surface of the half-space, both dilatational and shear are generated with the resulting mathematical expression for the wave propagation being more complicated than those classic questions. In according to Lamb classic theory (Lamb, 1904), harmonic loadings were considered and superposition techniques were used to obtain results for pulse loadings. The approach taken will be to consider the case of harmonic normal loading in the half space in some detail, as in according to Ewing *et al.* (1958), Miller and Pursey (1955, 1954) and finally to Graff (1991). Now, we consider a half-space subjected to a loading normal to the surface, applied parallel to the z axis. The load produces waves from a harmonic normal line force and so the resulting situation is one of plane strain. The basic equations follow:

$$(\lambda + 2\mu)\nabla^2 u - \mu\nabla \times \nabla \times u = \rho \frac{\partial^2 u}{\partial t^2} \quad (1)$$

Considering the harmonic time variation of the loading $\exp^{i\omega t}$ the Eq. (1) can be written:

$$(\lambda + 2\mu) \frac{\partial \Delta}{\partial x} + \mu \frac{\partial \Xi}{\partial y} + \rho \omega^2 u_x = 0 \quad (2)$$

$$(\lambda + 2\mu) \frac{\partial \Delta}{\partial y} - \mu \frac{\partial \Xi}{\partial x} + \rho \omega^2 u_y = 0$$

where, $\Xi = \nabla \times u$.

The resulting stresses may be expressed in terms of Δ and Ξ :

$$\sigma_{xx} = \frac{\mu^2}{\rho \omega^2} \left\{ 2 \frac{\partial^2 \Xi}{\partial x \partial y} - k^2 (k^2 - 2) \frac{\partial^2 \Delta}{\partial x^2} - k^4 \frac{\partial^2 \Delta}{\partial y^2} \right\} \quad (3)$$

$$\tau_{xy} = \frac{\mu^2}{\rho \omega^2} \left\{ \frac{\partial^2 \Xi}{\partial x^2} - \frac{\partial^2 \Xi}{\partial y^2} - 2k^2 \frac{\partial^2 \Delta}{\partial x \partial y} \right\} \quad (4)$$

where, $k^2 = (\lambda + 2\mu) = k_2^2/k_1^2$ and $k_1^2 = \omega^2/c_1^2$, $k_2^2 = \omega^2/c_2^2$.

According to Graff (1991) the formal solution follows:

$$\frac{d\Delta^*}{dy^2} - (\xi^2 - k_1^2)\Delta^* = 0 \quad \frac{d\Xi^*}{dy^2} - (\xi^2 - k_2^2)\Xi^* = 0 \quad (5)$$

Through the Fourier transform on the spatial variable x :

$$f(\xi) = \int_{-\infty}^{\infty} f(x) e^{-i\xi x} dx \quad f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(\xi) e^{i\xi x} d\xi \quad (6)$$

Considering the boundary conditions and their transformed:

$$\sigma_{yy}(x, 0) = \begin{cases} 1 & |x| < a \\ 0 & |x| > a \end{cases} \quad \tau_{xy}(x, 0) = 0 \quad (7)$$

$$\sigma_{yy}^*(\xi, 0) = 2 \sin \xi a / \xi \quad \tau_{xy}^*(\xi, 0) = 0 \quad (8)$$

The solutions of the Eq. (5) have the form:

$$\Delta^* = A \exp\left\{-\left(\xi^2 - k_1^2\right)^{1/2} y\right\} \quad \Xi^* = B \exp\left\{-\left(\xi^2 - k_2^2\right)^{1/2} y\right\} \quad (9)$$

For completeness, we will briefly illustrate the main aspects of a case closer to our problem: distribution of displacement and energy in dilatational, shear and surface waves from a harmonic normal load on a half-space.

Using the Helmholtz's theorem, we can introduce the scalar and vector potential Φ and Z , such that, for any displacement field u , the following form is valid:

$$u = \nabla \Phi + \nabla \times Z \quad \nabla \cdot Z = 0 \quad (10)$$

Using the polar coordinates the displacement vector hold:

$$u(r, z, t) = u_r e_r + u_z e_z \quad (11)$$

And in scalar and vector potential terms:

$$u = \nabla \Phi + \nabla \times (Z_\theta e_\theta) \quad (12)$$

The field equations assume the form:

$$\nabla^2 \Phi = \frac{1}{c_1^2} \frac{\partial^2 \Phi}{\partial t^2} \quad \nabla^2 Z_\theta = \frac{1}{r^2} Z_\theta = \frac{1}{c_2^2} \frac{\partial^2 Z_\theta}{\partial t^2} \quad (13)$$

Putting $Z_\theta = -\partial \aleph / \partial r$, the resulting displacements are given by:

$$u_r = \frac{\partial \Phi}{\partial r} + \frac{\partial^2 \aleph}{\partial r \partial z} \quad u_\theta = 0 \quad u_z = \frac{\partial \Phi}{\partial z} + \frac{\partial^2 \aleph}{\partial z^2} - \frac{1}{c_2^2} \aleph^* \quad (14)$$

The resulting stresses becomes:

$$\sigma_{rr} = \lambda \nabla^2 \Phi + 2\mu \left(\frac{\partial^2 \Phi}{\partial r^2} + \frac{\partial^2 \aleph}{\partial r \partial z} \right) \quad (15)$$

$$\sigma_{\theta\theta} = \lambda \nabla^2 \Phi + 2\frac{\mu}{r} \left(\frac{\partial \Phi}{\partial r} + \frac{\partial^2 \aleph}{\partial r \partial z} \right) \quad (16)$$

$$\sigma_{zz} = \lambda \nabla^2 \Phi + 2\mu \left(\frac{\partial \Phi}{\partial z} + \frac{\partial^2 \aleph}{\partial z^2} - \frac{1}{c_2^2} \aleph^* \right) \quad (17)$$

$$\tau_{rz} = \mu \frac{\partial}{\partial r} \left(2 \frac{\partial \Phi}{\partial z} + 2 \frac{\partial^2 \aleph}{\partial z^2} - \frac{1}{c_2^2} \ddot{\aleph} \right) \quad (18)$$

For the boundary conditions, it is possible the following form:

$$\sigma_{rz} = 0 \quad \sigma_{zz} = F(r)\aleph(t) \quad z=0 \quad (19)$$

where, $F(r)$ is an opportune and arbitrary function to represent as particular form of the Fourier-Bessel integral and, under opportune conditions, becomes:

$$F(r) = \frac{Z}{2\pi} \int_0^\infty \xi J(\xi r) d\xi \quad (20)$$

where, Z represents the magnitude of the applied force.

Finally, we will emphasize the scattering of compressional waves against an obstacle case, as an elastic inclusion in the elastic half-space. Doing so, it considers a plane harmonic compression wave impacting a ground immersed obstacle of spherical form. In potentials function terms we have:

$$u = \nabla \Phi + \nabla \times \left(\frac{\partial \aleph}{\partial \theta} e_\varphi \right) \quad (21)$$

In according to Graff (1991) briefly we recall the resulting displacements field:

$$u_r = \frac{\partial \Phi}{\partial r} + \frac{1}{r} (K_\theta \aleph) \quad u_\theta = \frac{1}{r} \frac{\partial \Phi}{\partial \theta} - \frac{\partial}{\partial \theta} (K_r \aleph) \quad (22)$$

where, K_r and K_θ :

$$K_r = \frac{1}{r} \frac{\partial}{\partial r} (r) \quad K_\theta = \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) \quad (23)$$

Specifying that: in the incident field $\Phi = \Phi^{inc}$ and $\aleph = 0$; in the scattered field $\Phi = \Phi^{sca}$ and $\aleph = \aleph^{sca}$, and in the refracted field $\Phi = \Phi^{ref}$ and $\aleph = \aleph^{ref}$.

Finite element model: The proposed model of rail track is the conventional one and consists of an axisymmetric scheme to reproduce a more realistic simulation. The principal elements of the model are reported in Table 1.

The dimensions and the material characteristics of the elements are those requested by the Italian standard (RFI) for high speed lines (Fig. 2).

To analyze the propagation process of the vibrations in the soil, the model assumed that the embankment was supported by soil to a depth of 7 m.

Table 1: Principal elements of the model

Element	Thickness (cm)
Protective compacted layer	30.00
Sub-ballast layer	12.00
Traditional ballast layer	35.00
Embankment	58.00

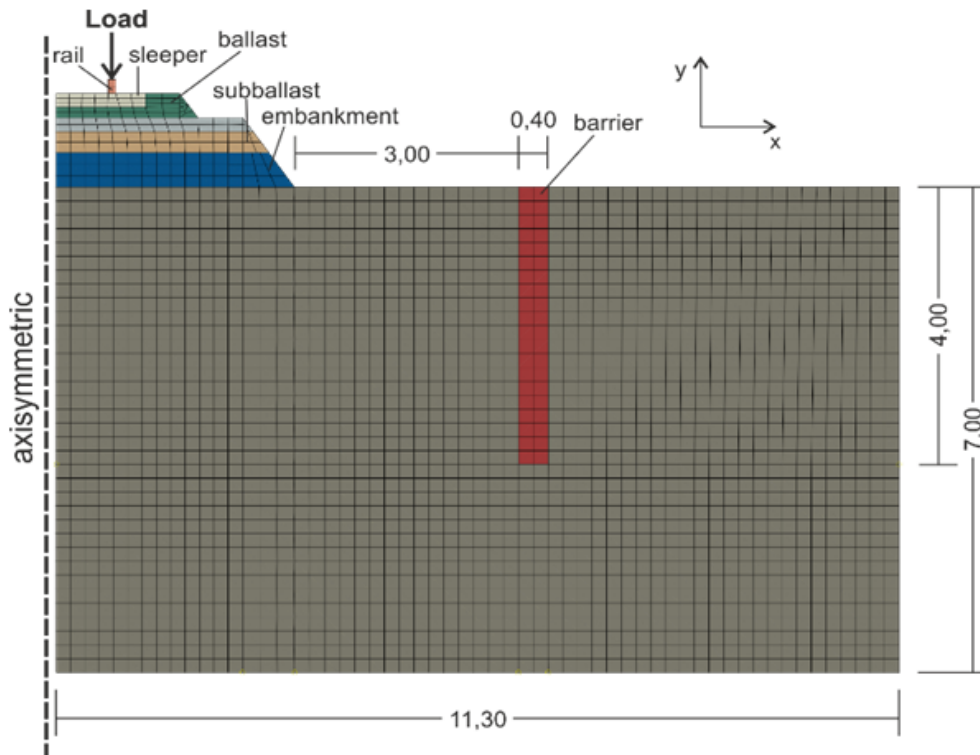


Fig. 2: Considered soil-railway system

Table 2: Material proprieties of the track-embankment-soil system

Mechanical characteristics	Rail UIC60 (steel)	Sleeper (concrete)	Ballast (crushed stone)	Sub-ballast (HMA)	Protective layer (sand/gravel)	Embankment (compacted soil)	Ground (natural soil)
Density ρ (kg/m ³)	7850	2400	1250	2200	2000	1400	2040
Modulus E (MPa)	210000	30000	130.00	6000	160	170	18.00
Poisson's ratio ν	0.30	0.15	0.30	0.40	0.45	0.36	0.20

Table 3: Material proprieties of the barrier

Material	Density ρ [kg/m ³]	Modulus E [MPa]	Poisson's ratio ν	Dumping ratio ξ
Compacted soil	2040	18.00	0.20	0.10
Concrete	2500	25000	0.15	0.05

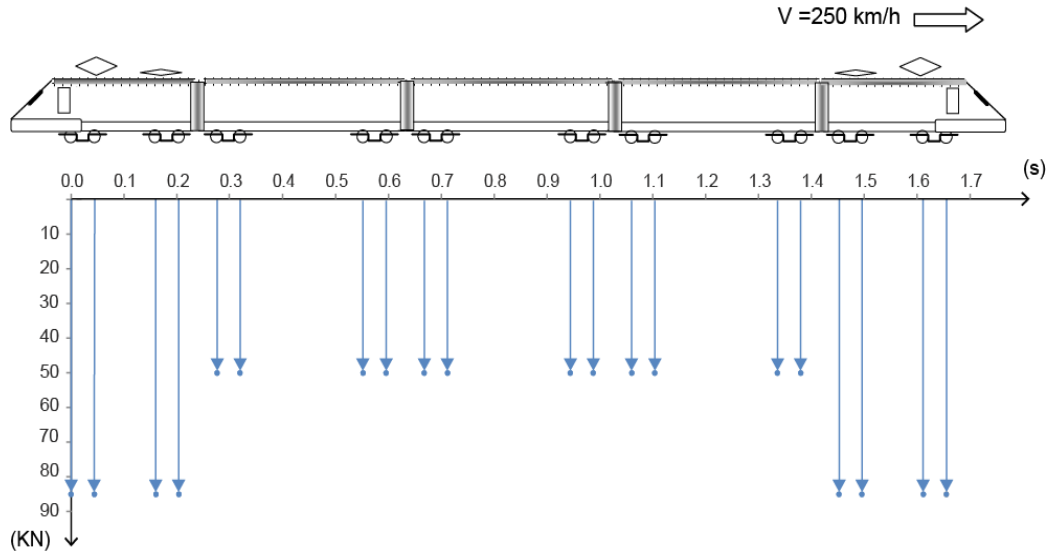


Fig. 3: Time history of loading of ETR 500 train travelling at speed of 250 km/h

The materials properties used in the model were derived from tests and available experimentations in literature (Buonsanti *et al.*, 2009b; Yang and Hung, 2009). The material properties of the soil and the track structure are tabulated in Table 2.

Finally, a discontinuity (40 cm thick and 4.0 m deep) in the semi-space at a given distance from the rail track (3.0 m from the embankment's end) was considered (Fig. 2).

In wave propagation problems, elements dimension are influenced by the highest frequency and the lowest velocity wave (V_R). Kuhlemeyer and Lysmer (1973) suggested the mesh size should be shorter than 1/8-1/10 of the wave length, following this recommendation the maximum element size was 0.2 m considering the Rayleigh wave velocity of the ground ($V_R = 0.54 \times V_p = 53.47$ m/s) and the considered highest frequency (100 Hz).

The axisymmetric 2-D model, shown in Fig. 2, was composed of 2159 elements and 2264 nodes. The elements comprised of 4-node, linear axisymmetric, solid and reduced-integration elements (CAX4R).

The train was simplified to a series of vertical loads (Yang and Hung, 2009; Younesian and Sadri, 2012), which were placed according to the geometry and the composition of the train (locomotives, bogies, wheels,

axles) and moving at a constant speed ν on the track. In the following, a simplified composition (two locomotives and three bogies) of the Italian high-speed train ETR 500 was considered (Fig. 3).

The forces are applied using a time function that represents the time history of the force in the considered node as shown in Fig. 3.

The effects of discontinuity are estimated considering two different barriers of Compacted Soil (CS) and concrete (Table 3).

As the calculations were performed in the time domain, the material damping was modelled with the Rayleigh damping model.

Following the Rayleigh method the damping matrix [C] can be obtained from the relation:

$$[C] = \alpha [M] + \beta [K] \tag{24}$$

where,

- [M] = The mass matrix
- [K] = The stiffness matrix
- α and β = Pre-defined constants

For a given mode i , the critical damping value ξ_i and the Rayleigh damping values, α and β , are related through:

$$\xi_i = \frac{\alpha}{2\omega_i} + \frac{b \cdot \omega_i}{2} \quad (25)$$

From Eq. (25) it can be observed that the damping ratio is proportional to the natural frequencies of the system.

In this study, considering the frequencies of interest, the method of determining the Rayleigh damping coefficient proposed by Chopra (2001) was used.

RESULTS AND DISCUSSION

The finite element simulations of train-induced ground vibration were developed, using the ABAQUS® software (Simulia Ltd., 2010), considering a 2D finite element model perpendicular to the track. All the finite element analyses in this study were performed in the time domain. A total period of 1.70 sec for 250 km/h speed was considered (Fig. 3). A maximum time increment (0.0014 sec) was considered according to the suggestions by Zerwer *et al.* (2003), though the automatic time option was activated to select the time increment.

The graphs of the outcomes of the analyses for a train speed of 250 km/h are shown in the following Fig. 4. From this figure, different wave fronts could be observed.

Effect of barriers in ground vibration: To investigate the effects of material properties of wave barriers several simulations were carried out. Furthermore, a benchmark model was run without any mitigation measure in order to have an appropriate reference for the comparison.

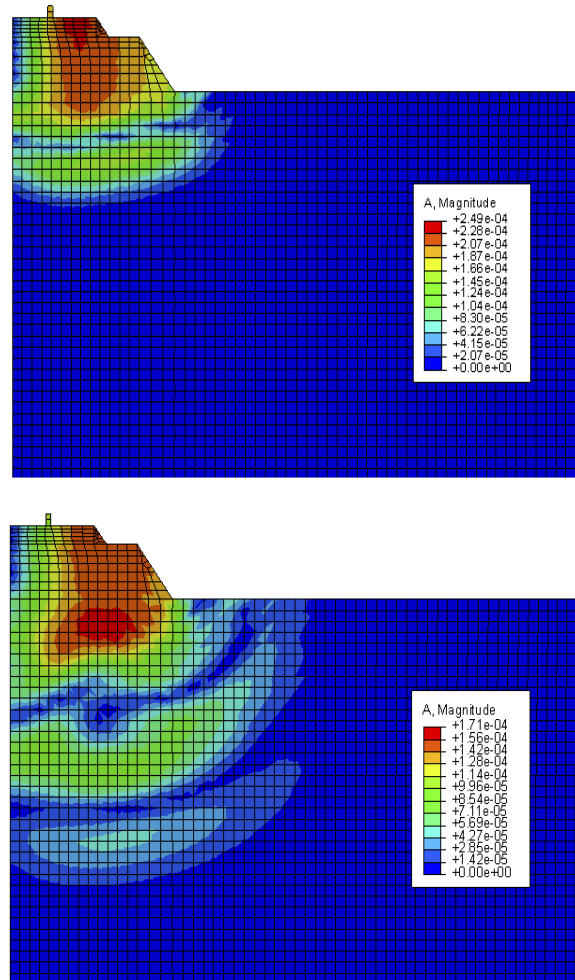


Fig. 4: Wave propagation acceleration in ground at different time steps (t = 0.0144 sec and t = 0.0432)

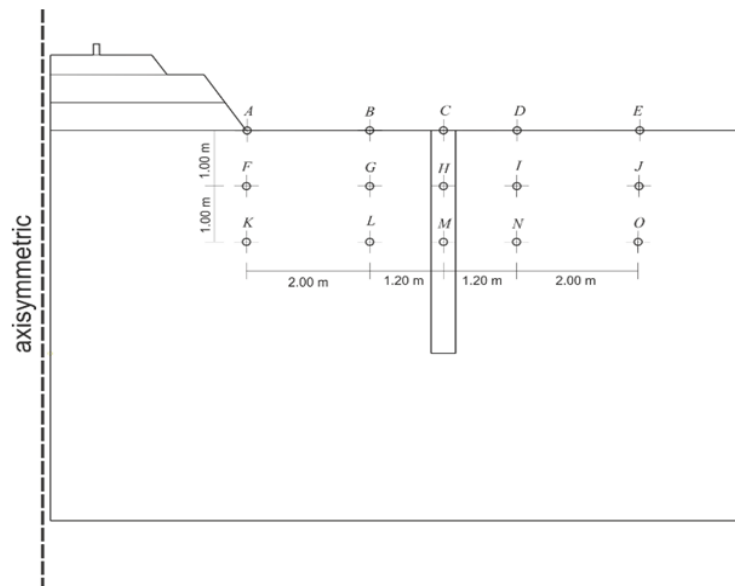


Fig. 5: Geometrical localization the considered nodes

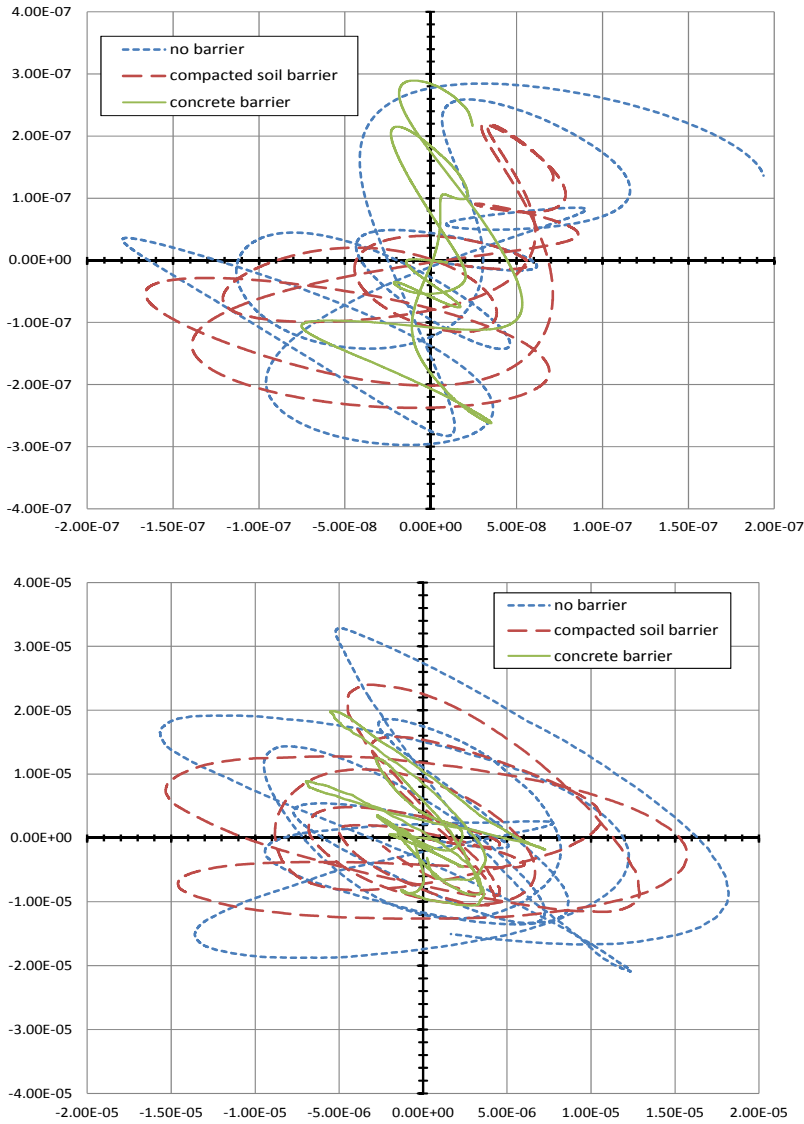


Fig. 6: Variation of velocities (m/s) and accelerations (m/s²) of node N

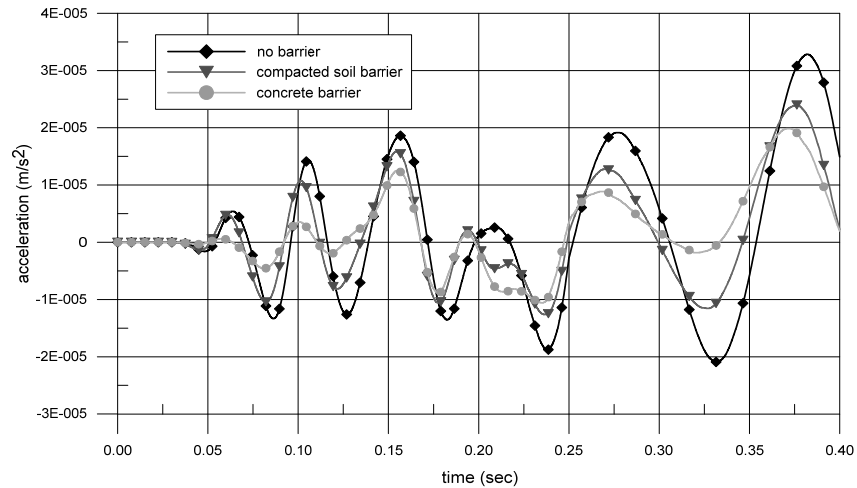


Fig. 7: Time histories of acceleration on ground surface (node N)

Figure 5 shows the localization of some particular nodes of the model considered for the following analysis.

Figure 6 shows the variation of velocity and acceleration (x-direction and y-direction) of the node N (Fig. 5) localized at a distance on 1 m form the barrier. The effects of barrier discontinuity comparing the acceleration evolution on ground surface (node N), are presented in Fig. 7.

In order to investigate the mitigation effects of the considered barriers on vibration wave the evolution of acceleration Fig. 8 and 9 show the attenuation curves of the maximum vertical acceleration at the ground

surface and at a depth of 2 m from the ground for different barriers in addition to the benchmark model.

As a general result of the analyses, as illustrated in Fig. 6 to 8, for the considered loads, it is concluded that, when the load is travelling with high speeds, concrete barriers provide a more efficient vibration reduction than the compacted soil barriers.

The results obtained supply useful information for the making and positioning of barriers, especially when it is required to respect regulation's limit for vibration, as expressed in terms of velocities and accelerations (i.e., Italian regulation UNI 9916/2220).

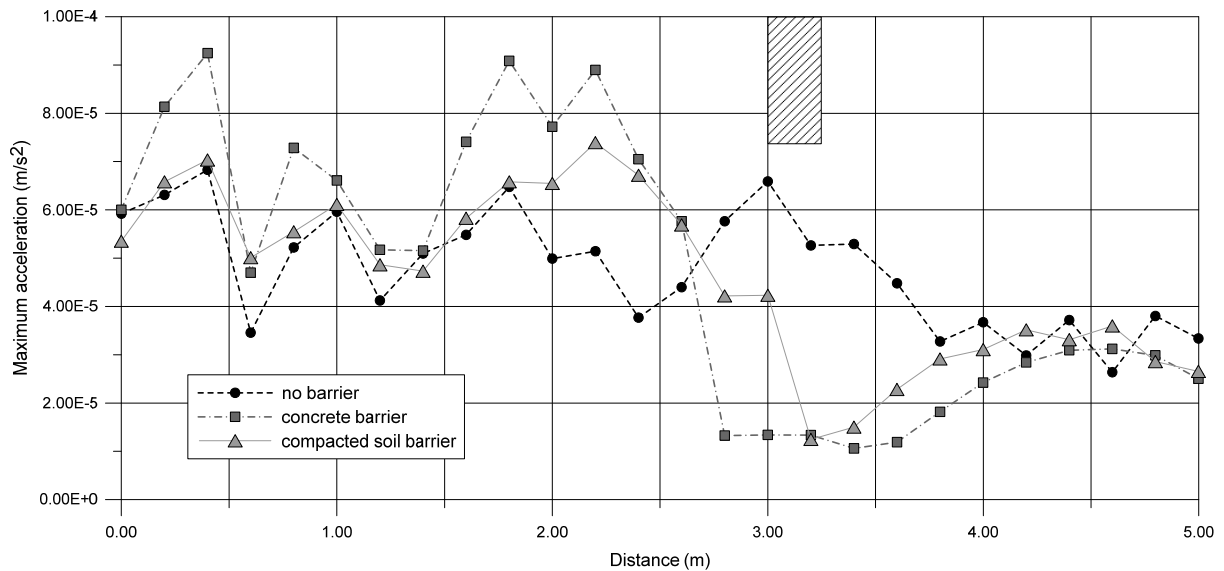


Fig. 8: Attenuation at the ground surface

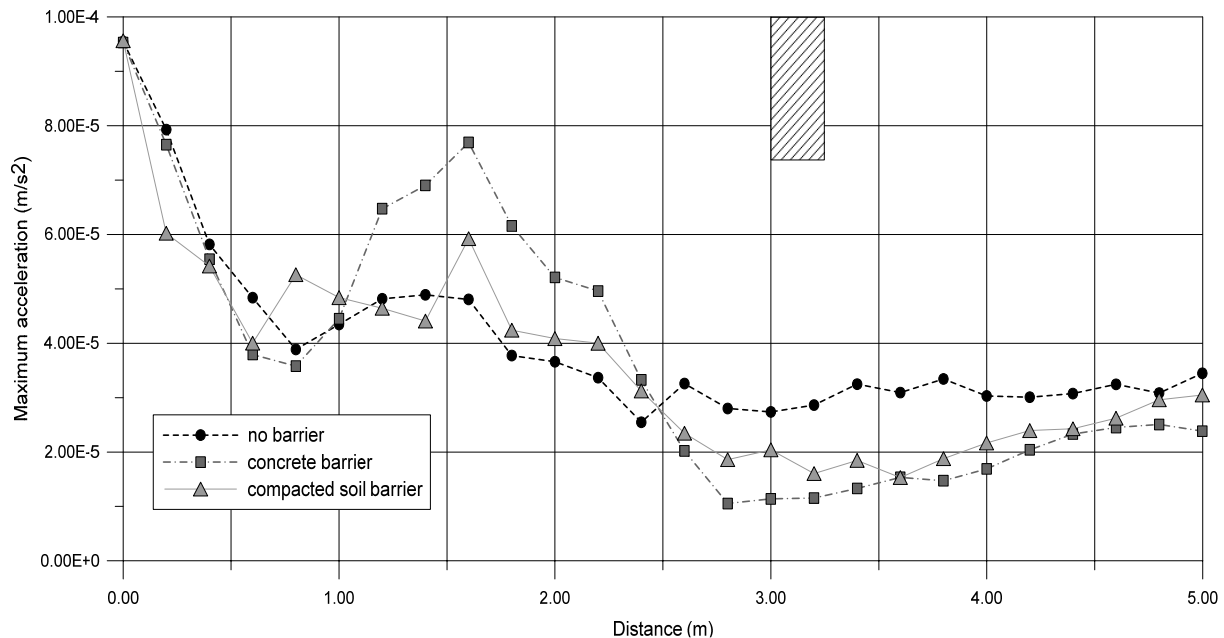


Fig. 9: Attenuation at a depth of 2 m from the ground surface

CONCLUSION

The results of the proposed elaborations have been finalized to the assessment of the incidence of the barrier on the vibration state induced from the passage of a high-speed train and the following conclusions can be made: concrete seems to provide a better reduction of the vibration than compacted soil. In spite of the greater density of the material, it involves an increase of the reflection phenomena and a consequent increase of the phenomenon at the top side of the barrier.

The proposed approach can be applied to a preliminary analysis of the influence of different types of barriers on the dynamic properties of vibration waves in the soil.

There is also the possibility to consider at the design stage these technical means (barriers and trenches) to limit the dynamic effects of railway transport.

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