Research Journal of Applied Sciences, Engineering and Technology 7(18): 3699-3708, 2014
DOI:10.19026/rjaset.7.724
ISSN: 2040-7459; e-ISSN: 2040-7467

Submitted: March 08, 2013 Accepted: April 22, 2013 Published: May 10, 2014

Research Article

Strength Reliability Analysis of Turbine Blade Using Surrogate Models

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Abstract: There are many stochastic parameters that have an effect on the reliability of steam turbine blades performance in practical operation. In order to improve the reliability of blade design, it is necessary to take these stochastic parameters into account. In this study, a variable cross-section twisted blade is investigated and geometrical parameters, material parameters and load parameters are considered as random variables. A reliability analysis method as a combination of a Finite Element Method (FEM), a surrogate model and Monte Carlo Simulation (MCS), is applied to solve the blade reliability analysis. Based on the blade finite element parametrical model and the experimental design, two kinds of surrogate models, Polynomial Response Surface (PRS) and Artificial Neural Network (ANN), are applied to construct the approximation analytical expressions between the blade responses (including maximum stress and deflection) and random input variables, which act as a surrogate of finite element solver to drastically reduce the number of simulations required. Then the surrogate is used for most of the samples needed in the Monte Carlo method and the statistical parameters and cumulative distribution functions of the maximum stress and deflection are obtained by Monte Carlo simulation. Finally, the probabilistic sensitivities analysis, which combines the magnitude of the gradient and the width of the scatter range of the random input variables, is applied to evaluate how much the maximum stress and deflection of the blade are influenced by the random nature of input parameters.

Keywords: Finite element method, Monte Carlo simulation, probabilistic sensitivity analysis, reliability analysis, surrogate model, turbine blade

INTRODUCTION

The turbine blade is one of the key components in a steam turbine. There are many unmeasurable and uncontrollable factors in the process of blade design, manufacturing, installation and operation that result in the randomness of structural responses. The traditional deterministic design methods (Yan *et al*., 2005; Liu and Meng, 1999) ignore these stochastic parameters effects, or make up the randomness through a conservative assumption (such as safety factor). So, it is difficult to explain why the blade is failed in normal operation as it is designed correctly by the traditional deterministic method and is also difficult to evaluate quantitatively how much the blade is safe. To realize the high reliability performance of the blade, it is necessary to consider these stochastic parameters and carry out reliability analysis based design.

In blade reliability analysis, structure responses of the blade (such as stress, deformation and frequencies) are obtained by a finite element method and the limit state functions are implicit with respect to basic random variables. Reliability analysis techniques, such as FORM and SORM (Choi *et al*., 2007; Grandhi and Wang, 1998), require limit state function gradients with respect to the basic random variables of finding most probable failure point at each iteration, but it is very difficult to obtain the gradients of the limit state function with respect to random variables when the limit state function is implicit.

Monte Carlo Simulation (MCS) can be applied to many practical problems, allowing the direct consideration of any type of probability distribution for the random variables; however, the computation time can be prohibitively high, especially when the structure exhibits non-linear behavior or the numerical model is rather complex. Although some variance reduction techniques (Park, 1994; Disciuva and Lomario, 2003), such as importance sampling and Latin hypercube sampling, have been proposed to reduce the number of samples and reduce the computational time to a certain extent, it is still not widely used in practical engineering.

Surrogate-based reliability analysis is considered to be an effective approximation approach for computationally expensive models with implicit limit functions (Queipo *et al*., 2005; Youn and Choi, 2004).

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The basic idea of the approach consists in the substitution of the real limit state function by approximate simple functions in the neighborhood of the design points. As a surrogate model commonly presents a simple form and sometimes is represented by an explicit expression, the computational cost of the operation can be reduced with respect to the cost required when the real limit state function is used. Any of the classical procedures for structural reliability evaluation can be applied on the surrogate limit state function. Response Surface Method (RSM) (Duan and Zhao, 2009; Liu and Moses, 1994; Herbert and Armando, 2004), Artificial Neural Network techniques (ANN) (Duan and Wang, 2010; Deng *et al*., 2005; Elheewy *et al*., 2006), spline and kriging (Rao, 2002) are examples of methods used to generate surrogates.

Sensitivity analysis can quantitatively specify how the random input variables parameters influence the structural response. But in deterministic sensitivity analysis (Xie *et al*., 2005), a deterministic variation of an input parameter that is used to determine the gradient usually does not take the physical range of variability into account and gradient information is local information only.

A variable-section twisted blade, which usually locates in intermediate pressure stage and low pressure stage of steam turbine, has more complex geometrical shape than an equal-section blade. It operates in an extremely harsh environment, i.e., high temperature, high pressure, large centrifugal force, steam force and steam-excited vibration. So far, there is seldom discussion on the strength reliability analysis of a variable-section twisted blade considering stochastic parameters effects and there is no discussion on how to quantitatively evaluate the sensitivities of structural responses with respect to the random input parameters considering the physical range of variability of the input parameter.

In this study, a variable-section twisted steam turbine blade is investigated and a finite element model is built parametrically. The geometrical parameters, material parameters and load parameters of the blade are considered as random input variables, while the maximum deflection and maximum equivalent stress are stochastic outputs. Design of Experiments (DOE) is applied to create sample points. A quadratic polynomial with cross terms and a feed-forward back-propagation network (BP network) are separately selected to construct an approximation function as a surrogate of the finite element solver according to sample points. Then the Monte-Carlo method is used to obtain the statistical characteristics and cumulative distribution function of the maximum deflection and the maximum equivalent stress of the blade. Probabilistic sensitivities analysis, which not only takes the gradient at a particular location into account, but also all the values of the random input parameter, is considered to evaluate how much the output parameters are influenced by the random input parameters. Scatter plots of structural responses with respect to the random input variables are illustrated to analyze how to optimize the random input variables to improve the reliability of blade.

MATERIALS AND METHODS

Response surface method: In the original conceptual form of the response surface technique, polynomials are used to approximate structural response functions. Polynomials employed in the response surface usually have a quadratic form. The response surface, approximated by a quadratic polynomial with cross terms, can be expressed as:

$$f(\mathbf{x}) = \beta_0 + \sum_{i=1}^{n}\beta_i x_i + \sum_{i=1}^{n}\sum_{j\le 1}^{n}\beta_{ij}x_i x_j \tag{1}$$

where, $f(\mathbf{x})$ is the quadratic polynomial with the cross terms used to represent the real structural response function, $\mathbf{x} = (x_1, x_2, \ldots, x_i, \ldots, x_n)$ $(i = 1, 2, \ldots, n)$ are design variables vector, n is the number of design variables, β_0, β_i, β_{ij} are called regression coefficient and among them β_0 is the coefficient of the constant term, β_i are the coefficient of the linear terms, β_{ij} are the coefficient of the quadratic terms.

For N_s sample points, the set of equations specified in Eq. (1) can be expressed in matrix form as:

$$\mathbf{f} = \mathbf{X\beta} + \boldsymbol{\varepsilon}, \; E(\boldsymbol{\varepsilon}) = 0, \; V(\boldsymbol{\varepsilon}) = \sigma^2 I \tag{2}$$

where,

X = An $N_s \times N_c$ matrix with the design variable values as the sampled points
N_c = The number of regression coefficients
β = The regression coefficient column matrix
ε = The error column matrix, the error expected value E (ε) is zero matrix
V (ε) = Variance of error

The estimated parameters $\hat{\boldsymbol{\beta}}$ (by least squares) are unbiased and have minimum variance. It can be found as:

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}^{\mathrm{T}}\mathbf{X})^{-1}\mathbf{X}^{\mathrm{T}}\mathbf{f} \tag{3}$$

The predicted response function is given by:

$$\hat{\mathbf{f}} = \mathbf{X}\hat{\boldsymbol{\beta}} = \mathbf{X}(\mathbf{X}^{\mathrm{T}}\mathbf{X})^{-1}\mathbf{X}^{\mathrm{T}}\mathbf{f} \tag{4}$$

Artificial neural network: An Artificial Neural Network (ANN) includes nodes and connections which link the nodes. Before a neural network can act as a surrogate, it has to be trained by adjusting these

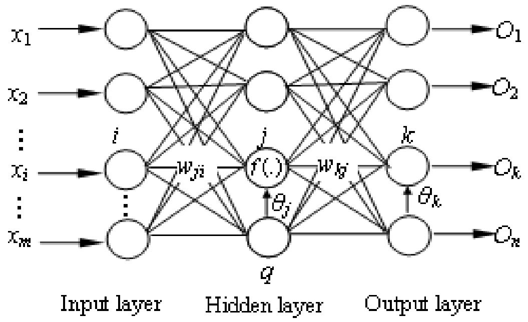


Fig. 1: BP neural network architecture

weights. The most widely used network type for approximation problems is the multi-layer perception, which is also called a feed-forward Back-Propagation network (BP network). In this study, a typical three-layer BP network is used and its architecture is shown in Fig. 1.

There are m nodes in the input layer, q nodes in hidden layer and n nodes in output layer and w_{ji} is the connecting weight from the j neuron in the hidden layer to the i neuron in the input layer, while θ_j is the threshold of the j neuron in the hidden layer. w_{kj} is the connecting weight from the k neuron in the output layer to the j neuron in the hidden layer and θ_k is the threshold of the k neuron in the output layer. In the reliability, the m neurons of the input layer represent the m random input variables x_i (i = 1, 2, … , m). These values are directly transmitted to the q neurons of the hidden layer affected linearly by weight w_{ji} and threshold θ_j. The total activation value of each neuron in this layer is:

$$a_j = \sum_{i=1}^{m} w_{ji} x_i + \theta_j \tag{5}$$

The output of each neuron in the hidden layer is a linear or nonlinear function of this activation value:

$$y_j = f(a_j) = f(\sum_{i=1}^{m} w_{ji} x_i + \theta_j) \tag{6}$$

where, f (.) is called the activation function. The most common non-linear activation function is the logistic sigmoidal function. It is given by:

$$f(x) = \frac{1}{1 + \exp(-\alpha x)} \tag{7}$$

where, α is a parameter defining the slope of the function, usually $\alpha = 1$.

Finally, the information is transformed by the neuron of the output layers in the same way. The activation function in the output layers is a linear function, so the expectant output value O_k is given by:

$$O_k = \sum_j w_{kj} y_j + \theta_k \tag{8}$$

Usually the predicted output value O_k from the network will not be the same as the actual output value t_k used in the training process. For each input-output pattern, the square of error E_p is written as follows:

$$E_p = \frac{1}{2} \sum_k (t_k - O_k)^2 \tag{9}$$

where,
k = The number of neurons in the output layer

The average system error is given by:

$$E = \frac{1}{2P} \sum_{p=1}^{P} E_p \tag{10}$$

where,
P = The number of training patterns

The standard back-propagation algorithm is to adjust the different weights and thresholds as well as the derivatives of E_p with respect to the input data to make the square of error least. More details of this process can be seen in the reference (Haykin, 1994).

Surrogate model-based Monte Carlo simulation: It is time consuming to perform finite element analysis if Monte Carlo Simulation (MCS) is used directly. In contrast, evaluating a surrogate model requires only a fraction of a second. Hence, the Monte Carlo simulation samples can be produced by an approximation function from the surrogate model and the structural responses can be simulated for thousands and thousands of times. The basic idea of MCS based on a surrogate model is that according to the distribution of random input variables, the locations and values of sampling points of input variables are created by design of experiments, such as Central Composite Design (CCD), or Latin-Hypercube Sampling (LHS) (Haldar and Mahadevan, 2000). The values of sampling points of output variables are obtained by FEM and a quadratic polynomial or a BP network is employed to fit these sample points and obtain the approximate function between the output responses and the input variables. When the errors of the approximate function are less than the desired requirement, the approximate function (surrogate model) is substituted for the FEM model and is used to create the Monte Carlo simulation. Furthermore, the statistical characteristics and cumulative distribution function of output variables are obtained and the reliability analysis can be carried out according to the limit state function.

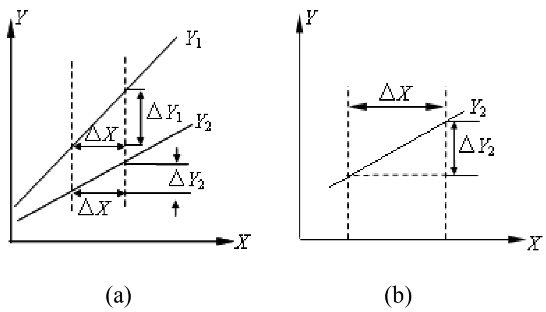


Fig. 2: Probabilistic sensitivity analysis

Probabilistic sensitivity analysis: Sensitivity analysis can quantitatively show how the random input variables parameters influence the structural response and denote how to modify which random input parameters to improve the structure reliability. But deterministic sensitivity analysis has some disadvantages. For example, to evaluate deterministic sensitivities, each input parameter can be varied by ±10% (one at a time) while keeping all other input parameters constant and then seeing how the output parameters react to these variations. An output parameter would be considered very sensitive with respect to a certain input parameter if a large change of the output parameter value is observed.

A deterministic variation of an input parameter that is used to determine the gradient usually does not take the physical range of variability into account. An input parameter varied by ±10% is not meaningful for the analysis if ±10% is too large or too little compared with the actual range of physical variability and randomness. Moreover, the gradient information in deterministic sensitivities is local information. It does not take into account that the output parameter may react more or less with respect to variation of input parameters at other locations in the input parameter space. However, in the probabilistic sensitivities, the physical range of variability is inherently considered because of the distribution functions for input parameters. The probabilistic sensitivities approach not only takes the magnitude at a particular location into account, but also all the values the random output parameter can have within the space of the random input variables. Probabilistic sensitivities measure how much the range of scatter of an output response is influenced by the scatter of the random input variables. Hence, both effects have an influence on probabilistic sensitivities: the magnitude of the gradient, plus the width of the scatter range of the random input variables. This is illustrated in the Fig. 2. If a random input variable has a certain given range of scatter ΔX, then the scatter of the corresponding random output response is larger and the larger the magnitude of the output response curve is, $\Delta Y_1 > \Delta Y_2$, denoted in Fig. 2a. But remember that an output response with a moderate magnitude can have a significant scatter if the random input variables have a wider range of scatter, shown in Fig. 2b.

The probabilistic sensitivity analysis is based on the results of Monte Carlo Simulation. A statistical significant test is used to judge probabilistic sensitivities. Suppose the probabilistic sensitivity of an output response Y_i with respect to random input variable X_i is denoted as $\partial Y_i/\partial X_i$; the hypothesis testing is:

$$\begin{aligned} H_0 &: \frac{\partial Y_i}{\partial X_i} = 0 \\ H_1 &: \frac{\partial Y_i}{\partial X_i} \neq 0 \end{aligned} \tag{11}$$

Given a confidence level γ, calculate the probability P_i that H_0 is true based on MCS results. If $P_i > 1-\gamma$, accept this hypothesis test, which means the Y_i is not sensitive to X_i. The sensitivity of Y_i with respect to the X_i can be negligible. Otherwise, accept the X_i and use $1-P_i$ to express the probabilistic sensitivity of Y_i with respect to X_i.

Performance functions of blade: When the steam turbine blade operates in a stable condition, the orientation and flow of steam remains almost unchanged and the steam flow is considered as a steady flow. According to the strength limit condition, when the maximum stress σ_{max} of the blade is less than the yield strength σ_s of the material, the blade can satisfy the static strength requirement, so the corresponding performance function G_1 (X) is:

$$G_1(\mathbf{X}) = \sigma_s - \sigma_{max} \tag{12}$$

where,

X = The random variables vector influencing σ_{max} and σ_s

According to the deformation limit condition, when the maximum deflection of blade δ_{max} is less than the allowable maximum deflection $[\delta_{max}]$, the blade can satisfy the deformation requirement, so the corresponding performance function G_2 (X) is:

$$G_2(\mathbf{X}) = [\delta_{max}] - \delta_{max} \tag{13}$$

A case study: A variable-section twisted blade of the 24th low pressure stage in steam path of some steam turbine is selected to be an example. The main parameters of the 24th low pressure stage are shown in Table 1.

Table 1: Main parameters of the 24th low pressure stage

Parameters name	Symbol	Value	Parameters name	Symbol	Value
Blade number	Z_b	94	Enthalpy drop (kJ/kg)	Δh_t^0	74.020
Nozzle number	Z_n	42	Degree of reaction*	Ω	0.414
Nozzle diameter (m)	d_n	1.677	Pressure before stage (N/m^2)	p_{s1}	553000
Blade diameter (m)	d_b	1.678	Pressure behind stage (N/m^2)	p_{s2} (Pa)	423000
Nozzle span (m)	l_n	0.426	Temperature before stage (°C)	T_1	374.100
Blade span (m)	l_b	0.432	Temperature behind stage (°C)	T_2	313.500
Nozzle angle* (°)	α_{1m}	13.550	Blade chord* (m)	B	0.070
Blade angle* (°)	β_2	20.160	Blade stagger angle (°)	βy	79.250

*: α_1, β_2 and Ω are the values at the blade average diameter; B: The chord of the first cross-section at the blade root

Table 2: Random parameters and statistical characteristics

Parameters	Mean value	Coefficient of variance	Distribution type
l_b (m)	0.432	0.02	Normal
B (m)	0.070	0.02	Normal
β_y (°)	79.000	0.01	Normal
α_{1m} (°)	13.550	0.01	Normal
E (N/m^2)	2.17×10^{11}	0.05	Normal
ρ (kg/m^3)	7850	0.05	Normal
Ω (r/min)	3000	0.01	Normal
σ_s (N/m^2)	370×10^6	0.05	Normal

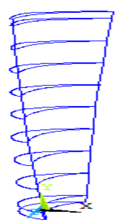


Fig. 3: Blade profile

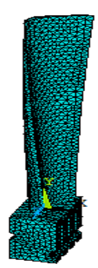


Fig. 4: Finite element model of blade

In this study, the blade span l_b, the blade chord B, stagger angular βy, nozzle steam angle α_{1m}, Young's modulus E, density ρ, rotation speed ω and material yield strength σ_s are considered as random variables. We calculate the statistical characteristics and cumulative distribution functions of structural responses, including the maximum stress σ_{max} and the maximum deflection δ_{max} of the blade and their sensitivities with respect to random input parameters and evaluate the blade reliability. The material of the blade is chromium alloy 1Cr13. The random parameters and their statistical characteristics are shown in Table 2.

Finite element parametric model of blade: The twisted blade is composed of blade profile and blade root. The blade profile is a complex shape assembled by several cross sections with molded lines, shown in Fig. 3. The blade root is a fork-type and is wedged tightly. The parameters l_b, B, β_y, α_{1m}, E, ρ and ω are selected as basic random input variables, which are also the variables in the process of finite element parametric model. The maximum stress σ_{max} and the maximum deflection δ_{max} are chosen as random output responses. The three-dimension solid element Solid 45 is used to mesh the blade, the whole model is composed of 2494 nodes and 7784 elements, shown in Fig. 4, here, the coordinate system x, y, z represent the blade peripheral direction, span direction and axial direction.

Steam force calculation: The steam force acting on the blade can be expressed by the peripheral component force P_x and axial component P_z, shown in Fig. 5. The 1-1 axis and the 2-2 axis are the minimum principal axes of inertial and maximum principal axes of inertial respectively.

For a variable cross-section twisted blade, the variation of steam force along the blade span must be considered because the steam flow will be changed a lot along the blade span. It is difficult to find analytical expressions to calculate the steam parameters of blade, so the steam force of blade is usually calculated by an approximate method. The blade is divided into n segments along its span and the total number of the cross section is n+1. The length of every segment is denoted as Δx_j (j = 1, 2, …, n), the peripheral component of steam force P_{xj} and the axial component of steam force P_{zj} at the j^{th} segment can be obtained by:

$$P_{xj} = \frac{\Delta G_j}{Z_b}(c_{1uj} + c_{2uj}) \tag{14}$$

$$P_{zj} = \frac{\Delta G_j}{Z_b}(c_{1zj} - c_{2zj}) + (p_{1j} - p_{2j})\Delta x t_{bj} \tag{15}$$

where,

t_{bj} = The pitch of the j section, $t_{bj} = \frac{\pi d_{bj}}{Z_b}$
d_{bj} = The diameter of the j section
c_{1uj}, c_{2uj} = Respectively the steam tangential component velocity at nozzle exit and at blade inlet in the j section
c_{1zj}, c_{2zj} = Respectively the axial component velocity at nozzle exit and at blade exit in the j section

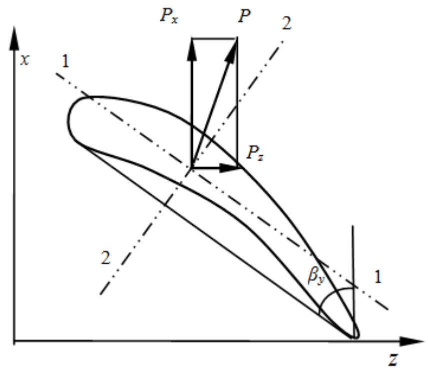


Fig. 5: Steam force on blade

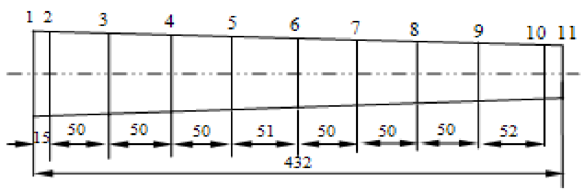


Fig. 6: Segments of twisted blade

p_{1j}, p_{2j} = Blade inlet pressure and outlet pressure in j section respectively

ΔG_j = The steam flow of the j^{th} segment and it can be calculated by:

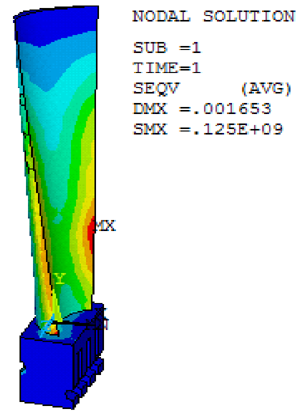
$$\Delta G_j = \left(\frac{\pi d_b \Delta x_j \sin \beta_2 \mu_b w_{2t}}{v_{2t}} \right)_j \tag{16}$$

where, μ_b is the blade flow coefficient, v_{2t} is the blade export isentropic specific volume and w_{2t} is the idea relative velocity of steam exporting from blade. In this study, the blade span is divided in to 10 segments and a total of 11 cross sections, shown in Fig. 6.

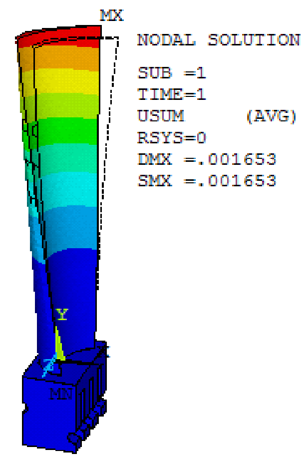
The steam flow parameters in every section, P_{xj} and P_{zj} in every segment are obtained according to the thermodynamic calculation, here, μ_b is given as 0.96, v_{2t} is 3.645 m^3/kg from p_{s1} expanding isentropic to p_{s2}. The details of process are presented in reference (Duan, 2009).

The loading boundaries of blade finite element model are: to put the steam force on every segment of blade to the corresponding loading surface and at the same time to load rotating speed to consider the role of centrifugal force. The displacement boundaries are: to constraint the x displacement and z displacement of fork-type blade root, i.e., $u_x = 0$, $u_z = 0$ and to constraint the all the degree of freedom in the pin holes, i.e., $u_x = u_y = u_z = 0$.

When all the basic input random variables are equal to mean values, the equivalent stress and the deformation distribution of the blade are shown in Fig. 7. The value of the maximum stress is 125 MPa and it is located near the middle of steam-out edge because of the twisted recovery produced by centrifugal force, which makes the distribution of maximum stress access to the middle profile. The value of the maximum deflection is 1.65 mm and it occurs on the tip of the blade.



(a) Equivalent stress of blade

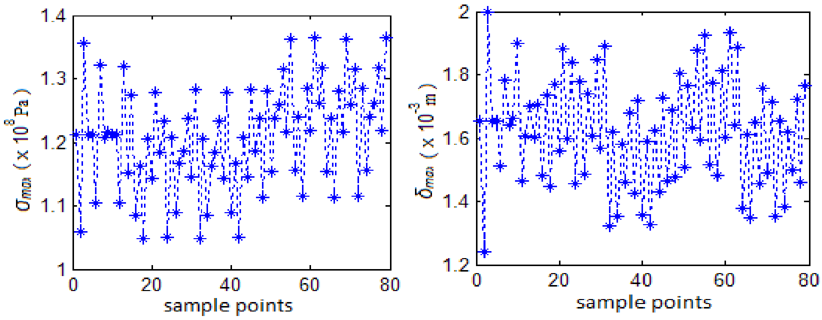


(b) Deformation of blade

Fig. 7: Equivalent stress and deformation of blade

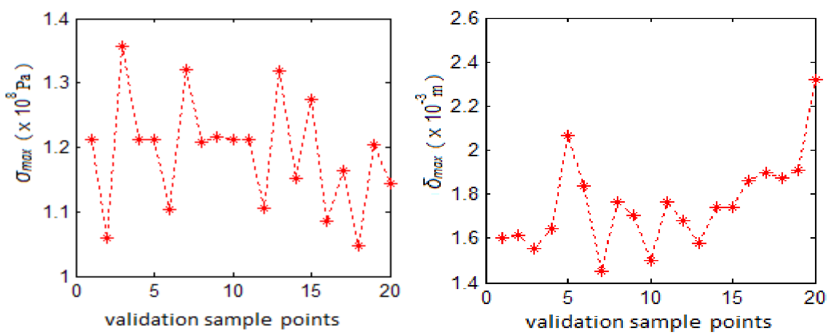
Design of experiments: As there are 7 random input variables, 79 sample points are created by CCD method and perform looping of finite element model for 79 times. The sample points to regress the approximate function are shown in Fig. 8.

In order to validate the BP network, 20 validation sample points, which are different from the former sample points created by CCD, are created by Latin-Hypercube Sampling method and corresponding sample points of σ_{max} and δ_{max} are obtained by performing loop of finite element model for 20 times. The validation sample points are shown in Fig. 9.



(a) Sample points for σ_{max} (b) Sample points for δ_{max}

Fig. 8: Sample points for constructing surrogate



(a) Validation sample points for σ_{max} (b) Validation sample points for δ_{max}

Fig. 9: Sample points for validation

Construct surrogate model:

Response surface: A quadratic polynomial with cross terms is used to fit these sample points. Using the forward-stepwise regression technology, a confidence level of 95% was used to filter out insignificant terms of the regression equation. As the variables dimension has large difference, all the random input variables are first to be carried out a linear centralization processing. The transformed random input variables are denoted as l_b', B', β_y', α_{1m}', E', ρ', ω'. The linear transformation is shown in Eq. (17):

$$\begin{aligned}
l_b' &= 1.27091\times10^{2} l_b - 54.9032 \\
B' &= 7.84332\times10^{2} B - 54.9032 \\
\beta_y' &= 79.4276\beta_y - 109.806 \\
\alpha_{1m}' &= 4.64549\times10^{2}\alpha_{1m} \\
E' &= 1.01204\times10^{-10} E - 21.9613 \\
\rho' &= 2.79762\times10^{-3}\rho - 21.9613 \\
\omega' &= 3.49702\times10^{-1}\omega - 109.806
\end{aligned} \tag{17}$$

The regression result is the sum of every random input variables multiplying the corresponding coefficient, that is:

$$\sum_{i=1}^{10} b_i X_i$$

Table 3: Regression terms and coefficients for σ_{max}

X	b	X	b
C	1.20730E+08	ω'	2.14720E+06
l_b'	5.42382E+06	$l_b'.\beta_y'$	-6.07942E+05
β_y'	4.30207E+06	$l_b'.\rho'$	2.03939E+05
α_{1m}'	1.41890E+05	$l_b'\omega'$	9.58327E+04
ρ'	3.73107E+06	$\beta_y'.\rho'$	1.61684E+05

Table 4: Regression terms and coefficients for δ_{max}

X	b	X	b
C	1.61907E-03	ω'	1.84203E-05
l_b'	1.37174E-04	$l_b'.l_b'$	-3.70115E-06
β_y'	5.37609E-05	$E'.E'$	3.90049E-06
E'	-7.40151E-05	$l_b'.E'$	-6.28007E-06
ρ'	1.60900E-05		

Table 3 and 4 show the input variables and their corresponding coefficients for regressing maximum stress σ_{max} and maximum deflection δ_{max}, respectively.

BP network: The input variables l_b, B, β_y, α_{1m}, E, ρ and ω act as network input corresponding to seven neurons of the input layer, while σ_{max} and δ_{max} are network outputs corresponding to two neurons of the output layer. The number of hidden neurons in the BP network is 25. The BP network of the blade is shown in Fig. 10.

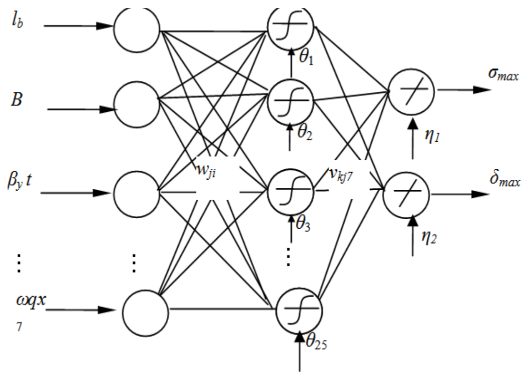


Fig. 10: BP model of blade

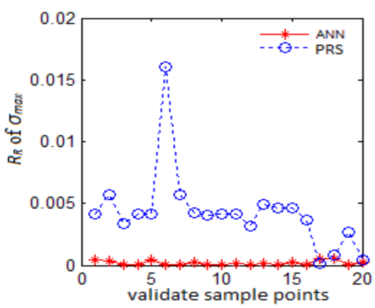


Fig. 11: R_R of σ_{max}

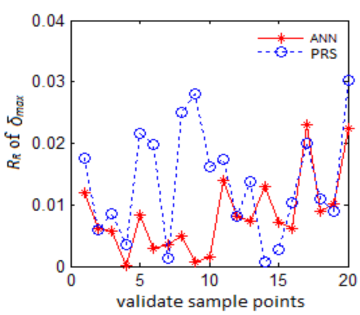


Fig. 12: R_R of δ_{max}

The mapping relationship between output responses and input variables is denoted as the following:

$$\sigma_{\max} = \eta_1 + \sum_{j=1}^{25}\left[v_{1j} f(\sum_{i=1}^{7} w_{ji}x_i + \theta_j) \right] \tag{18}$$

$$\delta_{\max} = \eta_2 + \sum_{j=1}^{25}\left[v_{2j} f(\sum_{i=1}^{7} w_{ji}x_i + \theta_j) \right] \tag{19}$$

The learning rate is given 0.05 and the minimum value of expected error is given 1×10^{-5}. Seventy nine training samples are used to train the BP network by Levenberg-Marquardt rule (Haykin, 1994). The training errors attained 9.50634×10^{-6} after 96 iterations. The weights and thresholds in the hidden layer and the output layer are also obtained.

Surrogate model evaluation: The relative residual error R_R is used to describe the error of predicted values with actual values in validation set, R_R is defined as:

$$R_R = \frac{y-\hat{y}}{y} \tag{20}$$

where,

y = The actual value obtained from FEM calculation
$\hat{y}$ = The predicted value obtained from the surrogate model

The 20 sample points produced by LHS are used as validation of surrogate model. Figure 11 and 12 show the values of R_R of σ_{max} and δ_{max} obtained by Polynomial Response Surface (PRS) and BP Artificial Neural Network (ANN) respectively. For the σ_{max}, the maximum values of R_R from the two surrogate model are 1.6e-2 (PRS) and 4.95e-4 (ANN) respectively; for the δ_{max}, the maximum values of R_R are 2.8112e-3 (PRS) and 2.3065e-3 (ANN) respectively. The precision of ANN model is little higher than PRS model.

RESULTS AND DISCUSSION

Substitute the surrogate model for the FEM model and create the 100000 Monte-Carlo simulation samples to carry out the statistical analysis for these simulation results, the following quantities can be obtained:

- Statistical characteristics of the maximum stress σ_{max} and the maximum deflection δ_{max}, shown in Table 5. It can be seen that the statistical parameters of σ_{max} and δ_{max} from the two methods are approached; the data discreteness from ANN-MCS is larger.
- The CDF of σ_{max} and the CDF of δ_{max} from the PRS-MCS and ANN-MCS are shown in Fig. 13.
- **Reliability evaluation of blade:** As the mean $\mu_{\sigma_{\max}}$ and the standard deviation $\sigma_{\sigma_{\max}}$ of the maximum stress σ_{max} are obtained in the above step and the statistical characteristic of σ_s is also given, the reliability index β_1 corresponding to G_1 (X) is:

$$\beta_1 = \frac{\mu_{\sigma_s} - \mu_{\sigma_{\max}}}{\sqrt{\sigma_{\sigma_s}{}^2 + \sigma_{\sigma_{\max}}{}^2}} \tag{21}$$

where,

μ_{σ_s}= The mean value of σ_s
σ_{σ_s}= The standard deviations of σ_s

The reliability of blade corresponding to G_1 (X) is:

$$P_1 = \Phi(\beta_1) \tag{22}$$

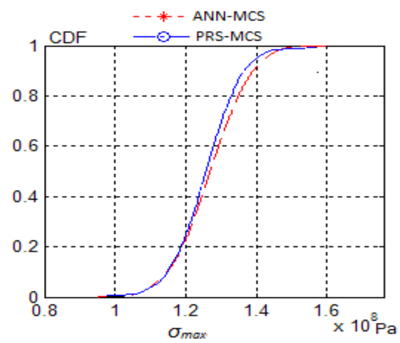
Table 5: Statistical characteristics of σ_{max} and δ_{max}

Parameters		Mean	S.D.	Min. value	Max. value
σ_{max}	PRS-MCS	120.7400	87.1590	154.2900	8.9510
(MPa)	ANN-MCS	126.9700	89.3310	177.3900	9.4345
δ_{max}	PRS-MCS	1.6194	0.9187	2.4289	0.1844
(10^{-3} m)	ANN-MCS	1.7000	1.2000	2.6000	0.1944

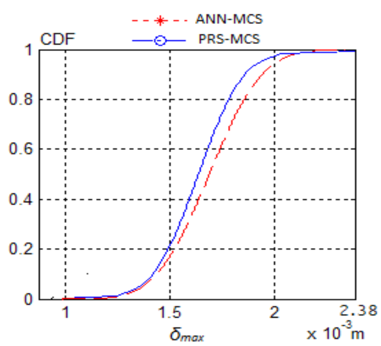
S.D.: Standard deviaton; Min.: Minimum; Max.: Maximum

Table 6: Strength reliability of blade

Reliability	β_1	P_1	β_2	P_2
PRS-MCS	20.55	1	2.854	0.9978
ANN-MCS	11.70	1	2.310	0.9896
MCS	11.45	1	2.031	0.9788



(a) CDF of σ_{max}



(b) CDF of δ_{max}

Fig. 13: CDF of σ_{max} and δ_{max}

where,

$\Phi(.)$ = The standard normal cumulative distribution function

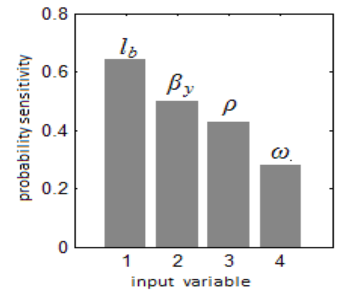
The reliability index β_2 corresponding to G_2 (X) is:

$$\beta_2 = \frac{\mu_{[\delta_{max}]} - \mu_{\delta_{max}}}{\sqrt{\sigma_{[\delta_{max}]}^2 + \sigma_{\delta_{max}}^2}} \tag{23}$$

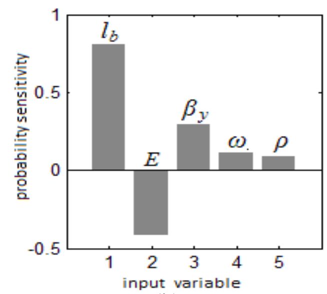
where,

$\mu_{\delta_{max}}, \mu_{[\delta_{max}]}$ = The mean values of δ_{max} and $[\delta_{max}]$ respectively

$\sigma_{\delta_{max}}, \sigma_{[\delta_{max}]}$ = The standard deviations of δ_{max} and $[\delta_{max}]$ respectively. Here, $[\delta_{max}]$ is $l_b/200$



(a)



(b)

Fig. 14: Probability sensitivities of σ_{max} and δ_{max}

Table 6 shows the results of β_1, β_2 and P_1, P_2 from the different methods PRS-MCS, ANN-MCS and direct MCS. For G_1 (X), the reliability P_1 from the three methods is the same, but β_1 from ANN-MCS is more approach to that from direct MCS. For G_2 (X), the value β_2 from ANN-MCS is also more approach to that from direct MCS.

- **Probability sensitivities of σ_{max} and δ_{max} with respect to the random input variables:** The confidence level γ is given to 0.025 which means the incorrect probability of hypothesis testing is 2.5%. According to the results of statistical significant test, the input random variables are divided into two groups: important group and unimportant group. The variables which accept the statistical significant test belong to unimportant group and the other variables refusing the test belong to important group. The variables in the important group are sorted by the absolute value of their probability sensitivity (1-P_i), shown in Fig. 14. The sequence of sensitivity of σ_{max} with respect to random input variables from high to low is l_b, β_y, ρ and ω, while the sequence of the sensitivity of δ_{max} with respect to random input variables is l_b, E, β_y, ω and ρ.

CONCLUSION

- This study combines the finite element, surrogate model (including quadratic polynomial response surface and BP artificial neural network) and Monte Carlo simulation

method to obtain the statistical characteristics and cumulative distribution functions of maximum stress and maximum deflection of a variable-section twisted blade of a steam turbine and carries out strength reliability analysis in the presence of random geometrical parameters, material parameters and load parameters.
- Comparison of PRS-MCS and ANN-MCS with direct MCS shows the surrogate-based reliability analysis approach can act as an ideal tool for the reliability analysis and design of a steam turbine blade. The approach is applied to complex structure reliability analysis with an implicit limit state function.
- Probability sensitivities analysis is used to quantitatively specify the degree of influence of random input variables on maximum deflection and maximum stress of the blade.

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