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Research Article

Ratio Type Exponential Estimator for the Estimation of Finite Population Variance under Two-stage Sampling

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Abstract: In this study we propose an improved exponential ratio type estimator in estimating the finite population variance in two stage sampling under two cases: (i) when sum of the weights cannot equal to one and (ii) when sum of the weights are equal to one. The bias and Mean Squared Error (MSE) are derived up to first order of approximation. The efficiency conditions, under which proposed estimator is more efficient than the usual sample variance estimator and regression estimator are obtained. Numerical and simulated studies are conducted to support the superiority of the estimators. Real data set is used to observe the performances of estimator.

Keywords: Bias, efficiency, exponential ratio type estimator, MSE, two-stage sampling

INTRODUCTION

In many situations, information on the auxiliary variable is required either at the designing stage or estimation stage or both stages, to increase precision of the estimators. Ratio, product and regression estimators are often used when advance knowledge of population mean of the auxiliary variable is readily available.

In application when sampling frame is not available, then it is expense or not feasible to obtain the sampling units directly from the population. Instead one can use the two stage sampling which is generally preferable for large scale surveys and at each stage frame is easily accessible.

Mahalanobis (1967) was the first one, who introduced the procedure of two-stage sampling which was further extended by Godambe (1951). A better approach for multi-stage design was introduced by Saxena *et al*. (1984). Mostly research papers appeared in two-stage sampling for estimation of population mean or total including Yunusa (2010), Saini (2013) and Singh *et al*. (2013). Many researchers like Wolter (1985), Das and Tripathi (1978), Isaki (1983), Upadhyaya *et al*. (2004), Shabbir and Gupta (2007), Singh and Vishwakarma (2008) and Shabbir and Gupta (2010) worked in estimating the finite population variance by using the simple random sampling. To best of our knowledge very few research papers exist in estimating the population variance, so in this study an attempt has been made for estimation of finite population variance by using the auxiliary information in two stage sampling.

Consider a finite population $U = (U_1, U_2, .., U_N)$ of *N* first stage units (*psus*) such that *i*-th *psu* $U_i = (1, 2, .., j, .., M_i)$ consist of M_i second stage units (*ssus*) and $M = \sum_{i=1}^{N} M_i$. Let y_{ij} and x_{ij} be the values of the study variable (y) and the auxiliary variable (x) respectively for the *j*-th *ssu* of the *i*-th *psu* $U_i = (j = 1, 2, .., M_i;\ i = 1, 2, .., N)$. To estimate S_y^2 under two-stage sampling scheme, it is assumed that S_x^2 known.

Let we define the following notations and symbols:

$$\gamma = \frac{1-f}{n};\ f = \frac{n}{N};\ \gamma_{2i} = \left(\frac{M_i}{\bar{M}}\right)^2 \frac{1}{nN}\left(\frac{1}{m_i} - \frac{1}{M_i}\right);$$
$$\bar{P} = \frac{1}{N}\sum_{i=1}^{N} \bar{P}_i, \text{ where } \bar{P}_i = \frac{1}{M_i}\sum_{j=1}^{M_i} P_{ij},$$
$$S_{1p}^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(\frac{M_i}{\bar{M}}\bar{P}_i - \bar{P}\right)^2;$$
$$S_{1pq} = \frac{1}{N-1}\sum_{i=1}^{N}\left(\frac{M_i}{\bar{M}}\bar{P}_i - \bar{P}\right)\left(\frac{M_i}{\bar{M}}\bar{Q}_i - \bar{Q}\right);$$
$$S_{2pi}^2 = \frac{1}{M_i-1}\sum_{j=1}^{M_i}\left(P_{ij} - \bar{P}_i\right)^2;$$
$$S_{2pqi} = \frac{1}{M_i-1}\sum_{j=1}^{M_i}\left(P_{ij} - \bar{P}_i\right)\left(Q_{ij} - \bar{Q}_i\right)$$

where, $P, Q = X, Y$; and $P \neq Q$; S_{1p}^2 and S_{2pi}^2 are the variances among *psus* means and variances among subunits for the *i*-th *psus*, while S_{1pq} and S_{2pqi} are their corresponding covariances.

Assume that a sample of *n psus* is drawn from *U* and then a sample of m_i from M_i *ssus* units i.e., from the *i*-th selected *psu* using simple random sampling without replacement at both stages.

We define the following relative error terms.

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Let $e_0 = \frac{s_{y(2s)}^2 - S_y^2}{S_y^2}$ and $e_1 = \frac{s_{x(2s)}^2 - S_x^2}{S_x^2}$ such that $E(e_0) = E(e_1) = 0$.
$E(e_0^2) = \gamma\beta_{2(1y)}^* + \sum_{i=1}^{N}\gamma_{2i}\beta_{2(2yi)}^*$; $E(e_1^2) = \gamma\beta_{2(1x)}^* + \sum_{i=1}^{N}\gamma_{2i}\beta_{2(2xi)}^*$;
$E(e_0 e_1) = \gamma\,\gamma_{22(1)}^* + \sum_{i=1}^{N}\gamma_{2i}\gamma_{2(1)}^*\gamma_{22(2i)}^*$

where,

$$\beta_{2(1y)} = \frac{\mu_{40(1)}}{\mu_{20(1)}^2};\ \beta_{2(1x)} = \frac{\mu_{04(1)}}{\mu_{02(1)}^2};\ \beta_{2(2yi)} = \frac{\mu_{40(2i)}}{\mu_{20(2i)}^2};\ \beta_{2(2xi)} = \frac{\mu_{04(2i)}}{\mu_{02(2i)}^2};$$
$$\beta_{2(1y)}^* = \beta_{2(1y)} - 1;\ \beta_{2(1x)}^* = \beta_{2(1x)} - 1;\ \gamma_{22(1)}^* = \gamma_{22(1)} - 1;$$
$$\beta_{2(2yi)}^* = \beta_{2(2yi)} - 1;\ \beta_{2(2xi)}^* = \beta_{2(2xi)} - 1;\ \gamma_{22(2i)}^* = \gamma_{22(2i)} - 1;$$
$$\gamma_{rs(1)} = \frac{\mu_{rs(1)}}{\mu_{20(1)}^{r/2}\mu_{02(1)}^{s/2}};\ \mu_{rs(1)} = \frac{\sum\left(\frac{m_i}{M}y_i - \bar{Y}\right)^r\left(\frac{m_i}{M}x_i - \bar{X}\right)^s}{N};$$
$$\gamma_{rs(2i)} = \frac{\mu_{rs(2i)}}{\mu_{20(2i)}^{r/2}\mu_{02(2i)}^{s/2}};\ \mu_{rs(2i)} = \frac{\sum_{j=1}^{M_i}\left(y_{ij} - \bar{Y}_i\right)^r\left(x_{ij} - \bar{X}_i\right)^s}{N}$$

The usual variance estimator for population variance in two-stage sampling is given by:

$$\hat{S}_{0(2s)}^2 = s_{y(2s)}^2 \tag{1}$$

The *MSE* of $\hat{S}_{0(2s)}^2$ is given by:

$$MSE\left(\hat{S}_{0(2s)}^2\right) \cong S_y^4\left(\gamma\beta_{2(1y)}^* + \sum_{i=1}^{N}\gamma_{2i}\beta_{2(2yi)}^*\right) \tag{2}$$

The traditional regression estimator for population variance under two-stage sampling is given by:

$$\hat{S}_{reg(2s)}^2 = s_{y(2s)}^2 + b\left(S_{x(2s)}^2 - s_{x(2s)}^2\right) \tag{3}$$

where, *b* is the sample regression coefficient in two stage sampling.
The *MSE* of $\hat{S}_{reg(2s)}^2$ is given by:

$$MSE\left(\hat{S}_{reg(2s)}^2\right) \cong S_y^4\left\{\gamma\beta_{2(1y)}^*(1 - \rho_1^2) + \sum_{i=1}^{N}\gamma_{2i}\beta_{2(2yi)}^*(1 - \rho_{2i}^2)\right\} \tag{4}$$

where, $\rho_1^2 = \frac{\gamma_{22(2i)}^{*2}}{\beta_{2(1y)}^*\beta_{2(1x)}^*}$ and $\rho_{2i}^2 = \frac{\gamma_{22(2i)}^{*2}}{\beta_{2(2yi)}^*\beta_{2(2xi)}^*}$.

PROPOSED ESTIMATOR

On the lines of Gupta and Shabbir (2008), we propose the following an improved exponential ratio type estimator for population variance (S_y^2) in two-stage sampling, given by:

$$\hat{S}_{P(2s)}^2 = \left\{k_1 s_{y(2s)}^2 + k_2(S_x^2 - s_{x(2s)}^2)\right\}exp\left\{\frac{S_x^2 - s_{x(2s)}^2}{S_x^2 + s_{x(2s)}^2}\right\} \tag{5}$$

where, k_1 and k_2 are suitably chosen constants. We discuss the two cases:

- When $k_1 + k_2 \neq 1$
- When $k_1 + k_2 = 1$

To find the properties of our proposed estimator ($\hat{S}_{P(2s)}^2$), we consider the following two cases.

Case 1: When $k_1 + k_2 \neq 1$.

Using notations from above section, we have:

$$\hat{S}^2_{P(2s)} \cong \left\{k_1 S_y^2(1+e_0) - k_2 S_x^2 e_1\right\}\left\{1 - \frac{e_0}{2} + \frac{3}{8}e_1^2 - \ldots\right\}$$

To first order of approximation, we have:

$$\left(\hat{S}^2_{P(2s)} - S_y^2\right) \cong (k_1 - 1)S_y^2 + k_1 S_y^2\left(e_0 - \frac{e_1}{2} + \frac{3}{8}e_1^2 - \frac{1}{2}e_0 e_1\right) - k_2 S_x^2\left(e_1 - \frac{1}{2}e_1^2\right) \quad (6)$$

Using (6), the bias of $\hat{S}^2_{P(2s)}$ to first order of approximation, is given by:

$$Bias\left(\hat{S}^2_{P(2s)}\right) \cong \Big[(k_1 - 1)S_y^2 + k_1 S_y^2\Big\{\gamma\left(\frac{3}{8}\beta^*_{2(1x)} - \frac{1}{2}\gamma^*_{22(1)}\right) + \sum_{i=1}^N \gamma_{2i}\left(\frac{3}{8}\beta^*_{2(2xi)} - \frac{1}{2}\gamma^*_{22(2i)}\right)\Big\} + \frac{1}{2}k_2 S_x^2\left\{\gamma\beta^*_{2(1x)} + \sum_{i=1}^N \gamma_{2i}\beta^*_{2(2xi)}\right\}\Big] \quad (7)$$

Using (6), the *MSE* of $\hat{S}^2_{P(2s)}$to first order of approximation, is given by:

$$MSE\left(\hat{S}^2_{P(2s)}\right) \cong S_y^4\Big[1 + k_1^2\Big\{1 + \gamma\left(\beta^*_{2(1y)} + \beta^*_{2(1x)} - 2\gamma^*_{22(1)}\right) + \sum_{i=1}^N \gamma_{2i}\left(\beta^*_{2(2y)i} + \beta^*_{2(2x)i} - 2\gamma^*_{22(2i)}\right)\Big\} + k_2^2\varphi^2\left\{\gamma\beta^*_{2(1x)} + \sum_{i=1}^N \gamma_{2i}\beta^*_{2(2x)i}\right\} - 2k_1\Big\{1 + \frac{1}{2}\Big(\gamma\left(\frac{3}{4}\beta^*_{2(1x)} - \gamma^*_{22(1)}\right) + \sum_{i=1}^N \gamma_{2i}\left(\frac{3}{4}\beta^*_{2(2x)i} - \gamma^*_{22(2i)}\right)\Big)\Big\} - 2k_2\varphi\left\{\gamma\beta^*_{2(1x)} + \sum_{i=1}^N \gamma_{2i}\beta^*_{2(2x)i}\right\} + 2k_1k_2\varphi\Big\{\gamma\left(\beta^*_{2(2x)} - \gamma^*_{22(1)}\right) + \sum_{i=1}^N \gamma_{2i}\left(\beta^*_{2(2x)i} - \gamma^*_{22(2i)}\right)\Big\}\Big]$$

where, $\varphi = \frac{S_x^2}{S_y^2}$

or:

$$MSE\left(\hat{S}^2_{P(2s)}\right) \cong S_y^4\left(1 + k_1^2 A_1^* + k_2^2 B_1^* + 2k_1 k_2 C_1^* - 2k_1 D_1^* - 2k_2 E_1^*\right) \quad (8)$$

where,

$$A_1^* = 1 + \gamma\left(\beta^*_{2(1y)} + \beta^*_{2(1x)} - 2\gamma^*_{22(1)}\right) + \sum_{i=1}^N \gamma_{2i}\left(\beta^*_{2(2yi)} + \beta^*_{2(2xi)} - 2\gamma^*_{22(2i)}\right)$$

$$B_1^* = \varphi^2\left(\gamma\beta^*_{2(1x)} + \sum_{i=1}^N \gamma_{2i}\beta^*_{2(2xi)}\right)$$

$$C_1^* = \varphi\left\{\gamma\left(\beta^*_{2(1x)} - \gamma^*_{22(1)}\right) + \sum_{i=1}^N \gamma_{2i}\left(\beta^*_{2(2xi)} - \gamma^*_{22(2i)}\right)\right\}$$

$$D_1^* = 1 + \frac{1}{2}\left\{\gamma\left(\frac{3}{4}\beta^*_{2(1x)} - \gamma^*_{22(1)}\right) + \sum_{i=1}^N \gamma_{2i}\left(\frac{3}{4}\beta^*_{2(2xi)} - \gamma^*_{22(2i)}\right)\right\}$$

$$E_1^* = \frac{1}{2}\varphi\left(\gamma\beta^*_{2(1x)} + \sum_{i=1}^N \gamma_{2i}\beta^*_{2(2xi)}\right)$$

Now differentiating (8) with respect to k_1 and k_2, we get:

$$k_{1opt} = \frac{B_1^* D_1^* - C_1^* E_1^*}{A_1^* B_1^* - C_1^{*2}} \text{ and } k_{2opt} = \frac{A_1^* E_1^* - C_1^* D_1^*}{A_1^* B_1^* - C_1^{*2}}$$

Substituting the optimum values of k_1 and k_2 in (8), we get the minimum *MSE* of $\hat{S}^2_{P(2s)}$, given by:

$$MSE\left(\hat{S}^2_{P(2s)}\right)_{min} \cong S_y^4\left\{1 - \frac{\left(A_1^* E_1^{*2} + B_1^* D_1^{*2} - 2C_1^* D_1^* E_1^*\right)}{A_1^* B_1^* - C_1^{*2}}\right\} \quad (9)$$

Case 2: When $k_1 + k_2 = 1$

Under Case 2, the proposed estimator becomes:

$$\hat{S}^{*2}_{P(2s)} = \left\{k_1 s^2_{y(2s)} + (1 - k_1)\left(S_x^2 - s^2_{x(2s)}\right)\right\} exp\left\{\frac{S_x^2 - s^2_{x(2s)}}{S_x^2 + s^2_{x(2s)}}\right\} \quad (10)$$

Putting $k_2 = 1 - k_1$ in (7) and (8), we get the bias and *MSE* of $\hat{S}^{*2}_{P(2s)}$ respectively as given by:

$$Bias\left(\hat{S}^{*2}_{P(2s)}\right) \cong \Big[(k_1 - 1)S_y^2 + k_1 S_y^2\Big\{\gamma\frac{1}{2}\left(\beta^*_{2(1x)}\left(\frac{3}{4} - \varphi\right) - \gamma^*_{22(1)}\right) + \sum_{i=1}^N \gamma_{2i}\frac{1}{2}\left(\beta^*_{2(2xi)}\left(\frac{3}{4} - \varphi\right) - \gamma^*_{22(2i)}\right)\Big\} + \frac{1}{2}\varphi\left\{\gamma\beta^*_{2(1x)} + \sum_{i=1}^N \gamma_{2i}\beta^*_{2(2xi)}\right\}\Big] \quad (11)$$

And

$$MSE(\hat{S}^{*2}_{P(2s)}) \cong S_y^4\{1 + k_1^2(A_1^* + B_1^* - 2C_1^*) - 2k_1(B_1^* - C_1^* + D_1^* - E_1^*) + B_1^* - 2E_1^*\} \quad (12)$$

Now differentiating (12) with respect to k_1, we get:

$$k^*_{1opt} = \frac{(B_1^* - C_1^* + D_1^* - E_1^*)}{(A_1^* + B_1^* - 2C_1^*)}$$

Substituting the optimum value of k_1 i.e., k^*_{1opt} in (12), we get the minimum *MSE* of $\hat{S}^{*2}_{P(2s)}$, given by:

$$MSE\left(\hat{S}^{*2}_{P(2s)}\right)_{min} \cong S_y^4\left\{1 + B_1^* - 2E_1^* - \frac{(B_1^* - C_1^* + D_1^* - E_1^*)^2}{(A_1^* + B_1^* - 2C_1^*)}\right\} \quad (13)$$

EFFICIENCY COMPARISONS

We compare the proposed estimator with usual sample variance estimator and regression estimator in two-stage sampling as follows:

Condition (1): By (2) and (9):

$$MSE(\hat{S}^2_{P(2s)})_{min} < MSE\left(\hat{S}^2_{0(2s)}\right) \text{ if}$$
$$\gamma\beta^*_{2(1y)} + \sum_{i=1}^N \gamma_{2i}\beta^*_{2(2yi)} + \frac{\left(A_1^* E_1^{*2} + B_1^* D_1^{*2} - 2C_1^* D_1^* E_1^*\right)}{A_1^* B_1^* - C_1^{*2}} - 1 > 0$$

Condition (2): By (4) and (9):

$$MSE(\hat{S}^2_{P(2s)})_{min} < MSE\left(\hat{S}^2_{reg(2s)}\right) \text{ if}$$
$$\gamma\beta^*_{2(1y)}(1 - \rho_1^2) + \sum_{i=1}^N \gamma_{2i}\beta^*_{2(2yi)}(1 - \rho_{2i}^2) + \frac{\left(A_1^* E_1^{*2} + B_1^* D_1^{*2} - 2C_1^* D_1^* E_1^*\right)}{A_1^* B_1^* - C_1^{*2}} - 1 > 0$$

Condition (3): By (2) and (13):

$$MSE(\hat{S}^{*2}_{P(2s)})_{min} < MSE\left(\hat{S}^2_{0(2s)}\right) \text{ if}$$
$$\gamma\beta^*_{2(1y)} + \sum_{i=1}^N \gamma_{2i}\beta^*_{2(2yi)} - \left\{1 + B_1^* - 2E_1^* - \frac{(B_1^* - C_1^* + D_1^* - E_1^*)^2}{(A_1^* + B_1^* - 2C_1^*)}\right\} > 0$$

Condition (4): By (4) and (13):

$$MSE(\hat{S}^{*2}_{P(2s)})_{min} - MSE\left(\hat{S}^2_{reg(2s)}\right) \text{ if } \gamma\beta^*_{2(1y)}(1 - \rho_1^2) + \sum_{i=1}^N \gamma_{2i}\beta^*_{2(2yi)}(1 - \rho_{2i}^2) - \Big\{1 + B_1^* - 2E_1^* -$$
B1∗−C1∗+D1∗−E1∗2A1∗+B1∗−2C1∗>0

Note: The proposed estimator will be more efficient than the usual variance and regression estimators in two-stage sampling under two cases, when above Conditions (1)-(4) are satisfied.

Numerical illustration: We use the following real data sets to observe the performances of estimators.

Population: (Sarndal *et al*., 1992)

y : Revenues from the 1985 municipal taxation
x : 1975 population for M = 284 municipalities (*ssus*) divided into N = 50 clusters (*psus*)

We use the following expression to obtain the Percent Relative Efficiency (*PRE*):

$$PRE = \frac{MSE\left(\hat{S}^2_{0(2s)}\right)}{(.)} \times 100$$

where, (.) denote the $MSE\left(\hat{S}^2_{0(2s)}\right)$, $MSE\left(\hat{S}^2_{reg(2s)}\right)$, $MSE(\hat{S}^2_{P(2s)})_{min}$ and $MSE(\hat{S}^{*2}_{P(2s)})_{min}$

Table 1: Percent relative efficiency of different estimators with respect to $\hat{S}^2_{0(2s)}$

Population	m_i	n	$\hat{S}^2_{0(2s)}$	$\hat{S}^2_{reg(2s)}$	$\hat{S}^2_{P(2s)}$	$(\hat{S}^{*2}_{P(2s)})$
Numerical study	3	10	100	724.86	1, 734.11	409.15
		20	100	777.87	1, 009.95	363.55
		30	100	877.59	957.11	352.62
	4	10	100	641.99	1, 324.65	396.15
		20	100	671.00	841.55	353.40
		30	100	727.40	785.75	342.38
Simulated study	3	10	100	560.66	2, 918.10	224.94
		20	100	309.53	1,232.71	215.86
		30	100	261.57	895.86	220.65
	4	10	100	460.29	2, 403.23	226.64
		20	100	282.77	1, 034.35	221.71
		30	100	242.28	698.62	221.29

For simulation study, we selected 10,000 independent first-stage samples of different sizes from a population. From every selected *psus*, a second-stage sample of different sizes, *ssus* was again selected. Thus, we had 10,000 independent samples each of different sizes. For each sample from 1 to 10,000, values of the estimators were computed and then on the basis of these values simulated *MSE* of different estimators were calculated. Percentage Relative Efficiency (*PRE*) of different estimators with respect to $S^2_{2s(0)}$ for different sample sizes are given in Table 1.

CONCLUSION

We proposed an improved exponential ratio type estimator for population variance in two stage sample under two cases:

- When sum of the weights cannot equal to one
- When sum of the weights are equal to one

Percentage Relative Efficiency (*PRE*) of different estimators for different sample sizes are given in Table 1. Both numerical and simulation studies show the same behavior of results.

From Table 1, we observed that the proposed estimator $(\hat{S}^2_{P(2s)})$ under Case 1 performs better than the usual sample variance estimator $\hat{S}^2_{0(2s)}$ and regression estimator $\hat{S}^2_{reg(2s)}$. The proposed estimator $(\hat{S}^{*2}_{P(2s)})$ under Case 2 is better than the usual sample variance estimator $\left(\hat{S}^2_{0(2s)}\right)$ but show the weaker performance than the traditional regression estimator $\hat{S}^2_{reg(2s)}$. So it is preferable to use the estimator $(\hat{S}^2_{P(2s)})$ under Case 1 for future study.

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