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Research Article

Switched Control for the Walking of a Compass Gait Biped Robot

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Abstract: This study presents a new approach for modeling and controlling of a compass gait biped robot based on the use of the switched systems. The linearization of the equations stemming from the formalism of Lagrange allows the construction of a set of local models used to describe the behavior of this non linear system. The selection of each model depends on its activation function depending on the system states. The synthesis of the stability of the walking robot is based on the use of second method of Lyapunov. The synthesis approach leads to a set of bilinear matrices inequalities non resolvable by actual numerical solvers. To come over these difficulties, some relaxations are brought to get useful and exploitable numerical solutions.

Keywords: Bilinear Matrix Inequalities (BMI), Lagrange formulation, Linear Matrix Inequalities (LMI), lyapunov method, relaxation, stability, switched system

INTRODUCTION

Biped Robots establish an interesting class of mobile robots, thanks to their capacity to move walking on different terrains (slippery, rocky and steep). The design of a regulator is very difficult. The description of the walking motion of a biped robot raises specific problems at the level of the mechanical architecture and of the optimization of the structure concerning the dorad degree of freedom desired unrestrained (Escobar *et al*., 2012). The problems of conception are also electric because the performance of the system is not linear. Certain researchers opted for the linearization about several operating points (Ahmed, 1992; Micheau *et al*., 2003; Terumasa *et al*., 2007; Fauteux *et al*., 2005).

Raibert (1986) proposed a simple yet strong command law of the running of its monopod, following the style of a kangaroo. It is mainly based on the regulation of the positioning of the leg during the flight phase. Thus, he stabilizes the horizontal speed of the monopod. Raibert suggests then to control multiple legged robotic systems through simple decoupling of the movements, by using the Principe of symmetric movement.

Charles (1996) proved the existence of passive cycles of walking on a monopod moving on a horizontal ground. Natural periodic systems were obtained from an approximation of the complete model of the robot.

The Theoretical analysis of the control structure stabilizing the obtained cycles shows that the phases of flight and support, taken separately, were not controllable. His command is based on an impulsive excitation of the monopod. It would be enough to carry out two impulses on the hip during the phase of flight and an impulse on the leg during the phase of support to stabilize the running of the monopod around the cycles.

Other researchers (Koditschek and Buhler, 1991; Ahmadi and Buhler, 1995) were interested in the theoretical analysis of the stability of Raibert monopod, using of the tools of dynamic systems analysis. Koditschek and Buhler (1991) proposed an analytical study of the compass robot provided with its command. They proved that the closed-loop system had overall convergence properties towards a stable system. For this reason, they have numerically build the sections of Poincaré (Vakakis and Burdick, 1990).

Many approaches have been also proposed to study modeling aspects and biped robots command and stability (Golliday and Hemami, 1977; Hemami and Wyman, 1979). We shall approach these questions from the perspective of passive walking through the most significant works in this sense.

In this study we propose a new approach for the synthesis of motion controllers for a walking compass robot. This approach is based on modeling robot like switched systems. The calculation of state feedback gains of the controller is based on the use of polyquadratic Lyapunov functions.

The Other works in this field, to our knowledge, are based on the use of predictive control or optimal control leading to the calculation of a single gain from the Ricatti equation for a single model. The generated

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stabilization conditions are usually too conservative and difficult to apply in practice.

We propose in this study, a new methodology of controller synthesis by considering the compass robot as a switched system described by a set of LTI models and not a single model. The use of the polyquadratic approach could mitigate the conservatism of others methods like the quadratic approach.

MODELING OF THE COMPASS GAIT BIPED ROBOT

Description: We have considered as a walker simplified model a bi pendulum formed of point masses, containing only a single articulation in the hip, which is capable of reproducing the running. It is called compass gait robot. The figure below presents this geometrical conception of the compass robot (Fig. 1).

The compass gait model: The equations of the compass during the phase of simple support are obtained by using the following Equations of Euler-Lagrange:

$$\frac{d}{dt}\left(\frac{\partial L(\dot{\theta},\theta)}{\partial\dot{\theta}}\right)-\left(\frac{\partial L(\dot{\theta},\theta)}{\partial\theta}\right)=F \tag{1}$$

With: $L(\dot{\theta},\theta)$: the Lagrangian of the system :

$$L(\dot{\theta},\theta)=E_c(\dot{\theta},\theta)-E_p(\dot{\theta},\theta) \tag{2}$$

F: External forces applied to the system.

The correspondent relations between the actuator pairs and the robot degrees of freedom are represented as follows:

$$\frac{\partial P_u}{\partial\dot{\theta}}=Torc_i \tag{3}$$

The virtual Power P_u is given by the following relation:

$$P_u=T_c\dot{\theta}_s+T_h(\dot{\theta}_s-\dot{\theta}_{ns}) \tag{4}$$

$$\frac{\partial P_u}{\partial\dot{\theta}_s}=T_c+T_h \tag{5}$$

$$\frac{\partial P_u}{\partial\dot{\theta}_{ns}}=T_c-T_h \tag{6}$$

The equations of Euler-Lagrange are written by:

$$M(\theta)\ddot{\theta}+N(\theta,\dot{\theta})+G(\theta)=J*Torc_i \tag{7}$$

With,

$$M(\theta)=\begin{pmatrix} mb^2 & -mbl\cos(\theta_s-\theta_{ns}) \\ -mbl\cos(\theta_s-\theta_{ns}) & m_h l^2+m(a^2+l^2) \end{pmatrix} \tag{8}$$

$$N(\theta)=\begin{pmatrix} -mbl\sin(\theta_s-\theta_{ns})\dot{\theta}_{ns}^{\,2} \\ mbl\sin(\theta_s-\theta_{ns})\dot{\theta}_s^{\,2} \end{pmatrix} \tag{9}$$

$$G(\theta)=\begin{pmatrix} mgb\sin(\theta_{ns}) \\ -(m_h l+mg(a+l)\sin(\theta_{ns})) \end{pmatrix};J=\begin{pmatrix} -1 & 0 \\ 1 & 1 \end{pmatrix} \tag{10}$$

$$Torc_i=\begin{pmatrix} T_h \\ T_c \end{pmatrix} \tag{11}$$

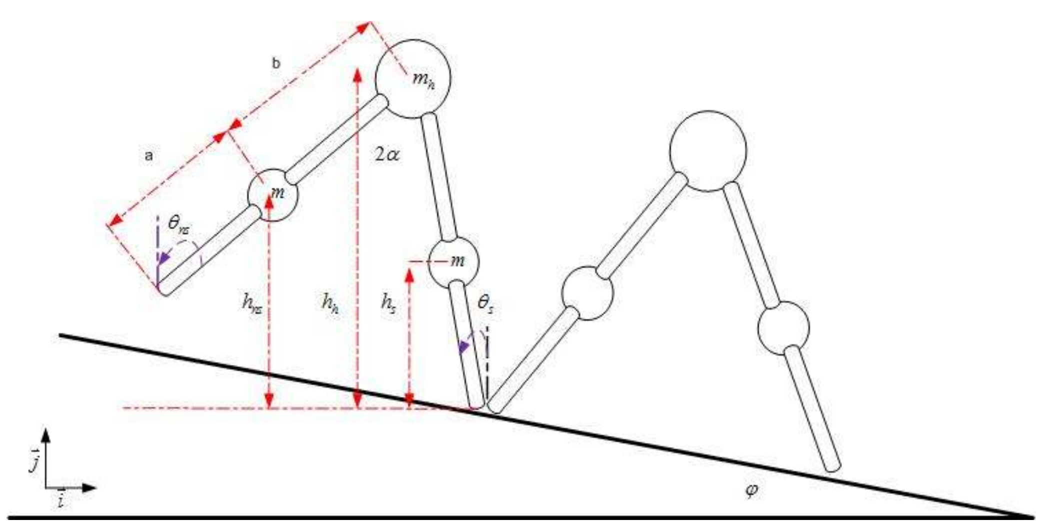

Fig. 1: Compass robot

θ_s: Absolute angle of the leg in touch with the ground, (indication 's' is for support leg); θ_{ns}: Absolute angle of the leg during flight, (indication 'ns' is for non swing leg); α: The half inter leg angle; φ: The slope angle; h_{ns}, h_s: Height separating both legs with regard to the point of biped contacting the ground; h_h: Height between hip and the point contacting the sole of compass; m, m_h: Mass of the pendulums which represents the leg and the hip

The torc is applied to the hip and the ankle.

The Lagrange Eq. (7) can, thus, be written in the following form:

$$\begin{pmatrix} mb^2 & -mbl\cos(\theta_s - \theta_{ns}) \\ -mbl\cos(\theta_s - \theta_{ns}) & m_h l^2 + m(a^2 + l^2) \end{pmatrix} \begin{pmatrix} \ddot{\theta}_{ns} \\ \ddot{\theta}_s \end{pmatrix} + \begin{pmatrix} -mbl\sin(\theta_s - \theta_{ns})\dot{\theta}_{ns}{}^2 \\ mbl\sin(\theta_s - \theta_{ns})\dot{\theta}_s{}^2 \end{pmatrix} + \begin{pmatrix} mgb\sin(\theta_{ns}) \\ -(m_h l + mg(a+l)\sin(\theta_{ns})) \end{pmatrix} = J\begin{pmatrix} T_h \\ T_c \end{pmatrix} \tag{12}$$

The state vector:

$$q = \begin{pmatrix} \theta_{ns} \\ \theta_s \\ \dot{\theta}_{ns} \\ \dot{\theta}_s \end{pmatrix}$$

The linear representation of the compass model by the jacobian method is thus written as:
With,

$$\dot{q} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \multicolumn{4}{c}{\ldots A_{jacob} \ldots} \end{pmatrix} \begin{pmatrix} \theta_{ns} \\ \theta_s \\ \dot{\theta}_{ns} \\ \dot{\theta}_s \end{pmatrix} \tag{13}$$

$$A_{jacob} = \frac{\partial}{\partial q}\left[(-M(\theta)^{-1}N(\theta,\dot{\theta}) - M(\theta)^{-1}G(\theta))\right]_{q=q_e=0} \tag{14}$$

The linear representation of the compass model is written as follows:

$$\dot{q} = \begin{pmatrix} \dot{\theta}_{ns} \\ \dot{\theta}_s \\ \ddot{\theta}_{ns} \\ \ddot{\theta}_s \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ (m_h l^2 + m(a^2 + l^2))mg & -mbl(m_h l + m(a+l)g) & 0 & 0 \\ m^2 b^2 l\,\mathrm{g} & -mbl(m_h l + m(a+l)g) & 0 & 0 \end{pmatrix} \begin{pmatrix} \theta_{ns} \\ \theta_s \\ \dot{\theta}_{ns} \\ \dot{\theta}_s \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \multicolumn{2}{c}{..J..} \end{pmatrix} \begin{pmatrix} T_h \\ T_c \end{pmatrix} \tag{15}$$

According to the linear representation of the compass model (15), the system corresponds to a switching system having three possible transitions; The transition is generated by a switching of the command varying the state equation which represents the system, moving thus from one mode to another according to the selection matrix J. Therefore, the torc applied by actuators commute, between the hip, the ankle and the pair ankle hip at the same time.

Command of the hip: In this case where only the hip is commanded the selection matrix J is then:

$$J = \begin{pmatrix} -1 & 0 \\ 1 & 0 \end{pmatrix} \tag{16}$$

The model of compass is:

$$\dot{q} = \begin{pmatrix} \dot{\theta}_{ns} \\ \dot{\theta}_s \\ \ddot{\theta}_{ns} \\ \ddot{\theta}_s \end{pmatrix} = A\begin{pmatrix} \theta_{ns} \\ \theta_s \\ \dot{\theta}_{ns} \\ \dot{\theta}_s \end{pmatrix} + B_h\begin{pmatrix} T_h \\ T_c \end{pmatrix} \text{with } B_h = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ -1 & 0 \\ 1 & 0 \end{pmatrix} \tag{17}$$

The ankle is commanded: The selection matrix is: $j = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$. The model command matrix B becomes:

$$B_c = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{18}$$

The ankle and the hip are commanded: The selection matrix in this case becomes:

$$J = \begin{pmatrix} -1 & 0 \\ 1 & 1 \end{pmatrix} \text{ and } \mathrm{B}_{ch} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ -1 & 0 \\ 1 & 1 \end{pmatrix} \tag{19}$$

The state vector of the system after the impact can be expressed in terms of the previous state vector of the robot by the following relation:

$$q^+ = \begin{pmatrix} j & 0 \\ H(\alpha) & 0 \end{pmatrix} q^- \tag{20}$$

The non support leg, located by the indications, in q^- is the one which ends its flight phase, while the non supporting leg, located by ns in q^+ is the one which will to leave the ground. The non support leg of in q^- and in $q^+$are thus switched around:

$$H(\alpha) = Q^{+(-1)}(\alpha) * Q^{-(-1)}(\alpha);\ j = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{21}$$

With,

$$q^+ = \begin{pmatrix} \theta_{ns}^+ \\ \theta_s^+ \\ \dot{\theta}_{ns}^+ \\ \dot{\theta}_s^+ \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & H_1 & H_2 \\ 0 & 0 & H_3 & H_4 \end{pmatrix} \begin{pmatrix} \theta_{ns}^- \\ \theta_s^- \\ \dot{\theta}_{ns}^- \\ \dot{\theta}_s^- \end{pmatrix} \tag{22}$$

$$\theta_{ns}^+ = \theta_s^-;\ \theta_s^+ = \theta_{ns}^- \tag{23}$$

$$\dot{\theta}_{ns}^{+} = H_1\dot{\theta}_{ns}^{-} + H_2\dot{\theta}_{s}^{-};\dot{\theta}_{s}^{+} = H_3\dot{\theta}_{ns}^{-} + H_4\dot{\theta}_{s}^{-} \quad (24)$$

During the phase of double-support, the orientations of both legs confirm that:

$$\theta_{ns}^{-} - \theta_{s}^{-} = \theta_{s}^{+} - \theta_{ns}^{+} = 2\alpha \quad (25)$$

With α is the between leg angle of the Compass.

Stability of the switching system: Let's consider the following switching system:

$$x(k+1) = \sum_{i=1}^{N}\mu_i(k)A_i(k)x(k) + \sum_{i=1}^{N}\mu_i(k)B_i(k)u(k) \quad (26)$$

where, the parameters μ_i (k) replace the commutative law such as $\sum_{i=1}^{N}\mu_i = 1$ the feedback control is written in the following form:

$$u(k) = \sum_{i=1}^{N}\mu_i(k)K_i(k)x(k) \quad (27)$$

The closed loop system is given by the following equation:

$$x(k+1) = \sum_{i=1}^{N}\mu_i(k)(A_i + B_iK_i)x(k) \quad (28)$$

The polyquadratical stability of the switching systems was proposed by Daafouz *et al*. (2002). It is possible to write the system (28) under the same following expression:

$$\xi_i = \begin{cases} 1: \text{If the model is described by the matrix } A_i \\ 0: \text{otherwise} \end{cases}$$

$$A(\xi_k) = \sum_{i=1}^{N}\xi_k^{\ i}A_i; \xi_k^{\ i} \geq 0; \sum_{i=1}^{N}\xi_k^{\ i} = 1 \quad (29)$$

We can thus write the system according to the following form:

$$x(k+1) = \sum_{i=1}^{N}\xi_k^{\ i}A_i \quad (30)$$

The system (30) is polyquadratically stable only if there are N symmetric matrices defined positively S_1... S_N and N matrices G_1... G_N of appropriate dimensions confirming:

$$\begin{bmatrix} G_i + G_i^T - S & G_i^T A_i \\ A_i^T G_i & S \end{bmatrix} > 0, \ \forall(\text{i,j}) \in (\text{e*e}) \quad (31)$$

The Lyapunov function used is written as:

$$\text{V}(x(\text{k})\xi(k)) = x^t(k)\sum_{i=1}^{N}P_i(\xi_k^{\ i})x(k) \quad (32)$$

With: $P_i = S_i^{-1}$.

Replacing A by (A_i + B_iK_i) and linearizing the matrix disparity by the change of variable R = GK. We reach the following condition expressed in LMI terms:

$$\begin{pmatrix} G_i + G_i^t - S_i & G_i^t A^t + R^t B_i^t \\ AG_i + B_iR_i & S_j \end{pmatrix} > 0; i......j = 1,.....n \quad (33)$$

The closed loop system is asymptotically stabilizable by state feedback if there are symmetric matrices $S_{ij}\succ 0$, Matrices R_iG_i of appropriate dimensions such as the gain of return of state is given by:

$$K_i = R_iG_i^{-1} \quad (34)$$

The search for solutions to the matrix disparity (35) with the equality constraints is a non convex problem. An imposed solution K exists only if the following conditions are confirmed (Halabi, 2005):

$$rank(K_i) = rank(K_iG_i^{-1}) \quad (35)$$

Some relaxations are introduced, allowing for the elimination of the non-convexity of the problem.

The calculation program starts by calculating K_i, stabilizing the pairs (A_i, B_i), for 1 … n, respecting the constraint of rank (35).

Stability of the compass model: We are interested in this part in the robot control, in other terms when the robot is in the swaying phase. We have more particularly studied the stability of the linear system when the pair applied by actuators, commutes between the hip, the ankle, or the pair of ankles and the hip at the same time. This switch is described by the matrix of selection *J*. The system is poly quadratically stable the following are set *LMI* condition:

- Ankle is commanded:

$$\begin{bmatrix} G_c + G_c^T - S_c & G_c^T A \\ A^T G_c + B_c R_1 & S_c \end{bmatrix} > 0: \text{Ankle command}$$

$$\begin{bmatrix} G_c + G_c^T - S_c & G_c^T A \\ A^T G_c + B_c R_1 & S_h \end{bmatrix} > 0: \text{Ankle switching towards Hip}$$

$$\begin{bmatrix} G_c + G_c^T - S_c & G_c^T A \\ A^T G_c + B_c R_1 & S_{ch} \end{bmatrix} > 0: \text{Ankle Switching towards (hip and ankle)}$$

$$\Rightarrow K_c = R_1G_c^{-1} \quad (36)$$

- Hip is commanded:

$$\begin{bmatrix} G_h + G_h^T - S_h & G_h^T A \\ A^T G_h + B_h R_2 & S_h \end{bmatrix} > 0 : Hip \text{ command}$$

$$\begin{bmatrix} G_h + G_h^T - S_h & G_h^T A \\ A^T G_h + B_h R_2 & S_c \end{bmatrix} > 0 : \text{Hip switching towards Ankle}$$

$$\begin{bmatrix} G_h + G_h^T - S_h & G_h^T A \\ A^T G_h + B_h R_2 & S_{ch} \end{bmatrix} > 0 : Hip \text{ Switching towards (hip and ankle)}$$

$$\Rightarrow K_h = R_2 G_h^{-1} \tag{37}$$

- (hip and ankle) are commanded:

$$\begin{bmatrix} G_{ch} + G_{ch}^T - S_{ch} & G_{ch}^T A \\ A^T G_{ch} + B_{ch} R_3 & S_{ch} \end{bmatrix} > 0 : \text{(hip and ankle) command}$$

$$\begin{bmatrix} G_{ch} + G_{ch}^T - S_{ch} & G_{ch}^T A \\ A^T G_{ch} + B_{ch} R_3 & S_h \end{bmatrix} > 0 : \text{ (hip and ankle)switching towards Hip}$$

$$\begin{bmatrix} G_{ch} + G_{ch}^T - S_{ch} & G_{ch}^T A \\ A^T G_{ch} + B_{ch} R_3 & S_c \end{bmatrix} > 0 : \text{(hip and ankle) switching towards ankle}$$

$$\Rightarrow K_{ch} = R_3 G_{ch}^{-1} \tag{38}$$

SIMULATION AND RESULTS

The compass robot described the owing discrete time system, respecting Shannon Theory $T_e \prec 2T_c; (T_e = 0.0001s)$.
With,

$$a = b = 0.5m, l = 1m, m = 5Kg, m_h = 10Kg, g = 9.8m/s^2, \varphi = \alpha = 3^\circ$$

$$\begin{cases} x(+1) = A_d x(k) + B_d u(k) \\ y(k) = C_d x(k) + D_d u(k) \end{cases} \tag{39}$$

With,

$$A_d = \begin{pmatrix} 1 & -1.4e^{-07} & 0.0001 & -4.667e^{-012} \\ -2e^{-08} & 1 & -6.667e^{-013} & 0.0001 \\ -0.0026 & -0.0028 & 1 & -1.4e^{-07} \\ -0.0004 & -0.0014 & -2e^{-08} & 1 \end{pmatrix}$$

$B_d = B_{dh} or B_d = B_{dc} or B_d = B_{dch}$

$$B_{dh} = \begin{pmatrix} -5e^{-009} & 0 \\ 5e^{-009} & 0 \\ -0.0001 & 0 \\ 0.0001 & 0 \end{pmatrix}, B_{dc} = \begin{pmatrix} 0 & -1.167e^{-016} \\ 0 & 5e^{-009} \\ 0 & -4.667e^{-012} \\ 0 & 0.0001 \end{pmatrix},$$

$$B_{dch} = \begin{pmatrix} -5e^{-009} & -1.167e^{-016} \\ 5e^{-009} & 5e^{-009} \\ -0.0001 & -4.667e^{-012} \\ 0.0001 & 0.0001 \end{pmatrix}, C_d = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, D_d = 0$$

By applying the approach of the switching system to find the command matrix Ki in a way that the closed loop system is stable, by using MATLAB © to resolve the disparities LMI, the results are the following:

- Ankle command:

$$K_c = 1e^3 \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0.0040 & 0.0134 & -0.0141 & -9.999 \end{pmatrix}$$

- Hip command:

$$K_h = 1e^4 \begin{pmatrix} 0.0004 & 0.0014 & -0.0916 & -1.7672 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- (hip and ankle) Command:

$$K_{ch} = 1e^8 \begin{pmatrix} 0.0001 & -0.0003 & -0.0026 & 6.7149 \\ -0.0001 & 0.0003 & 0.0026 & -6.7672 \end{pmatrix}$$

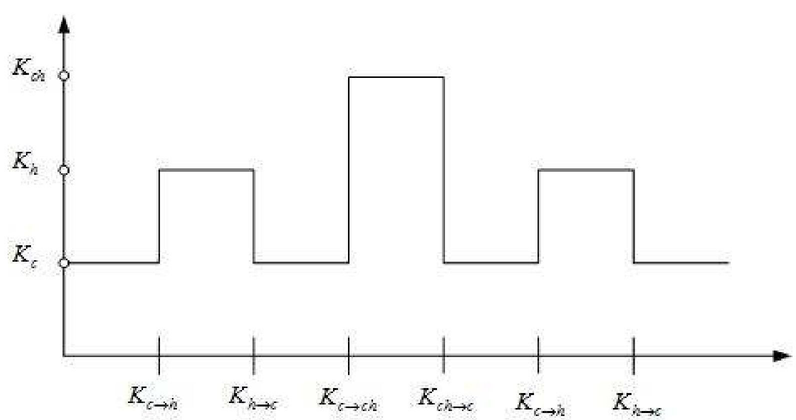

Fig. 2: Various modes of switching

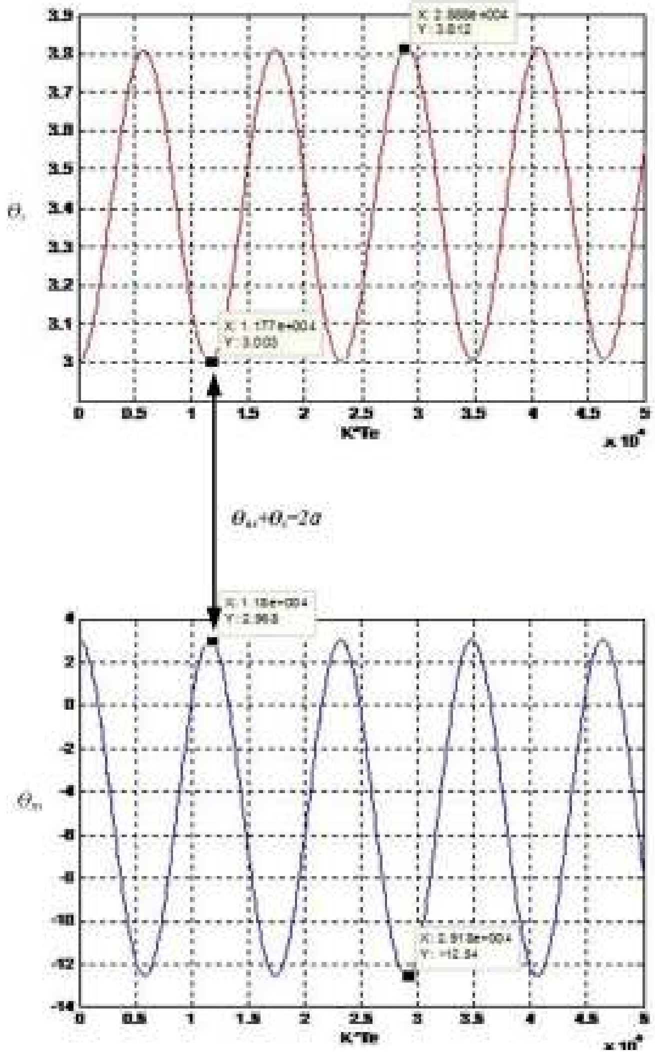

Fig. 3: Angle of the leg θ_s, θ_{ns}

The simulation under MATLAB © of the system during the swing phase in the various modes of switching gives the following signals (Fig. 2 and 3).

According to the curves of signals θ_S, θ_{ns}, the system remains stable and the gains by state feedback K_i Calculated by the method of the switching system stabilize the closed loop system in the three cases of command. This method of calculation offers thus a domain of important stability as it confirms in real time, the conditions of the stability of the compass before the flight: $\theta_S + \theta_{ns} = -2\varphi$ and $\theta_S + \theta_{ns} = 2\alpha$ and the condition after one step of shifting $\theta_{ns}^- - \theta_s^- = \theta_s^+ - \theta_{ns}^+ = 2\alpha$.

CONCLUSION

In this study we presented a new approach to synthesis of controllers for a biped robot. The used models for the synthesis of controllers are derived from the Lagrange formalism. The approach is based on the progress in research on switched systems. We have demonstrated that it is possible to stabilize robustly the walking robot by using three switched controllers which act alternately.

The approach of synthesis is also based on the use of linear matrix inequalities *LMI* type for the design of robust controllers. In some cases the generated matrix inequalities are *BMI* types. We proposed in this study, some techniques of relaxation to transform these *BMI* into simple *LMI* easy to be solved by existing numerical solvers.

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