

## Research Article

### The Multi-stage Environment Economic Dispatch of Power System under Conditional Risk Constraints

<sup>1</sup>Liu Chenggui, <sup>1,2</sup>Tong Xiaojiao, <sup>1</sup>Liu Yajuan and <sup>1</sup>Yin Kun

<sup>1</sup>College of Electrical and Information Engineering, Changsha University of Science and Technology, Changsha, 410004, China

<sup>2</sup>Hunan First Normal University, Changsha 410205, China

**Abstract:** In this study we adopt three-stage dispatch method to solve the electrical power systems Economic Dispatch (ED) problem. Due to the new uncertainties in large-scale wind power integration, Conditional Value-at-Risk (CVaR) was introduced to describe the safe and stable operation of the power system. Minimizing both the fuel cost and emission of atmospheric pollutants of thermal generators and load shedding cost were considered as objective functions, construct a considering risk constraints three-stage dispatch model of power systems. When computing, we incorporated the risk constraints into the objective function by penalty to get the optimization model in this study and the smoothing function method is used to make the optimization model have a continuous features, last the sample average method is used to calculate. Showing the advantage of this method in this study by calculating and analysis a 30 node illustrative system, besides, the last conclude demonstrates that the model proposed is feasibility.

**Keywords:** Conditional Value-at-Risk (CVaR), economic dispatch, multi-stage, penalty function, smoothing function

## INTRODUCTION

As a kind of renewable green energy, the wind power have a great significance to relieve the energy and environment crisis. But the randomness and intermittency of the wind power added a new uncertainties to the economic dispatch. In order to balance environmental protection and economic benefit and the environmental economic dispatch will be considered in this study.

At present, there have some research about EED associated wind power. Chen *et al.* (2011), the wind speed as random variables obey weibull distribution and puts forward the concept of energy and environment benefits, finally, use the improved particle swarm algorithm based on tabu search thought to solve the power system contains wind power multi-objective optimization problem; Yuan *et al.* (2010) use the genetic algorithm to solve the dynamic clean and economic optimization scheduling problem about power system contains wind power and joined corresponding penalty cost in objective function when the output of wind power more or less the available wind. The above reference adopt the traditional dispatch method that just make only one decision prior the run time, however we know that the more accurate information we will get when closer the run time. So it

is very necessary to make many decisions before the run time. Currently, Varaiya *et al.* (2011) present a three-stage dispatch method, in this study we will adopt this method to resolve multi-stage dispatch problem. The biggest difference between traditional dispatch method is that it will make three times decisions before run time. The currently study about three-stage dispatch is relatively less. Lee and Baldick (2013), adopt the improved L-shape algorithm based on the system frequency stability to solve two combinatorial optimization problems and join the purchase price of stored energy in the objective function. Yang and Wen (2005) by using the genetic algorithm to solve the multi-stage transmission system planning problem based on risk constraint, in this study, the confidence level is used to describe the objective function and constraint conditions to meet the given probability.

To describe the operation risk of power system and quantitatively processing the uncertainty factors, not only need more accurate methods, also consider the efficiency of its calculation. In recent years, as the Conditional Value at Risk (CVaR) (Su *et al.*, 2010) have a good control of tail risk and a good mathematical properties, it have been used in the field of electric power to deal with random problem. The reference (Rockafellar and Uryasev, 2000; Zhou *et al.*,

**Corresponding Author:** Liu Chenggui, College of Electrical and Information Engineering, Changsha University of Science and Technology, Changsha, 410004, China

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2012) put the safety risk value as the index of grid risk will effected the cost of the system and build the optimization goal contains risk constraint. But they all adopted the traditional dispatch method and this study will describe the three-stage grid security conditional risk value base on the conditional value at risk theory.

Based on the literature (Varaiya *et al.*, 2011), this study constructed three-stage EED optimization model of power system under conditional risk constraints and take the minimum value of cost of coal-fired generating units, pollutant discharge fees and the cost of cutting load as the objective function. In calculating, the conditions risk constraint by penalty function and be incorporated into the objective function as an optimization model, the smooth processing technology will be used for the model is not available for the micro and multi-peak characteristics and will be tested by IEEE30 node system, the results verify the feasibility of the proposed method.

### THREE-STAGE EED MODEL WITH RISK CONSTRAINT

**Three-stage dispatch model:** Most ED problem adopt the traditional dispatch method to determine the output of the Conventional unit. With the rapid development of smart grid, advanced monitoring and measurement technology has be used more widely and mature, that means the times more closer to the runtime the information we got is more accurate. So the three-stage dispatch method has been presented in literature (Varaiya *et al.*, 2011) (Fig. 1).

The biggest characteristic of this method is that it will make three times decision to optimize the output of the regular unit. As well as traditional decision, in Scheduling the output of the regular unit will be dispatched, with the advanced monitoring and measurement technology has be used more widely and mature, that means the times more closer to the runtime the information we got is more accurate, this need us do multiple decision to optimize the output of the generator. In Recourse, we use the obtained information to adjust the load to let the whole system achieved the most maximum social benefits under the operating constraints, in this study, we first node m controllable load as decision variable in this stage. The Emergency stage means that a emergency accident has been occurred in a few minutes before the runtime and security constraints have not been meted, in this stage

security operation and quantitative analysis the different we generally take the load shedding approach to let system meet the safe operation constraints, the first node n controllable load will been as decision variable in this stage.

**Three-stage EED optimization model of power system with risk constraint:** The goal of this study is adopt the three-stage method to reasonable distribute the output of generator and set the total cost is minimum in a scheduling cycle under the load and Security constraints was been met.

#### Objective function:

**Traditional generation cost:** According to the literature (Qiu *et al.*, 2011) that the cost of conventional generation cost  $F_c$  (\$/h) can be expressed as:

$$F_c = \sum_{k=1}^N F_{ck}(p_k^G) = a_{1k}(p_k^G)^2 + b_{1k}p_k^G + c_{1k} \quad (1)$$

where,

- N = The number of generator unit
- $p_k$  = The output of first k units, MW
- $a_{1k}, b_{1k}, c_{1k}$  = The generation cost constant

**Pollution cost function:** According to the literature Qiu *et al.* (2011) that the pollution cost function  $F_e$  (\$/h) can be expressed as:

$$F_e = \sum_{k=1}^N F_{ek}(p_k^G) = a_{2k}(p_k^G)^2 + b_{2k}p_k^G + c_{2k} \quad (2)$$

where,

- N = The number of generator units
- $p_k$  = The output of first k units, MW
- $a_{2k}, b_{2k}, c_{2k}$  = The constant of air pollution cost

#### Objective function:

$$\min F(p^k, d_m, d_n) = F_c + F_e - c_2 d_m - c_3 d_n \quad (3)$$

where,  $d_m, d_n$  are the variation of first m, n nodes controllable load, respectively in second and third stage and  $d_m, d_n \leq 0$ ;  $c_2, c_3$  are the price of the load variation and  $c_2 \ll c_3$ , \$/MW.

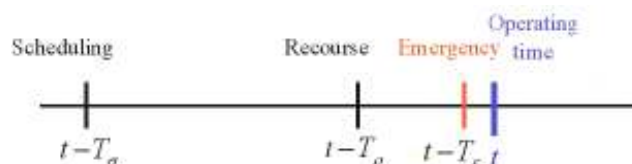


Fig. 1: The model of three-stage

**Security constraints:**  
**Balance constraints:**

$$\sum_{k=1}^N P_k^G + \sum_{n=1}^{N_w} P_n^W = P^D + d_m + d_n \quad (4)$$

where,

- $N_w$  = The numbers of wind turbines
- $P_n^W$  = The output of first n units, MW
- $P^D$  = The load demand

In the first stage  $d_m = d_n = 0$  and the second stage  $d_m \neq 0, d_n = 0$  and the third stage  $d_m = 0, d_n \neq 0$ .

**The constraints of conventional unit output:**

$$P_{k \min}^G \leq P_k^G \leq P_{k \max}^G \quad (5)$$

where,  $P_{k \max}^G, P_{k \min}^G$  are upper and lower limits respectively of the first k conventional units.

**Transmission line constraints:** In DC model,  $P_{ij}$  express the active power in transmission line and  $P_{ij} = (\delta_i - \delta_j) / X_{ij}$ , where  $X_{ij}$  is the lines reactance between first i and j node;  $\delta_i, \delta_j$  are the voltage phase angle of first i and j nodes, respectively. So the DC power flow can be expressed as:

$$P = B \cdot \delta$$

where,

- $B$  = The n-1 dimensional matrix
- $P$  = The node power injection vector
- $\delta$  = Node phase Angle matrix

From the above expression, we obtain the  $\delta = B^{-1} P$  and according the demand balance we can also obtain  $P_i = P_i^G - P_i^W - d_i$ , where d is the load forecasts. For the three-stage dispatch method is adopted in this study, so the load d is a variable and have a different value in different stage, in the second and third stage, the load d must subtract the shedding load in this stage.

S is a set of all lines connected with wind power, For transmission lines, the system will be reaches an unsafe condition at this line which have a biggest unsafe degree, so for complex power systems, only seek this line which have a greatest degree of risk, that can easily analyze the unsafe restraint of system. So there have the security index function in different stage respectively:

$$\varphi_1(P^G, P^W, d_m, d_n) = \max_{l \in S} \frac{P_{ij}(P^G, P^W) - P_{ij \max}}{P_{ij \max}}$$

$$\varphi_2(P^G, P^W, d_m, d_n) = \max_{l \in S} \frac{P_{ij}(P^G, P^W, d_m) - P_{ij \max}}{P_{ij \max}}$$

$$\varphi_3(P^G, P^W, d_m, d_n) = \max_{l \in S} \frac{P_{ij}(P^G, P^W, d_m, d_n) - P_{ij \max}}{P_{ij \max}} \quad (6)$$

where,

- $d_m, d_n$  = Decision variables in the second and third stage respectively
- $P_{ij \max}$  = The largest circulated power between nodes i and j

When  $\varphi(P^G, P^W, d_m, d_n) < 0$  means that the system operation is secure, otherwise is dangers. Because, when more closer to the runtime the information obtained is more accurate, we only choice the security index function in the third stage as the transmission constraints. In traditional optimization scheduling, control and state variables are presented as a determining variable, while the new energy generation in the system may be a random variation (Jabr and Pal, 2009), So lots of literature using the chance-constrained programming which mostly based on the intelligent algorithm or the combination of intelligent algorithms to this problem, however there will produce a large amounts of computing. The output of wind power in Conditional Value at Risk model will be simulated by Monte Carlo, with discrete points to approximate the integral function and put original problem into a convex optimization problem by construct a function and finally introduce a auxiliary variables zk to simplify the model. Considering the operational risk due to uncertainties in the new energy generation and load, we use CVaR (Yang and Wen, 2005) theory and transform the security constraints into risk constraint for quantitative analysis.

In the first stage, the decision variables are conventional units and the CVaR constraints of the transmission lines is:

$$q_1(P^G, P^W, z_1) = z_1 + \frac{1}{(1-\alpha_1)N} \sum_{i=1}^N (\varphi_1(P^G, P^W) - z_1)^+ \leq -r_1 \leq 0 \quad (7)$$

where,

- $z_1$  = The maximum threshold of the transmission component collection in the first stage
- $\alpha_1$  = The probability level as the system is safety operation in the first stage
- $-r_1$  = The risk range in the first stage and it is a constant

Similarly, we can obtained the CVaR constraints of the transmission lines in the second and third stage, respectively:

$$\begin{aligned}
 q_2(P^G, P^W, d_m, z_2) &= z_2 + \frac{1}{(1-\alpha_2)N} \sum_{i=1}^N (\varphi_2(P^G, P^W, d_m) - z_2)^+ \leq -r_2 \leq 0 \\
 q_3(P^G, P^W, d_m, d_n, z_3) &= z_3 + \frac{1}{(1-\alpha_3)N} \sum_{i=1}^N (\varphi_3(P^G, P^W, d_m, d_n) - z_3)^+ \leq -r_3 \leq 0
 \end{aligned}
 \tag{8}$$

where,

$z_2, z_3$  = The maximum threshold of the transmission component collection in the second and third stage

$\alpha_2, \alpha_3$  = The probability level as the system is safety operation in the second and third stage

$-r_2, -r_3$  = The risk range in the second and third stage and they are constant

**The three-stage EED model with CVaR constraint:** According to above analysis, we can build the three-stage EED model with CVaR constraints:

$$\left\{ \begin{aligned}
 &\min F = \min_{(P^G, d_m, d_n)} E[F_c + F_e - c_2 d_m - c_3 d_n] \\
 &z_1 + \frac{1}{(1-\alpha_1)N} \sum_{i=1}^N (\varphi_1(P^G, P^W) - z_1) \leq -r_1 \leq 0 \\
 &z_2 + \frac{1}{(1-\alpha_2)N} \sum_{i=1}^N (\varphi_2(P^G, P^W, d_m) - z_2) \leq -r_2 \leq 0 \\
 &z_3 + \frac{1}{(1-\alpha_3)N} \sum_{i=1}^N (\varphi_3(P^G, P^W, d_m, d_n) - z_3) \leq -r_3 \leq 0 \\
 &\sum_{k=1}^N P_k^G + \sum_{n=1}^N P_n^W = P^D + d_m + d_n \\
 &P_{\min}^G \leq P^G \leq P_{\max}^G
 \end{aligned} \right.
 \tag{9}$$

**THE COMPUTING FOR NEW DISPATCH MODEL**

**Simplify model by penalty function:** Form the (9) we can see there have three different security constraints in this Optimization model, in order to turn this Optimization model become a unconstrained optimization problems, we will processing this model by penalty function:

$$\left\{ \begin{aligned}
 &\min_{(P^G, d_m, d_n, z_1, z_2, z_3)} E[F_c + F_e - c_2 d_m - c_3 d_n] \\
 &+ pn_1 \left[ z_1 + r_1 + \frac{1}{(1-\alpha_1)N} \sum_{i=1}^N (\varphi_1(P^G, P^W) - z_1) \right] \\
 &+ pn_2 \left[ z_2 + r_2 + \frac{1}{(1-\alpha_2)N} \sum_{i=1}^N (\varphi_2(P^G, P^W, d_m) - z_2) \right] \\
 &+ pn_3 \left[ z_3 + r_3 + \frac{1}{(1-\alpha_3)N} \sum_{i=1}^N (\varphi_3(P^G, P^W, d_m, d_n) - z_3) \right] \\
 &s.t \\
 &\sum_{k=1}^N P_k^G + \sum_{n=1}^N P_n^W = P^D + d_m + d_n \\
 &P_{\min}^G \leq P^G \leq P_{\max}^G
 \end{aligned} \right.
 \tag{10}$$

**Simplify model by smoothing function:** We assume the function:

$$g(x) = \max_{1 \leq i \leq m} \{g_i(x)\}
 \tag{11}$$

where,  $g_i(x) : R^n \rightarrow R$  is twice continuously differentiable.

Obviously,  $g(x)$  function is not smooth at some point, e.g.,  $g_i(x) = g_j(x)$ , Given a parameter  $t$ , we use the same method to define its smoothing function:

$$g(t, x) = t \ln \left( \sum_{i=1}^m \exp(g_i(x) / t) \right)
 \tag{12}$$

For (11) due to its non-differentiable, we can't obtain each of its first derivative, According to the definition of smoothing function, we can processing this model by smooth technology:

$$\left\{ \begin{aligned}
 &\min_{(P^G, d_m, d_n, z_1, z_2, z_3)} E[F_c + F_e - c_2 d_m - c_3 d_n] \\
 &+ t_1 \bullet pn_1 \ln \left( 1 + \exp \left[ \frac{r_1 + z_1 + \frac{1}{(1-\alpha_1)N} \sum_{k=1}^N t_1 \ln(1 + \exp(\frac{\varphi_2(P^G, P^W) - z_1}{t_1}))}{t_1} \right] \right) \\
 &+ t_2 \bullet pn_2 \ln \left( 1 + \exp \left[ \frac{r_2 + z_2 + \frac{1}{(1-\alpha_2)N} \sum_{k=1}^N t_2 \ln(1 + \exp(\frac{\varphi_2(P^G, P^W, d_m) - z_2}{t_2}))}{t_2} \right] \right) \\
 &+ t_3 \bullet pn_3 \ln \left( 1 + \exp \left[ \frac{r_3 + z_3 + \frac{1}{(1-\alpha_3)N} \sum_{k=1}^N t_3 \ln(1 + \exp(\frac{\varphi_3(P^G, P^W, d_m, d_n) - z_3}{t_3}))}{t_3} \right] \right) \\
 &s.t \\
 &\sum_{k=1}^N P_k^G + \sum_{n=1}^N P_n^W = P^D + d_m + d_n \\
 &P_{\min}^G \leq P^G \leq P_{\max}^G
 \end{aligned} \right.
 \tag{13}$$

where,

$pn_1, pn_2, pn_3$  = The parameter of penalty function  
 $t_1, t_2, t_3$  = The parameter of smooth function

**SIMULATION AND RESULTS**

**The select of simulation system and data:** With 30 nodes system as an example, make a theoretical analysis to the model and node 1 is the balance bus and the first 7, 8 node is the controllable load respectively in the second and third stage, that  $m = 7, n = 8$ . The wind power is inject at the first 20 node and its value can be predicted by the reference (Hetzer *et al.*, 2008), the system total active load is 2.084 pu, Power reference value is 100 MVA. To simplify the calculation we assume that the parameter  $pn_1 = pn_2, = pn_3 = 10^3$  and  $t_1 = t_2, t_3 = 10^{-3}$ , the price of the shedding load in the second and third stage will be made according to local policy and in this study we assume the value of  $c_2, c_3$  is 100 and 1000 \$/MW.

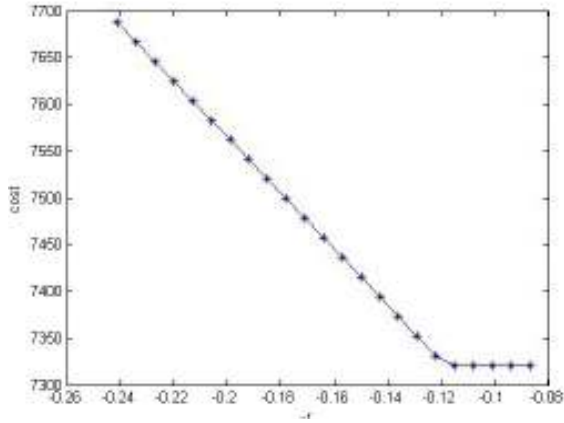


Fig. 2: The relationship of CVaR value and total cost

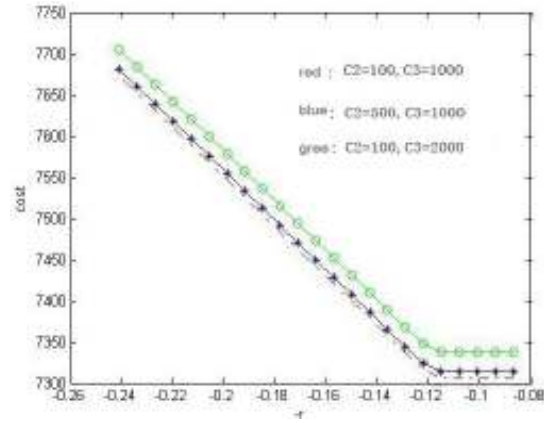


Fig. 5: The influence of different parameters on total cost

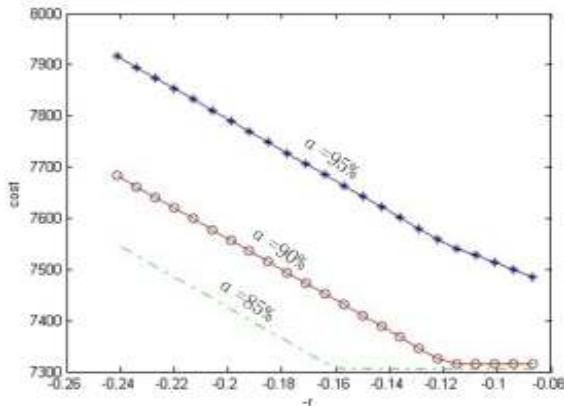


Fig. 3: The relationship of different confidence level and total cost

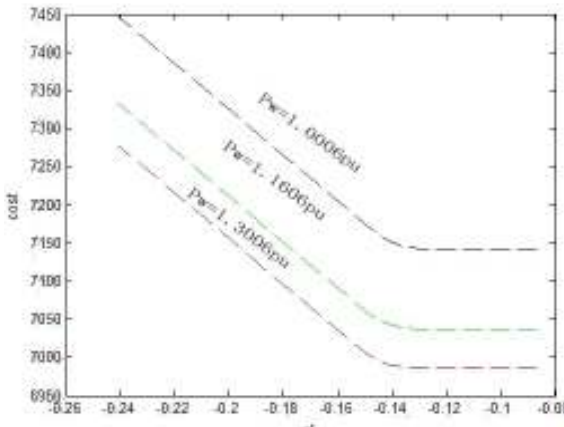


Fig. 4: The influence of wind power output on total cost

**Results and analysis:** For the range of grid security CVaR value in different stage, we can obtain by linear weighting method (Sun *et al.*, 2009) and Combined with the simulated IEEE30 node system and slip transmission power, we can assume (Fig. 2):

$$-r_1 = -r_2 = -r_3 \in [-0.241, -0.08]$$

**The relationship of different confidence level and total cost:** As the all confidence level are 90% and  $-r_1 = -r_2 = -r_3 \in [-0.241, -0.08]$ , we can get a range of optimal solutions in the Fig. 3. Form this picture we can see that with the increase of grid security CVaR value, i.e., the lower security of the grid, the cost of generation will be corresponding reducing, at the same time the security of the system will be more lower. When focus on the security operation of system, the smaller security CVaR value operation mode will be selected; when focusing on economic operation, the bigger security CVaR value will be selected. Therefore, form the Fig. 3 we can intuitive judgments the relationship between system security and economy, then provides a good theoretical basis for the decision makers to find a good balance of system security operation and economy.

**The comparison of different confidence levels:** Firstly, adopt Monte Carlo simulation to predict the wind power output is 1.0006 pu, then obtain the different total generation cost according to the confidence level changes. As can be seen from Fig. 4, for the same grid security CVaR value, when the confidence level is higher, the total generation cost will be added. Indicate that when the requirement of the system security is higher and the economics of system will be worse.

**The influence of wind power output on total cost:** Firstly, we assume that all confidence level of different stage are 90% and  $-r_1 = -r_2 = -r_3 \in [-0.241, -0.08]$ , the Fig. 5 shows that when the output of wind power is increasing, the total generation cost of the system will be reduced under the total load remains unchanged.

**The influence of different parameters on total cost:** Firstly, we assume that the output of wind power is the same in different parameters and the all confidence level are 90%. From the Fig. 5, when the parameters  $c_2, c_3$  is increasing, the total generation cost will be gained. Because the price of the shedding load is increased,

then the cost will be gained. For the third stage is more closer the runtime, so  $c_2$ ,  $c_3$ , in this study we assumed the  $c_3$  is 10 times of  $c_2$ . Therefore, we can see the changes of the value of  $c_3$  have more impact to the curves than the  $c_2$ .

## CONCLUSION

In this study, three-stage dispatch method have been adopt to solve the three-stage EED problem of power system, compare to the traditional method, the biggest difference is that it will make three times decisions before the runtime and the traditional method just make only once scheduling. With the wind power and other new energy incorporated and load uncertainty has increased the difficulty of the power grid scheduling, so, it must be accurately and quantitative to describe the safety operation of the grid. Conditions risk theory has good mathematical properties in study on random question, so it been used to quantitative research safety problem of grid, because adopt the three-stage dispatch method, requires systems must meet grid security constraints at each stage. For the model have non-differentiable and multimodal characteristics, using a smoothing processing technology make the Optimization goals has continuously differentiable characteristics. The main conclusions are as follows:

- For the three-stage dispatch method, adopt the different forecast value of the wind power to reflect the information we obtained is different in each stage and for the random nature of wind power output and load, the grid safety value in each stage are different and build the three-stage EED model under the risk constraint. And analyze the relations between the security and economic of the grid, conform to the development concept of the smart grid.
- For the three-stage model with risk constraints, using a penalty function processing and makes it to be a inequality constrained optimization model, while for non-smooth characteristics of the model, using smoothing function to converted it into a convex programming problem.
- Developing the application of three-stage method in power scheduling and providing a reference for the behind research of multi-stage schedule. While developing the application of conditions risk theory to deal with the random security issues in power system. Experiments show that the three-stage EED model contains wind power have high practical value.

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