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Research Article

Solution of Two-dimensional Transient Heat Conduction in a Hollow Sphere under Harmonic boundary condition

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Abstract: In this study, an analytical modeling of two dimensional heat conduction in a hollow sphere, subjected to time dependent periodic boundary condition at the inner and the outer surfaces, is performed. The thermo physical properties of the material are assumed to be isotropic and homogenous. Also, the effects of the temperature oscillations frequency on the boundaries, the thickness variation of the hollow sphere and thermo physical properties of the ambient and the sphere involved in some dimensionless numbers are studied. The results show that the obtained temperature distribution contains two characteristics, the dimensionless amplitude and the dimensionless phase difference. Comparison between the present results and the findings of the previous study as related to a two-dimensional solution of the hollow sphere subjected to the simple harmonic condition shows a good agreement.

Keywords: Convective heat transfer, fourier transforms, sphere, transient heat conduction

INTRODUCTION

The heat conduction analysis in spherical solids is important, because these geometries have some special features such as symmetry and minimum surface energy. Also, the transient periodic heat conduction is encountered in these shapes in very different forms such as heat-treatment of metals, air conditioning and food processing (Dincer, 1995a; Stela *et al*., 2005).

Heat transfer problems in transient form are very common in engineering applications, e.g., hydro cooling of spherical food products (Dincer, 1995b) or fast transient heat conduction in sphere subjected to the sudden and violent thermal effects on its surface used in many engineering fields such as aeronautics, electronics, metallurgy (Baïri and Laraqi, 2003; Dincer, 1995c). Transient heat conduction is studied in sphere by using Laplace transforms (Youming *et al*., 2003; Ostrogorsky, 2008) or in polar coordinates with multiple layers in radial direction (Suneet *et al*., 2008). These samples are some of useful examples in investigating transient heat conduction. The heat conduction problems with periodic boundary conditions have some applications in engineering, like periodic heat conduction through composite spheres consisting of shells (Lit, 1987), periodic radial heat conduction through a sphere (Sengupta *et al*., 1993) and in a solid homogeneous finite cylinder (Cossali, 2009). Moreover, the influence of combined periodic heat flux and convective boundary condition through semi-infinite and finite media is analytically studied (Khaled, 2008). Some experimental methods are considered for specifying the heat transfer coefficient and thermal diffusivity of material and the temperature field (Zudin 1995; Verein Deutscher Ingenieure, 2002; Khedari *et al*., 1995, 1996).

Analytical methods have been considered significant in solving the heat conduction problems. According to the differential equations characteristics such as the linearity which is govern on the heat conduction problems; these problems have been solved by means of analytical methods. For instance, analytical method to solve transient heat conduction in spherical coordinates with time-dependent boundary conditions (Prashant *et al*., 2010), the problem of evaluating the dynamic heat storage capacity of a solid sphere (Cossali, 2007) and the analytic solution of the periodic heat conduction in a homogeneous cylinder are some of the solutions in term of Fourier transform which are solved by researchers (Atefi *et al*., 2009). The heat conduction with time dependent in a hollow sphere with inner adiabatic boundary condition was investigated by Atefi and Moghimi (2006). The adiabatic boundary condition is a restriction which can reduce the applicant domain of the solution.

The main purpose of this study is to derive a general analytical solution for two-dimensional heat conduction in a hollow sphere subjected to a periodic boundary condition at the inner and the outer surfaces. It is also aimed to compare the obtained temperature distributions in a hollow sphere with some literature data taken from Atefi and Moghimi (2006) for model validation purposes. The convective heat transfer is imposed on the inner boundary to remove the restriction

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of the previous study. In addition, the effects of the inner boundary conditions on temperature distribution are discussed.

METHODOLOGY

Analysis: The governing heat conduction equation for a hollow sphere, with no heat generation and with uniform properties is defined below:

$$a^2 \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial r^2} + \frac{2}{r}\frac{\partial \theta}{\partial r} + \frac{1}{r^2}(cot\Psi \frac{\partial \theta}{\partial \Psi} + \frac{\partial^2 \theta}{\partial \Psi^2}) \quad (1)$$

The outer and inner boundary conditions are:

$$\begin{cases} \theta(r_o,\Psi,t) + \frac{k}{h}\frac{\partial \theta}{\partial r}|_{r_o,\Psi,t} = \Theta_o(\Psi,t) \\ \theta(r_i,\Psi,t) - \frac{k}{h}\frac{\partial \theta}{\partial r}|_{r_i,\Psi,t} = \Theta_i(\Psi,t) \end{cases} \quad (2)$$

where Θ_o (Ψ, t), Θ_i (Ψ, t) are considered to be periodic functions which are decomposed using Fourier series:

$$\Theta_o(\Psi,t) = g_o(t) f_o(\Psi)$$
$$g_o(t) = \sum_{m=1}^{\infty} \overline{\Theta}_{om} Sin\left(2m\pi \frac{t}{p}\right)$$
$$\Theta_o(\Psi,t) = \sum_{m=1}^{\infty} \overline{\Theta}_{om} Sin\left(2m\pi \frac{t}{p}\right) f_o(\Psi) \quad (3)$$

$$\Theta_i(\Psi,t) = g_i(t) f_i(\Psi)$$
$$g_i(t) = \sum_{m=1}^{\infty} \overline{\Theta}_{im} Sin\left(2m\pi \frac{t}{p}\right)$$
$$\Theta_i(\Psi,t) = \sum_{m=1}^{\infty} \overline{\Theta}_{im} Sin\left(2m\pi \frac{t}{p}\right) f_i(\Psi) \quad (4)$$

The initial temperature for hollow sphere is considered to be zero. Determination of the temperature field by considering Eq. (1) is not possible, directly (Trostel, 1956). So, the equation should be solved with assumption that, the boundary condition is time-independent. In this situation the boundary and initial conditions change as follow:

$$\begin{cases} \theta(r_o,\Psi,t) + \frac{k}{h}\frac{\partial \theta}{\partial r}|_{r_o,\Psi,t} = f_o(\Psi) \\ \theta(r_i,\Psi,t) - \frac{k}{h}\frac{\partial \theta}{\partial r}|_{r_i,\Psi,t} = f_i(\Psi) \end{cases} \quad (5)$$

$$\theta(r, \Psi, 0) = 0 \quad (6)$$

There are two ways to solve this problem; the first one is for steady state condition θ_0 (r, Ψ) and the second one is for the transient state condition θ_1 (r, Ψ, t):

$$\theta(r,\Psi,t) = \theta_0(r,\Psi) + \theta_1(r,\Psi,t) \quad (7)$$

The partial differential heat conduction equation in steady state condition is:

$$\frac{\partial^2 \theta_0}{\partial r^2} + \frac{2}{r}\frac{\partial \theta_\theta}{\partial r} + \frac{1}{r^2}\left(cot\Psi \frac{\partial \theta_\theta}{\partial \Psi} + \frac{\partial^2 \theta_0}{\partial \Psi}\right) = 0 \quad (8)$$

The boundary conditions are given by Eq. (5) and also the transient differential equation is:

$$a^2 \frac{\partial \theta_1}{\partial t} = \frac{\partial^2 \theta_1}{\partial r^2} + \frac{2}{r}\frac{\partial \theta_1}{\partial r} + \frac{1}{r^2}\left(cot\Psi \frac{\partial \theta_1}{\partial \Psi} + \frac{\partial^2 \theta_1}{\partial \Psi^2}\right) \quad (9)$$

The following conditions must be satisfied for transient case:

$$\begin{cases} \theta(r_o,\Psi,t) + \frac{k}{h}\frac{\partial \theta}{\partial r}|_{r_o,\Psi,t} = 0 \\ \theta(r_i,\Psi,t) - \frac{k}{h}\frac{\partial \theta}{\partial r}|_{r_i,\Psi,t} = 0 \end{cases} \quad (10)$$

$$\theta_1(r, \Psi, 0) = -\theta_0(r, \Psi) \quad (11)$$

Steady-state case: By using separation of variables method to solve Eq. (8), two differential equations are obtained, an Euler type and a Legendre type. Therefore, by applying Eq. (5), the solution of steady state is:

$$\theta_0(r,\Psi) = \sum_{n=0}^{\infty}\left(C_n^{(i)} n_{ni}(r) + C_n^{(o)} \eta_{no}(r)\right) P_n(\zeta) \quad (12)$$

with assumption of:

$$\eta_{ni}(r) = \left(a_n^{(i)} r^n + \beta_n^{(i)} r^{-(n+1)}\right)$$
$$\eta_{no}(r) = \left(a_n^{(o)} r^n + \beta_n^{(o)} r^{-(n+1)}\right)$$
$$C_n^{(0)} = \frac{2n+1}{2}\int_{-1}^{1} f_o(\zeta) P_n(\zeta) d\zeta$$
$$C_n^{(i)} = \frac{2n+1}{2}\int_{-1}^{1} f_i(\zeta) P_n(\zeta) d\zeta \quad (13)$$
$$\zeta = \cos\Psi$$

where,

$$\Delta^{(n)} = \left(r_o^n + n\frac{k}{h} r_o^{(n-1)}\right)$$
$$\left(r_i^{-(n+1)} + (n+1)\frac{k}{h} r_i^{-(n+2)}\right)$$
$$-\left(r_i^n + n\frac{k}{h} r_i^{(n-1)}\right)$$
$$\left(r_o^{-(n+1)} + (n+1)\frac{k}{h} r_o^{-(n+2)}\right) \quad (14)$$

$$\alpha_n^{(o)} = \frac{1}{\Delta^{(n)}}\left(r_i^{-(n+1)} + (n+1)\frac{k}{h} r_i^{-(n+2)}\right)$$
$$\alpha_n^{(i)} = -\frac{1}{\Delta^{(n)}}\left(r_o^{-(n+1)} - (n+1)\frac{k}{h} r_o^{-(n+2)}\right)$$
$$\beta_n^{(o)} = -\frac{1}{\Delta^{(n)}}\left(r_i^n - n\frac{k}{h} r_i^{(n-1)}\right)$$
$$\beta_n^{(i)} = \frac{1}{\Delta^{(n)}}\left(r_o^n + n\frac{k}{h} r_o^{(n-1)}\right) \quad (15)$$

Transient heat transfer case: Applying the separation of variables method to Eq. (9) and using boundary Eq. (10) the eigen values ω_{kn}, are obtained. Therefore, the final solution for this state is:

$$\theta_1(r,\Psi,t) = -\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\frac{1}{\delta_{kn}}\Phi(r\omega_{kn})\,P_n(\zeta)\, e^{\frac{-(\omega_k)2t}{a}}\int_{r_i}^{r_o} r^2\left(C_n^{(i)}\eta_{ni}(r) + C_n^{(o)}\eta_{no}(r)\right)\Phi_n(r\omega_{kn})dr \quad (16)$$

where, δ_{kn} is:

$$\delta_{kn} = \frac{r}{2\omega_{kn}^2}\left\{r^2\Phi_n{}^2\omega_{kn}^2 - n(n+1)\Phi_n{}^2 + r\omega_{kn}\Phi_n\Phi_n{}' + r^2\omega_{kn}^2\Phi_n{}'^2\right\}_{r_i}^{r_o} \quad (17)$$

Finally, the temperature distribution under the constant boundary condition is the summation of the steady and transient states:

$$\theta(r,\Psi,t) = \sum_{n=0}^{\infty}\{P_n(\zeta)(C_n^{(i)}\eta_{ni}(r) + C_n^{(o)}\eta_{no}((r) - \sum_{k=0}^{\infty}\frac{1}{\delta_{kn}}e^{-\left(\frac{\omega_{kn}}{a}\right)2t}\Phi_n(r\omega_{kn})\int_{r_i}^{r_o} r^2 P_n(\zeta)\left(C_n^{(i)}\eta_{ni}(r) + C_n^{(o)}\eta_{no}(r)\right)\Phi_n(r\omega_{kn})\,dr\} \quad (18)$$

where,

$$\Phi_n(\omega_{kn}r) = \frac{1}{\sqrt{r}}\left\{\left(J_{-(n+1/2)}(\omega_{kn}r_o) + \right.\right.$$

k2hro2ωknroJ−n+1/2'ωknro−J−n+1/2ωknro Jn+12ωknr−Jn+1/2ωknro+ k2hro2ωknroJn+1/2'ωknro−Jn+1/2ωknro J−n+1/2(ωknr) (19)

TEMPERATURE DISTRIBUTION UNDER TIME VARYING BOUNDARY CONDITION

Here, Eq. (18) can be expressed as:

$$\theta(r,\Psi,t) = \sum_{n=0}^{\infty}\begin{pmatrix}\eta_{ni}(r) - \sum_{k=0}^{\infty}\frac{1}{\delta_{kn}}e^{-\left(\frac{\omega_{kn}}{a}\right)2t}\Phi_n(r\omega_{kn}) \\ \int_{r_i}^{r_o} r^2\eta_{ni}(r)\ \Phi_n(r\omega_{kn})dr\end{pmatrix} C_n^{(i)}P_n(\zeta) + \sum_{n=0}^{\infty}\begin{pmatrix}\eta_{no}(r) - \sum_{k=0}^{\infty}\frac{1}{\delta_{kn}}e^{-\left(\frac{\omega_{kn}}{a}\right)2t}\Phi_n(r\omega_{kn}) \\ \int_{r_i}^{r_o} r^2\eta_{no}(r)\ \Phi_n(r\omega_{kn})dr\end{pmatrix} C_n^{(o)}P_n(\zeta) \quad (20)$$

Which is obtained from time-independent boundary conditions? In order to derive the time-dependent temperature distribution, the following are written:

$$dC_n^{(o)} = \frac{dC_n^{(o)}}{d\tau}d\tau$$
$$dC_n^{(i)} = \frac{dC_n^{(i)}}{d\tau}d\tau \quad (21)$$

Based on the Duhamel's theorem, it can be considered that the change which happens at time τ is constant. Thus, the temperature distribution after time t-τ, can be expressed as (Özisik, 1993):

$$\theta(r,\Psi,t) = \sum_{n=0}^{\infty}\begin{pmatrix}\eta_{ni}(r)\sum_{k=0}^{\infty}\frac{1}{\delta_{kn}}e^{-\left(\frac{\omega_{kn}}{a}\right)^{2t}}\Phi_n(r\omega_{kn}) \\ \int_{r_i}^{r_o} r^2\eta_{ni}(r)\ \Phi_n(r\omega_{kn})dr\end{pmatrix}\frac{dC_n^{(i)}}{d\tau}d\tau P_n(\zeta) + \sum_{n=0}^{\infty}\begin{pmatrix}\eta_{no}(r) - \sum_{k=0}^{\infty}\frac{1}{\delta_{kn}}e^{-\left(\frac{\omega_{kn}}{a}\right)^{2t}}\Phi_n(r\omega_{kn}) \\ \int_{r_i}^{r_o} r^2\eta_{no}(r)\,\Phi_n(r\omega_{kn})dr\end{pmatrix}\frac{dC_n^{(o)}}{d\tau}d\tau P_n(\zeta) \quad (22)$$

Thus, the temperature field is obtained by the summation of $dC_n^{(i)}$ and $dC_n^{(o)}$ during $d\tau$ and the influence of $C_n^{(i)}(0)$ and $C_n^{(o)}(0)$, respectively. The following equation is proven by the method of integration by parts:

$$C_n^{(o)}(0)\ e^{-\left(\frac{\omega_{kn}}{a}\right)^{2t}} + \int_0^t e^{-\left(\frac{\omega_{kn}}{a}\right)^2(t-\tau)}\frac{dC_n^{(o)}}{d\tau}d\tau = C_n^{(o)}(t) - \left(\frac{\omega_{kn}}{a}\right)^2\int_0^t C_n^{(o)}(\tau)e^{-\left(\frac{\omega_{kn}}{a}\right)^2(t-\tau)}d\tau$$
$$C_n^{(i)}(0)\ e^{-\left(\frac{\omega_{kn}}{a}\right)^{2t}} + \int_0^t e^{-\left(\frac{\omega_{kn}}{a}\right)^2(t-\tau)}\frac{dC_n^{(i)}}{d\tau}d\tau = C_n^{(i)}(t) - \left(\frac{\omega_{kn}}{a}\right)^2\int_0^t C_n^{(i)}(\tau)e^{-\left(\frac{\omega_{kn}}{a}\right)^2(t-\tau)}d\tau \quad (23)$$

Using Eq. (23), we obtain the simplified temperature distribution as:

$$\theta(r,\Psi,t) = \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} D_{kn}\,\Phi_n(\omega_{kn},r)P_n(\zeta)T_{kn}(t)$$
$$\zeta = \cos\Psi \quad (24)$$

where,

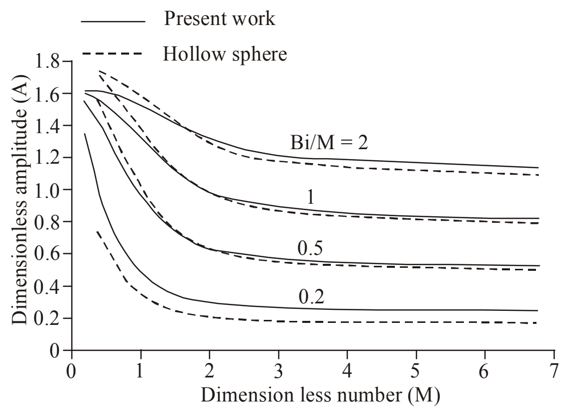

Fig. 1: Comparison between the results of the amplitude of two-dimensional temperature field of a hollow sphere at the outer surface presented in (Atefi and Moghimi, 2006) and obtained results under harmonic boundary condition

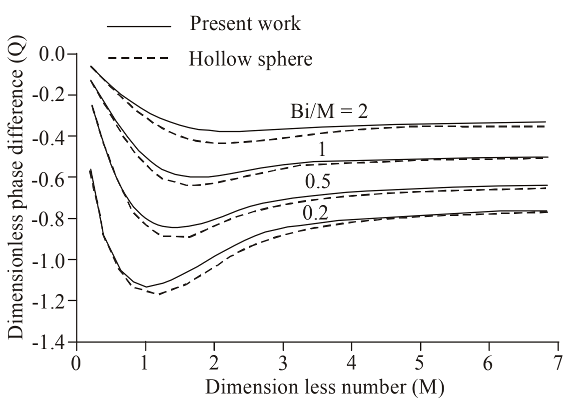

Fig. 2: Comparison between the results of the phase difference of two-dimensional temperature field of a hollow sphere at the outer surface presented in Atefi and Moghimi (2006) and obtained results under harmonic boundary condition

$$D_{kn} = \frac{\omega_{kn}^2}{a^2\delta_{kn}} T_{kn}$$
$$= \int_0^t \int_{r_i}^{r_o} r^2 \left(C_n^{(i)}(\tau)\eta_{ni}(r) + \right.$$
$Cno\tau\eta nor\Phi n\omega knre\!-\!\omega kna2t\!-\!\tau drd\tau$ (25)

The stored thermal energy in hollow sphere is then defined as:

$$\mathrm{Q} = -\int_0^{2\pi}\int_0^{\pi}\int_{r_i}^{r_o} \rho c\theta\,(r,\Psi,t)r^2 \mathrm{dr}$$
$$\sin\Psi d\,\Psi d\varphi \quad (26)$$

Therefore, the stored thermal energy becomes:

$$\mathrm{Q} = -2\pi\boldsymbol{\rho}\mathrm{c}$$
$$\sum_{n=0}^{\infty}\sum_{k=0}^{\infty} D_{kn}\int_{r_i}^{r_o}\int_{-1}^{1} P_n\,(\zeta) r^2 \Phi_n(\omega_{kn} r)$$
$$T_{kn}(t) dr d\varsigma \quad (27)$$

In order to plot the obtained results, some dimensionless numbers are defined as follows:

$$\bar{r} = \frac{r}{r_o}\ ,\quad \bar{t} = \frac{t}{p},\quad x = \frac{r_i}{r_o}$$
$$Bi = \frac{hr_o}{k}\ ,\quad Fo = \frac{p}{a^2 r_o^{\ 2}} \quad (28)$$

where, $\bar{r}$, $\bar{t}$, x, Bi, Fo are dimensionless radius, dimensionless time, dimensionless thickness, Biot and Fourier numbers, respectively. Moreover, functions $f_i(\Psi)$ and $f_o(\Psi)$ must be determined. These functions are arbitrary functions. In order to compare the obtained results with literature one (Atefi and Moghimi, 2006), it is possible to expose the boundary conditions which are assumed in Atefi and Moghimi (2006). In this reference, the function $f_o(\Psi)$ = 1+$Sin^2\Psi$ and $f_i(\Psi)$ is zero. Moreover, $\mathrm{g}_i(t)$ is zero and $\mathrm{g}_o(t)$ is defined by sin $(2\pi\bar{t})$ in harmonic state:

$$\begin{cases} \theta(1,\Psi,\bar{t}) + \dfrac{1}{Bi}\dfrac{\partial\theta}{\partial\bar{r}}\Big|_{1,\Psi,\bar{t}} = \Theta_o(\Psi,\bar{t}) = g_o\,(t) f_o(\Psi) \\ g_o(t) = \sin(2\pi\bar{t}) \\ \dfrac{\partial\theta}{\partial\bar{r}}\Big|_{x,\Psi,\bar{t}} = 0 \end{cases} \quad (29)$$

The results of this comparison are shown in Fig. 1 and 2. As it is explained before, in this study the inner surface of sphere is not insulated and consequently, the functions $f_i(\Psi)$ and $\mathrm{g}_i(t)$ is not zero. In order to plot the results in this case, it is necessary to assume functions for $f_i(\Psi)$ and $f_o(\Psi)$:

$$f_i(\Psi) = f_o(\Psi) = 2\cos\,(\Psi) \quad (30)$$

In this case, γ_{kni} and γ_{kno} are:

$$\gamma_{kni} = \int_x^{1-2} r\,\eta_{ni}(\bar{r})\Phi(\omega_{kn}\bar{r})\,\mathrm{d}\bar{r}$$
$$\gamma_{kno} = \int_x^{1-2} r\,\eta_{no}(\bar{r})\Phi(\omega_{kn}\bar{r})\,\mathrm{d}\bar{r} \quad (31)$$

From Eq. (27), $T_{kn}(\bar{t})$ becomes:

$$T_{kn}(\bar{t}) = \left(\tfrac{2n+1}{2}\right)\int_{-1}^{1}(2\zeta)\,P_n\,\zeta\,\mathrm{d}\zeta$$
$$\begin{pmatrix} \int_0^{\bar{t}} \gamma_{kni}\sum_{m=1}^{\infty}\bar{\Theta}_{im} Sin\,(2m\pi\bar{t}) e^{-\omega_{kn^2}Fo(\bar{t}-\tau)} d\tau + \\ \int_0^{\bar{t}} \gamma_{kno}\Phi_n(\omega_{kn}\bar{r})\sum_{m=1}^{\infty}\bar{\Theta}_{om} Sin \\ (2m\pi\bar{t}) e^{-\omega_{kn^2}Fo(\bar{t}-\tau)} d\tau \end{pmatrix} \quad (32)$$

when ($\bar{t} \to \infty$) the steady state result is obtained as:

$$T_{kn}(\bar{t}) = \frac{(\frac{2n+1}{2})}{Fo\omega_{kn}{}^2}\int_{-1}^{1}(2\zeta)\,P_n(\zeta)\,d\zeta\left[\sum_{m=1}^{\infty}\frac{(\gamma_{kno}\bar{\Theta}_{om}+\gamma_{kni}\bar{\Theta}_{im})}{\sqrt{1+\left(\frac{2mM^2}{\omega_{kn}^2}\right)^2}}Sin(2m\pi\bar{t}+\varphi_{kn})\right] \quad (33)$$

where the dimensionless parameters M and ϕ_{kn} are:

$$M = \sqrt{\frac{\pi}{Fo}}$$

$$\varphi_{kn} = \text{Arc tan}\left(-\frac{2mM^2}{\omega_{kn}^2}\right) \quad (34)$$

so, the temperature distribution of hollow sphere becomes:

$$\theta(\bar{r},\Psi,\bar{t}) = \sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\left(\frac{2n+1}{2}\right)\int_{-1}^{+1}2\zeta P_n\,\zeta\,d\zeta\left(\frac{\Phi(\bar{r}\omega_{kn})P_n(\zeta)}{\delta_{kn}}\right)\left[\sum_{m=1}^{\infty}\frac{(\gamma_{kno}\bar{\Theta}_{om}+\gamma_{kni}\bar{\Theta}_{im})}{\sqrt{1+\left(\frac{2mM^2}{\omega_{kn}^2}\right)^2}}Sin(2m\pi\bar{t}+\varphi_{kn})\right] \quad (35)$$

Substituting the eigenvalues into Eq. (35), the temperature distribution at the outer surface of the hollow sphere becomes:

$$\theta\,(1,\Psi,t) = \sum_{n=0}^{\infty}\sum_{k=0}^{\infty}\sum_{m=1}^{\infty}A_{knm}Sin(2m\pi\bar{t}+\varphi_{kn}) \quad (36)$$

where, A_{knm} is:

$$A_{knm} = \left(\frac{2n+1}{2}\right)\int_{-1}^{+1}2\zeta P_n(\zeta)d\zeta\left(\frac{\Phi(\omega_{kn})P_n(\zeta)}{\delta_{kn}}\right)\frac{(\gamma_{kno}\bar{\Theta}_{om}+\gamma_{kni}\bar{\Theta}_{im})}{\sqrt{1+\left(\frac{2mM^2}{\omega_{kn}^2}\right)^2}} \quad (37)$$

A_{knm} is the ratio of the oscillation amplitude of temperature distribution field in the hollow sphere to the ambient temperature with the same frequency and φ_{kn} is the phase difference.

The assumed functions $g_o(\bar{t}), g_i(\bar{t})$ in harmonic state are:

$$g_o(\bar{t}) = \sin(2\pi\bar{t})$$
$$g_i(\bar{t}) = -\sin(2\pi\bar{t}) \quad (38)$$

RESULTS AND DISCUSSION

The dimensionless amplitude A and the dimensionless phase difference φ are shown with respect to dimensionless numbers M and Bi/M. M is proportional to the square root of oscillations frequency of ambient temperature and inverse square root of Fourier number (Fo). Also Bi/M takes effect from environmental condition, period of oscillation and thermo physical properties of the hollow sphere. $g_o(\bar{t})$, $g_i(\bar{t})$ are harmonic functions. For special case, the dimensionless amplitude and dimensionless phase difference for harmonic state are plotted and compared with Atefi and Moghimi (2006). In this comparison the hollow sphere thickness increases to the possible limit (x = 0.2) in order to reduce the effect of the inner boundary condition. Comparison between our result and Atefi and Moghimi (2006) shows a good agreement as shown in Fig. 1 and 2. In order to provide the comparison between obtained results and the previous study (Atefi and Moghimi, 2006) the function $f_i(\Psi)$ assumes to be zero and the function $f_o(\Psi)$ = $1+Sin^2(\Psi)$. Also, the time-dependent function is assumed in harmonic state Eq. (38). Moreover, according to the Eq. (29), at small Bi number the assumed boundary condition becomes closer to the adiabatic one and results in the better comparison. In constant Bi/M and the low frequency region Bi number is small, so in this region the influence of the inner boundary condition on the temperature distribution at the outer surface is obvious. Effects of inner boundary condition play a crucial role in the temperature distribution variation trend.

The dimensionless amplitude and the phase difference are plotted versus M. It is possible to assume dimensionless number (M). The time dependent part of the inner boundary condition ($g_o(\bar{t}), g_i(\bar{t})$) is as same as the outer boundary condition but there is a phase difference between them which equals to 180°. In this case the maximum available temperature difference is imposed on the hollow sphere. For these two types of boundary conditions dimensionless amplitude and phase difference versus dimensionless number M are shown in Fig. 3 to 6. In this problem, by increasing the Bi number, the amount of stored energy in hollow sphere increases. Since, the thermal systems usually have slow responses; in low frequency region this effect is more obvious. In this region, the hollow sphere acts as low frequency storage. Also, in constant Bi/M, while the Bi number decreases, the value of dimensionless number (M) decreases which result in the gradual

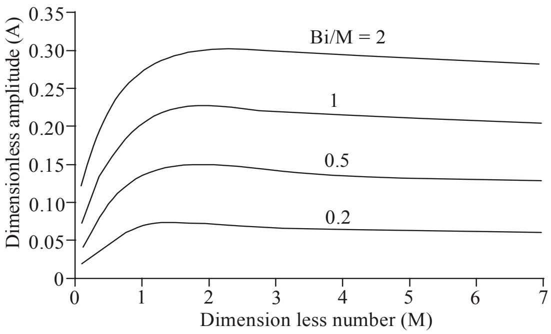

Fig. 3: Dimensionless amplitude, *A* when x = 0.4, $\Psi = \frac{\pi}{6}$ in harmonic state

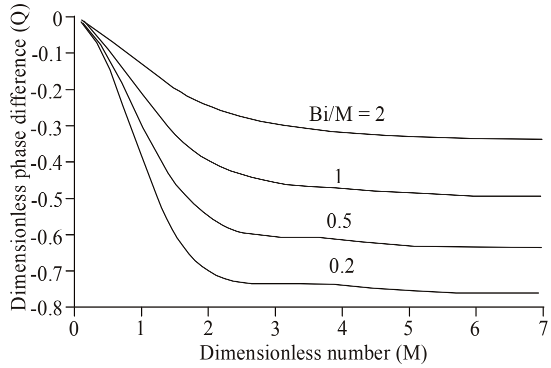

Fig. 4: Dimensionless phase difference, φ when x = 0.4, $\Psi = \frac{\pi}{6}$, in harmonic state

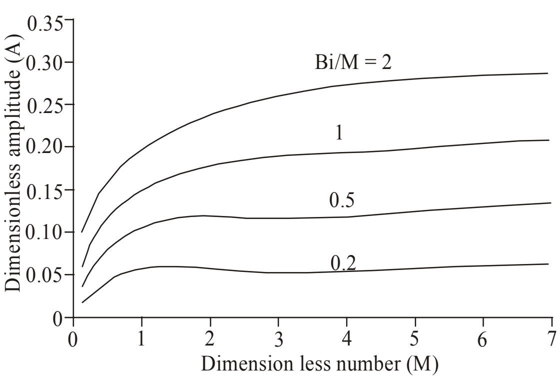

Fig. 5: Dimensionless amplitude, *A* when x = 0.7, $\Psi = \frac{\pi}{6}$, in harmonic state

penetration of the inner boundary effect. The more influences of the inner boundary lead to the more variations of dimensionless amplitude and phase difference in this region. At high frequencies region or high *M* numbers, the amount of energy which passes through the hollow sphere increases this is in inverse of what happens in low frequency region. In the hollow

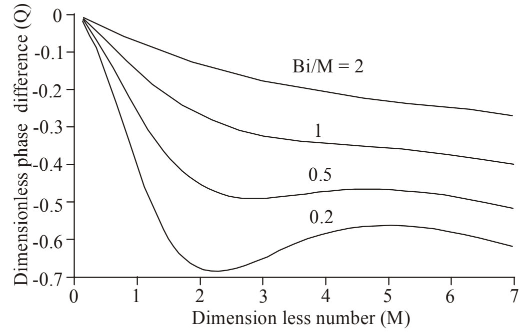

Fig. 6: Dimensionless phase difference, φ when x = 0.7, $\Psi = \frac{\pi}{6}$ in harmonic state

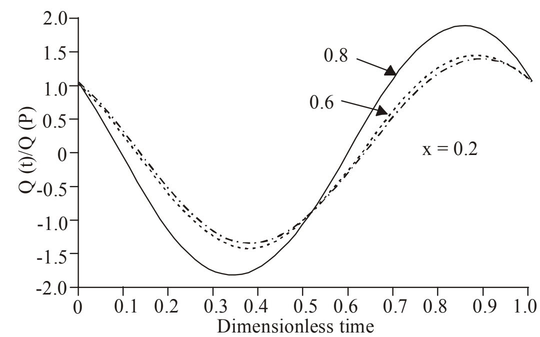

Fig. 7: $Q\ (\bar{t})\ /Q\ (P)$, when M = 2, *Bi* = 1 and $\Psi = \frac{\pi}{3}$ in harmonic state

sphere energy storing causes phase difference. Therefore, *Bi/M* ratio is a good criterion of the phase difference variations. Since *Bi/M* ratio increases, the phase difference decreases. In low frequency region, the phase difference tends to zero because system can follow the ambient temperature frequency variations easily. As it was mentioned before, dimensionless thickness (x) affects the resistance of the system and by increase in *x* value, the inner boundary effects penetrate more and more.

Figure 3 and 4 are plotted for x = 0.4, $\Psi = \frac{\pi}{6}$ in harmonic state. Figure 5 and 6 are plotted for x = 0.7, $\Psi = \frac{\pi}{6}$ in harmonic state. Comparison between Fig. 3 to 6 demonstrates the effect in *x* variation. In order to show the general trend of Figures which is independent of assumed functions $g_o\ (\bar{t})$, $g_i\ (\bar{t})$; Fig. 3 to 6 are plotted. These figures are plotted for same angle ($\Psi = \frac{\pi}{6}$). Comparison between Fig. 3 and 5 shows the same trend in variation of *A*.

The amount of stored energy inside the hollow sphere is plotted versus the dimensionless time ($\bar{t}$) in Fig. 7. As it is shown in this figure increase in *x* number leads to increase in stored energy. Especially in lower frequencies the amount of stored energy increases. Increase in the value of frequency (*M*), leads to decrease in the value of $Q(t)/Q(P)$.

CONCLUSION

In this study, the solution of two-dimensional temperature field distribution under time periodic boundary condition in a hollow sphere has been presented. Varying the parameters (e.g., Bi, M, x) results in a change in dimensionless amplitude and dimensionless phase difference. For instance, increase in Bi number leads to increase in the system resistance or increasing in dimensionless thickness (x) results in decreasing the dimensionless phase difference.

NOMENCLATURE

a^2	=	Inverse of thermal diffusivity, s/m^2
A, A_n	=	Dimensionless amplitude of temperature
B, B_n	=	Dimensionless amplitude of heat flux
Bi	=	Biot number
c	=	Specific Heat, J/kg.K
$f(\zeta)$	=	Space function
Fo	=	Fourier number
$g(t)$	=	Time function of boundary condition
h	=	Convection heat transfer coefficient, W/m^2.K
k	=	Thermal conductivity, W/m.K
M	=	Defined in Eq. (34)
P	=	Time period
Q	=	Stored heat
r	=	Radius, m
$\bar{r}$	=	Dimensionless radius
t	=	Time, sec
$\bar{t}$	=	Dimensionless time
V	=	Volume, m^3
x	=	Dimensionless thickness

Greek letters:

θ	=	Temperature field
ω	=	Eigen value
Φ, η	=	Eigen function
$\alpha_n^{(i)} \alpha_n^{(o)}$	=	Defined by Eq. (15)
$\beta_n^{(o)} \beta_n^{(i)}$	=	Defined by Eq. (15)
$C_n^{(i)} C_n^{(o)}$	=	Defined by Eq. (13)
$\Delta^{(n)}$	=	Defined by Eq. (14)
Ψ	=	Spherical Angle
γ_{kni}	=	Defined by Eq. (31)
δ_{kn}	=	Defined by Eq. (16)
φ_{kn}	=	Phase difference
ρ	=	Density, kg/ m^3
τ	=	Time, sec

Subscript:

0	= Steady
1	= Unsteady
i	= Inner
o	= Outer

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