Research Article

Headway Distributions Based on Empirical Erlang and Pearson Type III Time Methods Compared

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Abstract: The study is aimed at exploring empirical headway derived from Erlang and Pearson type III distribution and comparing the resultant distributions. The Erlang distribution is a simplified form of the Pearson type III. In circumstance where vehicle overtaking is very limited and the ensuing headway values are small, Pearson type IIIor the Erlang distribution may be used to represent the headway frequency distribution. Based on the hypothesis that headways derived from Erlang and Pearson type III distribution will not differ significantly, headway study was carried out in Kulai, Malaysia under dry and daylight conditions. Twenty four hour volume, speed and headways and vehicle type data were collected continuously for eight weeks. The survey data were supplemented with highway design information culled from the Malaysian Design Manual. Empirical headway data were computed using Pearson type III and Erlang distribution methods. Results show that the difference between estimated headways using both techniques is very marginal. The study concluded that in cases where overtaking induces minimal headways, the Pearson type III or the Erlang distribution may be used to represent the headway frequency distribution. However, Pearson type III distribution has to be modified in circumstances where the probability of observing headway increases.

Keywords: Erlang distribution, headway, Pearson type III

INTRODUCTION

Weaving between vehicles is expected in traffic streams. It's probably more pronounced when priority control is used to resolve vehicular conflicts. When a driver arrives at an intersection, the merging options are finite. The driver may elect to enter the major road when acceptable gaps are present or may reject the gaps are being too unsafe and wait for subsequent gaps. In any case the driver will eventually have to make decision correctly or otherwise and enter the prevailing traffic stream. It happens daily on many roadways. Whether, the driver is traversing along a road segment, attempting to negotiate a bend or approaching an intersection, there must be a time intervals often called headways between successive vehicles past a point on the roadway. Headway describes the arrival traffic pattern of vehicles and easy to obtain. Attempts have been made in previous studies to find statistical methods and probabilistic theory with sound and cogent theoretical distribution that can reasonably represent the observed headway distributions. At free flow, headway distribution may be approximated by negative exponential or Poisson distributions. A difficulty of the use of the negative exponential distribution is that the probability of observing headway increases as the size of headway decreases. In circumstance where overtaking is very limited and the headway values are

small, Pearson type III or the Erlang distribution may be used to represent the headway frequency distribution. The Erlang distribution is a simplified form of the Pearson type III. Based on the hypothesis that the headways derived using Erlang and Pearson type III distribution will not differ significantly.

LITERATURE REVIEW

Headway between successive vehicles is an important microscopic flow characteristic that affects the road safety and capacity, level of service, driver behavior. Roadway capacity is very sensitive to headway distribution among others parameters. When the arrival of vehicles at a particular point on the roadway is described, the distribution may either be the number of vehicles arriving in a time interval or simply the time interval between arrivals of successive vehicles. As shown in Eq. (1) and (2) headway, gap and traffic flow are related:

$$headway, h = \frac{3600}{flow} \qquad (1)$$

$$gap, g = h - \frac{l}{speed} \qquad (2)$$

where, l is average length of vehicle, Reaction time value for drivers responding to road situations is a

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fundamental design requirement. Driver's reaction time varies from 0.5 to 4.0 sec even though it is generally assumed that 2.5 sec is sufficient for safe headway between successive vehicles. In any case, several headway distribution models have been proposed, for example, Buckley (1968) semi-Poisson model, Cowan (1975) M3 distribution model, the log-normal distribution (Mei and Bullen, 1999) and the double displaced negative exponential distribution (Sullivan and Troutbeck, 1994). The gamma and shifted exponential distributions was proposed by Al-Ghamdi (1999). The Pearson distribution is a family of continuous probability distributions where Pearson density p can be derived using Eq. (3):

$$\frac{p'(x)}{p(x)} + \frac{a+x-\lambda}{b_2(x-\lambda)^2+b_1(x-\lambda)+b_0} = 0 \qquad (3)$$

For: $b_o = \frac{4\beta_2-3\beta_1}{10\beta_2-12\beta_1-18}\mu_2$

$a = b_1 = \sqrt{\mu_2}\sqrt{\beta_1}\frac{\beta_2+3}{10\beta_2-12\beta_1-18}$

and,

$b_2 = \frac{2\beta_2-3\beta_1-6}{10\beta_2-12\beta_1-18}$

Note that in Eq. (3), the parameter a determines a stationary point, since this is a first order linear differential equation with variable coefficients, then:

$$p(x)\ \alpha\ \exp\left(-\int \frac{x-a}{b_2x^2+b_1x+b_0}\,dx\right)$$

In any case, Pearson type III distribution is a gamma distribution:

$$\lambda = \mu_1 + \frac{b_o}{b_1} - (m+1)b_1 \qquad (4)$$

where,

$b_0 + b_1(x-\lambda)\ is\ \Gamma(m+1, b_1^2)$

Erlang distribution on the other hand is a continuous distribution. It has a positive value for all real numbers greater than zero and is given by two parameters: the shape k, which is a non-negative integer and the rate λ, which is a non-negative real number. The distribution is sometimes defined using the inverse of the rate parameter, the scale μ. It is the distribution of the sum of k independent exponential variables with mean μ. When the shape parameter k equals 1, the distribution simplifies to the exponential distribution. Because of the factorial function in the denominator, the Erlang distribution is only defined when the parameter k is a positive integer. In fact, this distribution is sometimes called the Erlang-k distribution (e.g., an Erlang-2 distribution is an Erlang

Table 1: Distribution and flow level

Flow (veh/h)	Flow range	Distribution family
Low	Less than 400	Negative exponential
Medium	400 to 1,200	Shifted exponential and gamma
High	Above 1,200	Erlang and normal

Flow is in vehicles per hour

distribution with $k = 2$). The Gamma distribution generalizes the Erlang by allowing k to be any real number, using the gamma function instead of the factorial function. The probability density function of the Erlang distribution is:

$$f(x; k; \lambda) = \frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!}\ for\ x, \lambda\ \geq 0 \qquad (5)$$

The parameter k is called the shape parameter and the parameter λ is called the rate parameter. Where (γ) is the lower incomplete gamma function and the Cumulative Distribution Function (CDF) may also be expressed as:

$$F(x; k; \lambda) = 1 - \sum_{n=0}^{k-1}\frac{1}{n!}e^{-\lambda x}(\lambda x)^n \qquad (6)$$

An alternative, but equivalent, parameterization uses the scale parameter μ which is the reciprocal of the rate parameter that is $(\mu = \frac{1}{\lambda})$. When the scale parameter μ equals 2, then distribution simplifies to the chi-squared distribution with $2k$ degrees of freedom. It can therefore be regarded as a generalized chi-squared distribution. Now, there is three type of traffic flow in the field as shown in Table 1. This grouping was helpful to roughly define the boundaries for this three flow states.

According to Summala (2000) If the traffic flow is assumed to be random then the probability of exactly n vehicles arriving at a given point on the roadway in any t second interval is obtained from the Poisson distribution bearing in mind that the Poisson distribution is primarily concerned with discrete variable number of random events per unit time or space:

$$P(n) = \frac{qt^n e(-qt)}{n!} \qquad (7)$$

where, q is the mean rate of arrival per unit time.

However it is the negative exponential distribution which is most commonly used when describing the roadway headway distributions. A difficulty of the use is that the probability of observing headway diminishes as the size of headway decreases because vehicles have a finite length and a minimum following headway. Where a small number of low-value headways are observed, Pearson type lll or Erlang distribution may be used to describe the headway frequency distribution. In any case Erlang distribution is a modified version of Pearson type lll distribution. A random variable such as headway is said to be distributed as the type lll distribution if its probability density is given by:
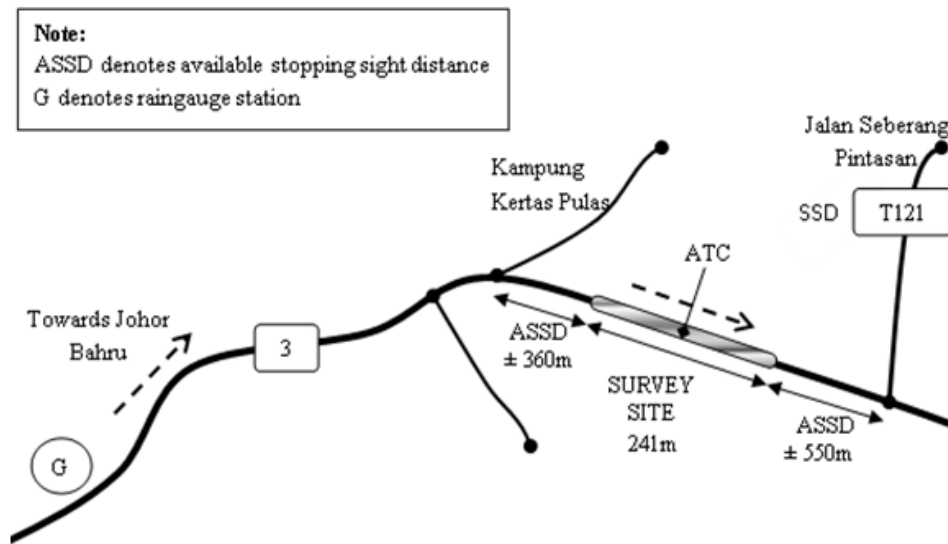
Fig. 1: Typical survey site setup of impact study

$$f(h) = \frac{\lambda}{\Gamma(k)} h^{(k-1)} e^{-\lambda h} \quad 0 < h < \infty \tag{8}$$

where, headways in a single lane are being observed then any headway less than the minimum following distance (h must be greater than α) cannot be observed and Pearson type lll distribution has to be modified to give:

$$f(h) = \frac{\lambda}{\Gamma(k)} [\lambda(h - \alpha)]^{(k-1)} e^{-\lambda(k-\alpha)} \tag{9}$$

where,

$$\Gamma(k) = \int_0^\infty z^{k-1} e^{-z} \, dz$$

Then Eq. (9) can be re-written as:

$$f(h) = \frac{\lambda}{\int_0^\infty z^{k-1} e^{-z} dz} [\lambda(h - \alpha)]^{(k-1)} e^{-\lambda(k-\alpha)} \tag{10}$$

For; α, is the shift parameter generally taken as 0.5 sec:

And $k = \left(\frac{h-\alpha}{h_s}\right)^2$

where,
$\hat{h}$ = The mean of the observed headway data
h_s = Standard deviation of the observed headway data

And $\lambda = \frac{k}{h-\alpha}$

Data collection: Study was carried out under dry weather and daylight conditions. Twenty four hour traffic volume, speeds, vehicle types, headways and gaps were recorded continuously for 8 weeks (January-March 2012) for both directions.

Over 500,000 vehicles/roadway direction were captured on the data logger. Federal Highway FT001 Senai, Kulai and FT005 Skudai, Johor Bahru, have been selected for the studies after careful strategic considerations. The roadway was divided into three sections (upstream, transition and downstream) in both directions. The upstream section was set at a distance greater than Stopping Sight Distance (SSD) so as to minimize influences on free flow speeds. Motorists along upstream section are assumed to be driving at free flow speed. Motorists at the downstream are constrained by U-turning movements occasioned by the presence of midblock facilities. Figure 1 above shows a typical layout of study site and Fig. 2 shows typical traffic flow profile.

DATA ANALYSIS AND RESULTS

Summary of headway distribution is shown below in Table 2. Where headways in a single lane are being observed then any headway less than the minimum following distance cannot be observed; and the Pearson type III distribution has to be modified. As shown in Table 2, the theoretical values are valid for use as substitute of observed headway values in a traffic stream. Student t-test was used to see whether the outcomes from Pearson lll and Erlang are significant in headway frequency distributions. The difference between estimated headways using both techniques is very marginal.

Chi-square test was applied at 5% level for 2° of freedom to see how closely the frequencies of the observed sample values agree with the expected Pearson lll and Erlang distribution frequencies. Results
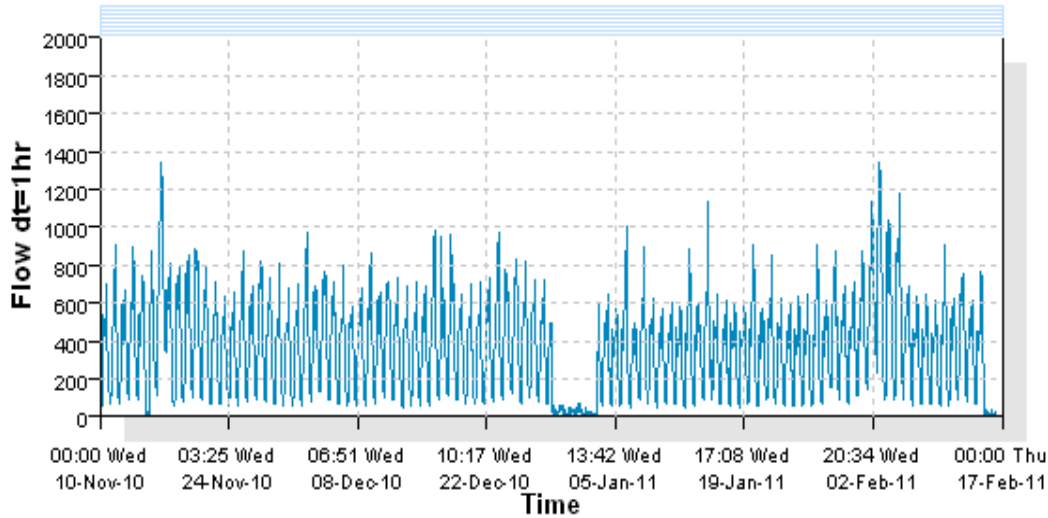
Fig. 2: Traffic flow profile for surveyed sites

Table 2: Summary of headway distribution

Site	Flow veh/h	Observed mean h_w (sec)	Pearson type III (sec)	Erlang (sec)
1	1631	1.5	0.61	0.80
2	1408	1.5	0.37	0.28
3	1557	1.5	0.36	0.26
4	887	2.0	0.25	0.52

Multilane road at free-flow speed; Tabulated t-test = 1.886 (α = 0.10, υ = 2) Obs.hw = Observed headway; Site: 1 JB to Kulai; Site 2: Kulai to JB; Site 3: JB to Senai; Site 4: Senai to JB; Reaction time is taken as 2.0±0.5 sec

Table 3: t-test results for headway data

	$H_o:\mu_o = \mu_p$ $H_1:\mu_o > \mu_p$		$H_o:\mu_o = \mu_p$ $H_1:\mu_o > \mu_p$	
Hypothesis/site	1	2	3	4
Observed headway (sec)	1.50	1.50	1.50	2.00
Pearson III, (sec)	0.61	0.36	0.37	0.25
Change	-0.89	-1.14	-1.13	-1.75
p-value	0	0	0	0
Reject H_o	Yes	Yes	Yes	Yes
Erlang, (sec)	0.80	0.26	0.28	0.52
Change	-0.70	-1.24	-1.22	-1.48
p-value	0	0	0	0
Reject H_o hypothesis?	Yes	Yes	Yes	Yes

Table 4: Chi-square test for headway data

Site	1	2	3	4
Observed headway (sec)	1.500	1.50	1.50	1.50
Pearson III (sec)	0.610	0.36	0.37	0.25
Chi-square	1.299	3.61	3.45	6.25
Chi-square tab = 3.14	<3.140	>3.14	>3.14	>3.14
Erlang, (sec)	0.800	0.26	0.28	0.52
Chi-square	0.613	5.92	5.32	4.21
Chi-square tab = 3.14	<3.140	>3.14	>3.14	>3.14

show that the data actually come from the type of distribution being considered. All the calculated Chi-square values are considerably less the tabulated value of 3.14 thus suggesting that there is no significant difference between the observed and theoretical headway distribution at 5% significance level.

Although headway distributions for free flow scenario may be approximated by the negative exponential distribution, it is clear that the distribution of headways will depend on the traffic volume and density of the roadway. If drivers cannot maintain their desired speed by overtaking slower moving vehicles then free-flow conditions no longer exist and the highway is prone to congestion.

In Table 3, it has been shown that the difference of mean headway from observed data and theoretical Pearson and Erlang was found to be statistically significant. The summary results for Chi-square test are shown in Table 4. Where a small number of low-value headways are observed, such as in the case when a limited amount of overtaking is possible, Pearson type lll or Erlang distribution may be used to represent the headway frequency distribution as shown in the study. In the study headways have used statistical methods and probability theory to find acceptable theoretical distribution that represent observed headway distributions. The extent to which the derived theoretical distribution act as substitute to observed value depends largely upon prevailing volume and density relationships. A difficulty of the use of the negative exponential distribution even under free-flow condition lies with the probability of observing headway increases and the size of headway decreases.

It should be noted that headways less than 1 sec, excess are hidden in class interval of 2 sec. Vehicle length is finite, traffic density is defined per length and traffic flow is a function of speed and density. Traffic congestion may increase until finally all vehicles are at locked step.

CONCLUSION

The study was based on the hypothesis that headways derived from Erlang and Pearson type III

distribution will not differ significantly and also that the distribution of headways may be presented by the exponential distribution. The study concluded that in cases where overtaking induces minimal headways, the Pearson type III or the Erlang distribution may be used to represent the headway frequency distribution.

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