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Research Article

A Multiple Dependent State Control Chart Based on Double Control Limits

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Abstract: This study is to propose and design a new X-bar control chart utilizing previous subgroup information called a multiple dependent state control chart based on double control limits. The in-control Average Run Length (ARL) and the out-of-control ARLs according to mean shifts are derived. The performance of the proposed control chart is compared with the traditional X-bar chart in terms of ARLs, which shows the outperformance of the proposed control chart. This idea can be easily extended to other types of control charts.

Keywords: Average run length, double control limits, multiple dependent state plan

INTRODUCTION

In a manufacturing process, the variation in the process may be caused due to several reasons for example, the external temperature, improper adjustment of manufacturing machines, human errors such as the operator error in recording or analyzing the data and defective raw material (He *et al*., 2002). The process is favorable when it is in control because in this state the process is manufacturing the acceptable products. On the other hand, if the process has shifted, then the process may produce products beyond the specification limits and so the industrial engineer wants the quick indication about this situation so as to search for the cause of the variation. The control chart is one of the important tools of statistical process control. The control chart helps industrial engineers to monitor the manufacturer process and point out when the process is going to be out of control.

Researchers are putting efforts to design efficient control schemes to improve the monitoring capability. Variable control charts are used when the variable of interest in the process is measurable. Among them, the Shewhart X-bar control charts have been widely used for the measurement process. Many authors have discussed the advantages and application of X-bar control charts including for example, Rahim (1989), Saniga (1989), Chiu (1995), Ben-Daya and Rahim (2000), Saniga and Davis (2001), Mehrafrooz and Noorossana (2011), Zhang *et al*. (2011), Chen *et al*. (2011), Fallah Nezhad and Niaki (2010) and Caballero Morales (2013).

By exploring the literature, we note that the control charts utilizing the single, double and triple sampling are available in the literature (He *et al*., 2002). It is known that the control chart based on double sampling or triple sampling is more efficient than the single sampling. One of the important sampling schemes called the Multiple Dependent State (MDS) sampling has been widely used in the area of acceptance sampling plans. Wortham and Baker (1976) introduced the MDS sampling scheme, where the decision on the deposition of a lot is taken on the basis of the current sample information and the previous lot information. In fact, MDS sampling does not require additional sampling but uses previous sample data. Readers may refer to Balamurali and Jun (2007) and Aslam *et al*. (2013).

In this study, we will introduce the MDS sampling in the area of control charts. The proposed control chart is based on double control limits and utilizes the sample information from the previous subgroups in addition to the current subgroup. So, it is expected that the proposed control chart will be more efficient than the traditional Shewhart control chart in term of average run length.

METHODOLOGY

MDS X-bar chart using double control limits: Assume that the quality characteristic of interest follows a normal distribution with mean μ and variance σ^2. It is also assumed that the process mean is m when the process is in control. We propose the MDS X-bar

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control chart using the double control limits. The operational procedure of the proposed chart is given in the following three steps:

Step 1: Select a random sample of size n from the current subgroup and compute $\overline{X}$.

Step 2: Declare the process to be in control if $LCL_2\,\overline{X} \leq UCL_2$. Declare the process to be out-of-control if $\overline{X} \geq UCL_1$ or $\overline{X} \leq LCL_1$. Otherwise, go to Step 3.

Step 3: Declare the process to be in control if i proceeding subgroups shows that the process is in control that is $LCL_2 \leq \overline{X} \leq UCL_2$). Otherwise, declare the process to be out-of-control.

The double control limits of the proposed control chart are composed of the outer control limits:

$$UCL_1 = m + k_1\sigma/\sqrt{\text{n}},\ LCL_1 = m - k_1\sigma/\sqrt{\text{n}}$$

and the inner control limits given by:

$$UCL_2 = m + k_2\sigma/\sqrt{\text{n}},\ LCL_2 = m - k_2\sigma/\sqrt{\text{n}}$$

The proposed chart involves two control constants k_1 and k_2 as well as the parameter i. The proposed chart is reduced to the traditional Shewhart X-bar control chart when $k_1 = k_2 = k$ and $i = 1$.

The probability that the process is in control for the proposed control chart is given as follows:

$$P_{in} = P\ (LCL_2 \leq \overline{X} \leq UCL_2) + \{P\ (LCL_1 < \overline{X} < LCL_2) + P\ (UCL_2 < \overline{X} < UCL_1)\}\ \{P\ (LCL_2 \leq \overline{X} \leq UCL_2)\}^i \quad (1)$$

The probability in Eq. (1) is obtained by:

$$P_{in} = (2\Phi(k_2) - 1) + 2\{\Phi(k_1) - \Phi(k_2)\}\{(2\Phi(k_2) - 1)\}^i \quad (2)$$

So, the Average Run Length (ARL) when the process is in control or the in-control ARL is given as:

$$ARL_0 = \frac{1}{1 - P_{in}} \quad (3)$$

Suppose that the process is shifted from m to $m + c\sigma$, where c is the shift constant in the process. Then, the probability that the process is declared as being in control can be given as follows:

$$P^1_{in} = (\Phi\ (k_2 - c\sqrt{n}) + \Phi\ (k_2 + c\sqrt{n})\ -1) + \{(\Phi\ (\ -(k_2 + c\sqrt{n})\ -\ \Phi\ (\ -(k_1 + c\sqrt{n})))) + \Phi\ ((k_1 - c\sqrt{n})\ -\Phi\ ((k_2 - c\sqrt{n})))\}\ \{\Phi\ (k_2 - c\sqrt{n}) + \Phi\ (k_2 + c\sqrt{n})\ -1\}^i \quad (4)$$

The ARL when the process has shifted to $m + c\sigma$ or the out-of-control ARL is given as:

$$ARL_1 = \frac{1}{1 - P^1_{in}} \quad (5)$$

To construct the tables, we used various values of sample size n, some specified in-control ARL and various shifts in the process. We wrote the R codes for the proposed control chart to complete the tables. These R codes are available from the authors upon request.

The values of ARL_1 are placed in Table 1 to 3. Table 1 is presented when ARL_0 = 200, Table 2 for ARL_0 = 300 and Table 3 for ARL_0 = 370. The control constants k_1 and k_2 as well as the number of the preceding subgroups (i) are determined to yield the specified ARL_0.

It is observed that the ARL decreases rapidly as the shift constant c increases and that the decreasing rate becomes faster as the sample size gets larger. There seems to be no trends in k_1, k_2 and i according to the sample size.

Comparative study: In this section, we compare the performance of the proposed control chart with the traditional X-bar control chart in terms of the ARL. As we have mentioned above the proposed control chart is the generalization of the Shewhart X-bar control chart. When $k_1 = k_2$ = k and i = 0, the proposed control chart becomes the Shewhart X-bar control chart. For the comparison purpose, we selected ARL_0 = 370 and considered various values of sample size. We placed the ARL_1 values for the both control charts in Table 4.

Table 1: ARL_1 for the proposed chart when r0 = ARL_0 =200

c	n = 10 k_1 = 2.943 k_2 = 2.273 i = 4	n = 20 k_1 = 3.166 k_2 = 2.232 i = 6	n = 30 k_1 = 3.037 k_2 = 2.207 i = 4	n = 40 k_1 = 3.021 k_2 = 2.087 i = 2	n = 50 k_1 = 2.927 k_2 = 2.290 i = 4
0	200.000	200.0000	200.000	200.000	200.000
0.1	129.200	85.6400	66.150	54.150	43.770
0.2	53.678	21.5500	13.680	9.908	7.315
0.3	21.820	7.0216	4.284	3.054	2.440
0.4	9.889	3.2650	2.108	1.582	1.422
0.5	5.181	2.0140	1.428	1.172	1.119
0.6	3.149	1.4840	1.163	1.046	1.026
0.7	2.184	1.2267	1.054	1.009	1.003
0.8	1.681	1.0980	1.015	1.001	1.000
0.9	1.398	1.0370	1.003	1.000	1.000
1	1.230	1.0120	1.000	1.000	1.000

Table 2: ARL_1 for the proposed chart when $r0 = ARL_0 = 300$

n	$n = 10$ $k_1 = 3.133$ $k_2 = 2.252$ $i = 3$	$n = 20$ $k_1 = 3.211$ $k_2 = 2.264$ $i = 4$	$n = 30$ $k_1 = 3.266$ $k_2 = 2.286$ $i = 5$	$n = 40$ $k_1 = 3.024$ $k_2 = 2.291$ $i = 2$	$n = 50$ $k_1 = 3.2241$ $k_2 = 2.1311$ $i = 2$
0	300.000	300.000	300.000	300.000	300.000
0.1	187.100	125.400	88.630	79.360	58.190
0.2	72.650	30.150	16.040	14.050	8.373
0.3	27.640	9.023	4.676	3.932	2.472
0.4	11.790	3.775	2.235	1.826	1.364
0.5	5.847	2.144	1.486	1.251	1.089
0.6	3.384	1.521	1.188	1.071	1.017
0.7	2.258	1.240	1.064	1.016	1.002
0.8	1.696	1.104	1.018	1.002	1.000
0.9	1.395	1.040	1.004	1.000	1.000
1	1.224	1.013	1.000	1.000	1.000

Table 3: ARL_1 for the proposed chart when $r0 = ARL_0 = 370$

c	$n = 10$ $k_1 = 3.150$ $k_2 = 2.255$ $i = 2$	$n = 20$ $k_1 = 3.282$ $k_2 = 2.250$ $i = 3$	$n = 30$ $k_1 = 3.148$ $k_2 = 2.256$ $i = 2$	$n = 40$ $k_1 = 3.229$ $k_2 = 2.199$ $i = 2$	$n = 50$ $k_1 = 3.131$ $k_2 = 2.274$ $i = 2$
0	370.000	370.000	370.000	370.000	370.000
0.1	230.700	152.800	117.500	87.820	72.420
0.2	90.270	35.890	22.790	13.710	10.360
0.3	34.480	10.340	6.204	3.697	2.855
0.4	14.560	4.115	2.568	1.735	1.465
0.5	7.027	2.224	1.536	1.216	1.118
0.6	3.909	1.531	1.188	1.059	1.025
0.7	2.494	1.237	1.061	1.013	1.003
0.8	1.799	1.101	1.017	1.002	1.000
0.9	1.436	1.039	1.003	1.000	1.000
1	1.238	1.013	1.000	1.000	1.000

Table 4: Comparisons of ARL_1 for both charts $r0 = ARL_0 = 370$

	n = 10		n = 20		n = 30		n = 40		n = 50	
c	k = 2.99	$k_1 = 3.15$ $k_2 = 2.25$ i = 2	k = 2.99	$k_1 = 3.28$ $k_2 = 2.25$ i = 3	k = 2.99	$k_1 = 3.14$ $k_2 = 2.25$ i = 2	k = 2.99	$k_1 = 3.220$ $k_2 = 2.199$ i = 2	k = 2.999	$k_1 = 3.130$ $k_2 = 2.274$ i = 2
0	370.000	370.030	370.000	370.010	370.000	370.010	370.000	370.010	370.000	370.032
0.1	243.800	230.750	177.500	152.830	137.010	117.570	109.860	87.822	90.568	72.421
0.2	109.80	90.278	56.547	35.893	35.135	22.795	24.154	13.715	17.719	10.368
0.3	49.570	34.488	20.550	10.348	11.432	6.205	7.398	3.698	5.267	2.856
0.4	24.150	14.561	8.851	4.115	4.777	2.568	3.133	1.735	2.315	1.465
0.5	12.810	7.028	4.493	2.225	2.519	1.536	1.771	1.216	1.421	1.119
0.6	7.398	3.909	2.661	1.531	1.632	1.188	1.271	1.059	1.120	1.025
0.7	4.632	2.494	1.811	1.237	1.253	1.062	1.083	1.013	1.026	1.004
0.8	3.133	1.799	1.392	1.102	1.091	1.017	1.020	1.002	1.004	1.000
0.9	2.278	1.436	1.180	1.039	1.028	1.004	1.004	1.000	1.000	1.000
1	1.771	1.239	1.076	1.013	1.007	1.001	1.000	1.000	1.000	1.000

From Table 4, we note that the for the same values of c and n, the ARLs of the proposed chart are much smaller than those of Shewhart X-bar control chart. The proposed control chart is also more efficient than the existing control chart for small shifts. For example, when $c = 0.1$ and $n = 50$, the value of ARL_1 from the existing X-bar control chart is 90.56 and 72.42. The difference between the values of ARL_1 increases as r_0 decreasing or shifts increasing.

CONCLUSION

The MDS X-bar control chart using double control limits is introduced in this study. The proposed control chart not only uses the current information from the sample but also uses the pervious subgroup information. The use of the pervious information increases the efficiency of the control chart in detection of the process mean shift. The efficiency of the proposed control is discussed with the Shewhart X-bar control chart in terms of ARL for the shifted process. From the comparison, we concluded that the proposed chart provides the smaller values of ARL as compared to the existing control chart. The use of the control chart in the industries improves the production process and quality of the product as quick indication will minimize the non-conforming products. The idea of MDS and double control limits can be applied to other types of control charts, which will be pursued in a future study.

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