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Research Article

On The Simple Derivation of Stress-strain Relationship in Composite Laminated Material of Plate and Shell Structures

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Abstract: This study aimed to develop a model to accurately predict the stress-strain relationship and proposed for laminated composite material. Lack of accuracy of Classical Shells Theory (CST) in predicting the influence of transverse deformation occurs due to the line normal to the surface is assumed remain straight and normal to the mid-plane before and after deformation. This assumption overestimates the structures too stiff and the deflections too small. Anyway, for very thin structures CST still suitable for isotropic homogeneous material, but the shear transverse deformations were neglected, hence provide inaccurate results for thicker structures. These lacks had been revised by Constant Shear or First Order Shear Deformation Theory (CSDT/FOSDT), but still suffer shear locking phenomenon, because always have constant value in the shear term. This matter had been corrected by Higher Order Shear Deformation Theory (HOSDT) using refined assumption that the line normal to the surface in a parabolic function and not normal to the mid-plane, but normal to the surfaces so it fulfill the zero strain in the surfaces. The stress-strain relationship of laminated composite material is applied by using Higher Order Lamination Theory (HOLT) that adopted from HOSDT that was accurate for any thicknesses variation and complex material.

Keywords: Higher order shear deformation, laminated composite material, plate and shell structures, simple derivation, stress-strain relationship

INTRODUCTION

In the last decade in civil engineering area has been born FRP as fibrous composite laminated material and it has been discussed by researcher such as Fremond and Maceri (2005), Lawrence Colin Bank (2006), Qasrawi (2007) and Tarek (2010). They investigate on the repairing or retrofitting the existing structures and development all-FRP material as new primary structures and predict the potency of FRP as the one of smartest civil engineering material.

A good example of an all-FRP possibility to be built was Aberfeldy Footbridge (Skinner, 2009; Busel, 2009) in Scotland that built over the River Tay in 1992 shown in Fig. 1 and this is the world's first all-plastic footbridge and it had good performance after 20 years of life. The bridge is a 113 m total span, three-span configuration of 25, 63 and 25 m, respectively cable-stayed structure, width of 2.23 m and two planes 40 cables in two pylons of 18 m height.

Pylons, girders, deck slab even its cables made from 14.5 ton composites. Girders, parapet and pylon made from GFRP with E-glass fiber and isophthalic polyester resin matrices, while cables made from Parafil, an aramid Kevlar fiber coated with polyethylene.

This study is a part of development of a FRP girder and the objective of this study is to propose a mathematical model to accurately predict the stress-strain relationship, that suitable for laminated composite plates and shells material. No numerical benchmarks examples are presented due to the complexity of the shell structures in order to keep the relationship remain general independent from loading, span and thickness.

MATERIALS

Composite and FRP: FRP or other laminated composite material made of two or more layered fine fiber with diameters of 5-15 μm that glued together in the resin material called matrix shown in Fig. 2. The fibrous composite can be Kevlar, Aramid, carbon or glass that each lamina is transversely isotropic in longitudinal direction as main reinforcement in handling the stresses due to its high-strength but lightweight characteristics.

Although laminated composite material have a lot of advantages compared to isotropic one such as steel and concrete, but its application need higher order knowledge to know how it should be treated, analyzed and behaviorally modeled to reach its optimum performance, especially in bending-axial thin walled component such hollow stiffened beam, plates and shells structures.

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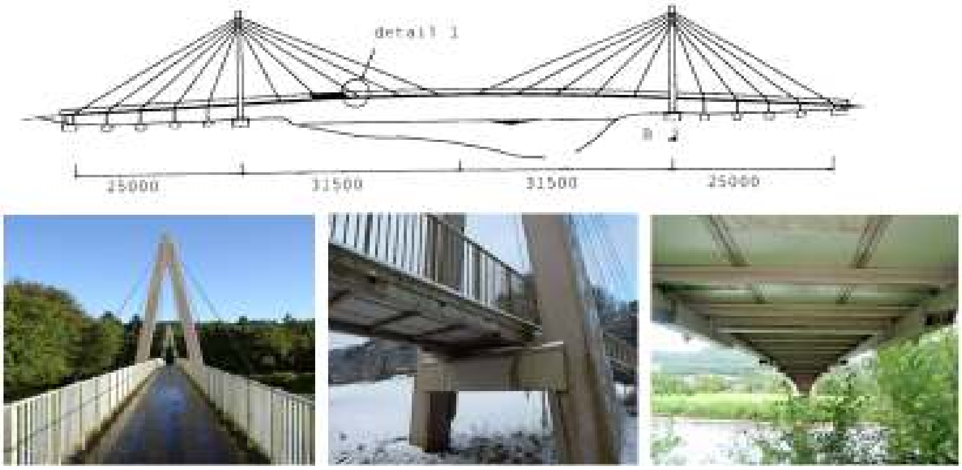

Fig. 1: A barely footbridge: one of all-FRP structures http://compositesandarchitecture.com and http://happypontist.blogspot.com

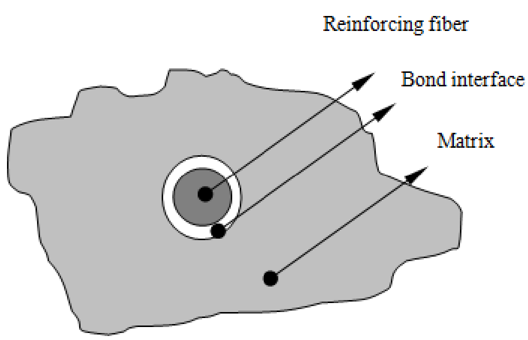

Fig. 2: Reinforcing fiber, matrix and bond interface (Springolo, 2005)

In the world of thin and moderate thick plates and shells analysis, there are three major streams of theory. First model is the one that called Classical Shell Theory (CST) that was is an extension of Euler-Bernoulli beam theory and was developed in Love (1888) using assumptions proposed by Kirchhoff (1887). In fact, for very thin plate ($^h/_a$>>20) with the homogeneous and isotropic material, CST theory still sufficient, but not for thicker ones. This model have a serious shear problem for thicker plates and shells, these arise from its hypothesis that before and after deformation, the line normal to the plane is remain straight and normal to the mid-plane. Therefore, the transverse shear deformation was neglected in this model, as the consequence of these will affected zero shear stress and strain in *xz* and *yz* plane. All of these simplifications can cause the plate and shell stiffness too large, the deflection was too small and its natural frequency get higher than it should be. For modern composite laminated material, where the ratio between elastic and shear modulus (E/G) getting higher, the structures will be so sensitive to the thickness effect because effective transverse shear modulus significantly less than elastic longitudinal modulus through the fiber direction. Hence, transverse shear stress become predominant large and the material will be sensitive to these stresses. This E/G ratio that hence causes CLST (Classical Laminated Shell Theory) is not appropriate for moderate to thick composite laminated plates and shells analysis.

SHEAR DEFORMATION THEORIES

For plates and shells (Reissner and Stein, 1951; Mindlin, 1951) theory were very similar to the Timoshenko and Woinowsky-Krieger (1959) theory where they had improved previous lacks using constant shear deformation assumption Called First Order Shear Deformation Theory (CSDT/FOSDT). Their hypothesis assumed the line normal to the surface remains straight but after deformation it is not normal to the mid-surface. Actually shear deformation has already considered in their model, but only in a constant term. Hence, CSDT/FOSDT need such as $^5/_6$ shear correction factor to fix the corresponding strain energy terms. This coefficient arise another problems that cannot be determined easily and therefore shear locking phenomenon can occurs and also zero shear strain requirement at the surfaces cannot be fulfilled ($\tau_{xz} \neq 0$).

Higher order shear deformation theory: Third order shear deformation by Levinson (1980) and then developed by Bert (1984), Reddy (1984, 1985), Kant and Mallikarjuna (1989) and Kant and Kommineni (1994) called Higher Order Shear Deformation Theory (HOSDT) was a better model by refined hypothesis that a parabolic function of the depth *z* should replaced instead of straight line normal to the surface and not normal to the mid-plane anymore, but normal to the surfaces hence it can meet zero strain requirement in the surfaces $\varepsilon_{xz}|_{z=\pm h/2} = 0$. For both thin and thick plates and shells, HOSDT model has higher accuracy both for

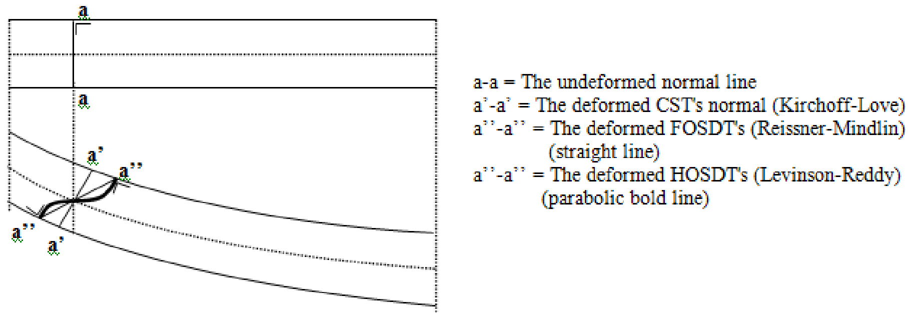

Fig. 3: The hypothesis differences of the shell normal line among three models (Levinson, 1980)

homogeneous isotropic or even layered anisotropic material. This last model involved higher order displacement field using Taylor series in the thickness coordinates and therefore more accurate in predicting global behavior and respond of plate and shell structures. Thus for laminated composite material called HOLT (Higher Order Lamination Theory), this model has close results to the 3D elasticity solution (Latheswary *et al*., 2004).

Based on these matters, the strains and stresses relationship can be found for composite laminated called HOLT (Higher Order Lamination Theory) model. Several restrictions is made for simplification aims, that is the properties of lamina is homogeneous, elastic linear and transversely isotropic with fiber angle and number of lamina variation. Free vibration and buckling eigen and also heat transfer problems were neglected in this analysis. Ignorance of normal stress in z-direction (σ_{zz} = 0) is also set and bonding between two adjacent lamina surfaces are assumed has a full matrix interaction each other and ensure to be strong enough to hold the shear stress or any delamination. This means that the displacement and the strain through the thickness are assumed distributed continuously.

Normal line of three hypothesis before and after deformation: The differences of the normal line hypothesis among three models for before and after deformed shell section given in Fig. 3.

Unlike the previous CST a'-a' and a"-a" model, the third model, HOSDT, is based on the assumption that the deformed normal plane is in the parabolic line to approach an actual deformation. This model have shear transverse value and always fulfill the zero shear strain in the surfaces because the normal line perpendicular in both of surfaces where the shell's depth $z = \pm^{h}/_{2}$.

METHODOLOGY

Proposed modified formulation for isotropic material: After considering thick shell effect, now in order to provide geometrically nonlinear effect in the case of thin shell, an addition coefficient $^{z}/_{h}$ is proposed. A 'tricky' easy and simple enough nonlinear effect is present instead of complicated incremental algorithm hence numerous iterations can be avoided. Therefore asymptotic curve of u and v displacement field through thickness h straightforwardly can be found. This trick will be useful in the finite element formulation.

The modified HOSDT displacement field using corresponding tricky nonlinear term is:

$$\begin{aligned} u(x,y,z) &= u_o(x,y,z) + z.\psi_x(x,y,z) + z^3\phi_x(x,y,z) + \frac{z}{h}.\psi_x(x,y,z) \\ v(x,y,z) &= v_o(x,y,z) + z.\psi_y(x,y,z) + z^3\phi_y(x,y,z) + \frac{z}{h}.\psi_y(x,y,z) \\ w(x,y,z) &= w_o(x,y,z) \end{aligned} \quad (1)$$

This simple term addition will nonlinearly increase accuracy without total or modified Lagrangian formulation or any corrotational approach and also eliminated such Newton-Raphson like that usualy make computational duration costly.

The relationship between displacement u, v and corresponding term $^{1}/_{h}$ shown in Fig. 4.

At the time as the thickness increases in the case of thick shells, the term would be close to zero, hence the shear behavior will be predominant. Conversely, as the thickness h decreases like in the case of thin shells, shear behavior will be recessive, it will create the geometrically nonlinear effect becomes predominant as expected.

Using strain-displacement relationship in the elasticity continuum, strain ε and shear angle γ in xz dan yz plane can be found as:

$$\begin{aligned} \varepsilon_{xz} &= \frac{1}{2}\gamma_{xz} = \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) \\ \varepsilon_{yz} &= \frac{1}{2}\gamma_{yz} = \frac{1}{2}\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) \end{aligned} \quad (2)$$

Setting the condition of transverse shear strain γ_{xz} and γ_{yz} in Eq. (2) on the top and bottom surface for $z = \pm h/2$, as zero:

$$\varepsilon_{xz}\left(z = \pm\tfrac{h}{2}\right) = \varepsilon_{xz}\left(z = \pm\tfrac{h}{2}\right) = 0$$

Hence the kinematic variable ϕ_x and ϕ_y can be stated as:

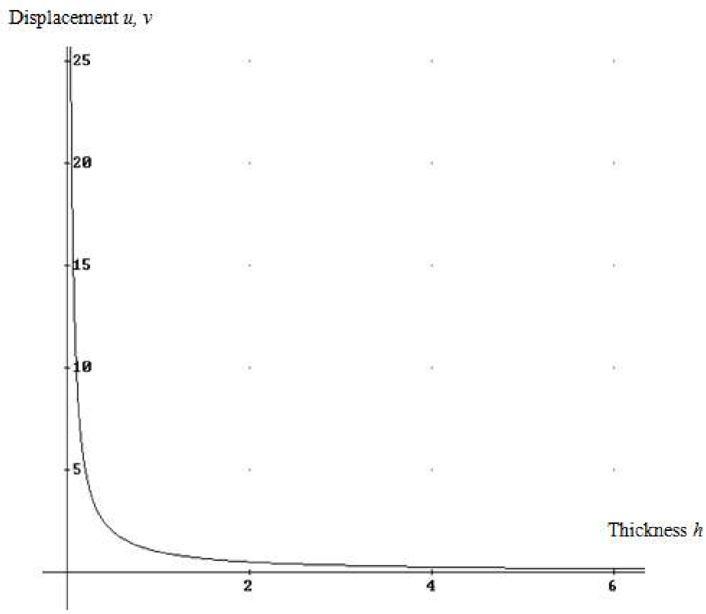

Fig. 4: Asymptotic relationship between displacement field *u* and *v* to thickness *h*

$$\phi_x = -\frac{4}{3h^2}\left((h+1).\psi_x + h.\frac{\partial w}{\partial x}\right)$$
$$\phi_y = -\frac{4}{3h^2}\left((h+1).\psi_y + h.\frac{\partial w}{\partial y}\right) \quad (3)$$

After ignore the derivation to z, both of kinematic variables in the Eq. (3) should be substituted into Eq. (2) and (1), the corresponding strain component $\{\varepsilon\}$ of HOSDT model results:

$$\varepsilon_x = \frac{\partial u}{\partial x} = \frac{\partial u_{ox}}{\partial x} + \left(1+\tfrac{1}{h}\right).z.\left(1-\frac{4.z^2}{3h^2}\right)\frac{\partial \psi_x}{\partial x} - \frac{4.z^3}{3.h^2}.\frac{\partial^2 w}{\partial x^2}$$

$$\varepsilon_y = \frac{\partial v}{\partial y} = \frac{\partial v_{oy}}{\partial y} + \left(1+\tfrac{1}{h}\right).z.\left(1-\frac{4.z^2}{3h^2}\right)\frac{\partial \psi_y}{\partial y} - \frac{4.z^3}{3.h^2}.\frac{\partial^2 w}{\partial y^2}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\partial u_{ox}}{\partial y} + \frac{\partial v_{oy}}{\partial x} + \left(1+\tfrac{1}{h}\right).z.$$
$$\left(1-\frac{4.z^2}{3h^2}\right).\left(\frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x}\right) - \frac{8.z^3}{3.h^2}\frac{\partial^2 w}{\partial x \partial y}$$

$$\gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = z.\left(1+\tfrac{1}{h}\right).\left(1-\frac{4.z^2}{h^2}\right).\psi_x + \left(1-\frac{4.z^2}{h^2}\right)\frac{\partial w}{\partial x}$$

$$\gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} = z.\left(1+\tfrac{1}{h}\right).\left(1-\frac{4.z^2}{h^2}\right).\psi_y + \left(1-\frac{4.z^2}{h^2}\right)\frac{\partial w}{\partial y} \quad (4)$$

Using isotropic material, the stress $\{\sigma\}$ of HOSDT model can be establish from the strain $\{\varepsilon\}$ in Eq. (5) that have been substituted into stress-strain elasticity continuum become:

$$\sigma_x = \frac{E}{1-\nu^2}\left(\varepsilon_x + \nu.\varepsilon_y\right)$$
$$= \frac{E}{1-\nu^2}\left[f_1(z).\left(\frac{\partial \psi_x}{\partial x} + \nu.\frac{\partial \psi_y}{\partial y}\right) - f_2(z).\left(\frac{\partial^2 w}{\partial x^2} + \nu.\frac{\partial^2 w}{\partial y^2}\right)\right]$$

$$\sigma_y = \frac{E}{1-\nu^2}\left(\nu.\varepsilon_x + \varepsilon_y\right)$$
$$= \frac{E}{1-\nu^2}\left[f_1(z).\left(\nu.\frac{\partial \psi_x}{\partial x} + \frac{\partial \psi_y}{\partial y}\right) - f_2(z).\left(\nu.\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)\right]$$

$$\tau_{xy} = 2.G.\varepsilon_{xy} = G.\gamma_{xy}$$
$$= G.\left[f_1(z).\left(\frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x}\right) - 2.f_2(z)\frac{\partial^2 w}{\partial x \partial y}\right]$$

$$\tau_{xz} = 2.G.\varepsilon_{xz} = G.\gamma_{xz}$$
$$= G.\left[f_3(z).\psi_x + f_4(z)\frac{\partial w}{\partial x}\right]$$

$$\tau_{yz} = 2.G.\varepsilon_{yz} = G.\gamma_{yz}$$
$$= G.\left[f_3(z).\psi_y + f_4(z)\frac{\partial w}{\partial y}\right] \quad (5)$$

where,

$$G = \frac{E}{2.(1+\nu)}$$

$$f_1(z) = z.(1+\tfrac{1}{h}).\left(1-\frac{4.z^2}{3.h^2}\right)$$

$$f_2(z) = \frac{4.z^3}{3.h^2}$$

$$f_3(z) = z.(1+\tfrac{1}{h}).\left(1-\frac{4.z^2}{h^2}\right)$$

$$f_4(z) = \left(1-\frac{4.z^2}{h^2}\right)$$

At last the moment, normal and shear internal forces of *M*, *N* and *Q* HOSDT model can be written into:

$$M_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x.z\ dz = \frac{1}{5}.D\left(\frac{\partial^2 w}{\partial x^2} + \nu.\frac{\partial^2 w}{\partial y^2}\right) + \frac{4}{5}.(1+\tfrac{1}{h}).D.\left(\frac{\partial \psi_x}{\partial x} + \nu\frac{\partial \psi_y}{\partial y}\right)$$

$$M_y = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_y.z\ dz = \frac{1}{5}.D\left(\nu.\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) + \frac{4}{5}.(1+\tfrac{1}{h}).D.\left(\nu.\frac{\partial \psi_x}{\partial x} + \frac{\partial \psi_y}{\partial y}\right)$$

$$M_{xy} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{xy}.z\ dz = \frac{1}{5}.D.(1-\nu).\left[2.(1+\tfrac{1}{h}).\left(\frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x}\right) - \frac{\partial^2 w}{\partial x \partial y}\right] \quad (6)$$

$$N_x = \int_{0}^{\frac{h}{2}} \sigma_x\ dz = \frac{1}{4.h}D.\left[5.(1+\tfrac{1}{h}).\left(\frac{\partial \psi_x}{\partial x} + \nu.\frac{\partial \psi_y}{\partial y}\right) - \left(\frac{\partial^2 w}{\partial x^2} + \nu.\frac{\partial^2 w}{\partial y^2}\right)\right]$$

$$N_x = \int_{0}^{\frac{h}{2}} \sigma_x\ dz = \frac{1}{4.h}D.\left[5.(1+\tfrac{1}{h}).\left(\nu.\frac{\partial \psi_x}{\partial x} + \frac{\partial \psi_y}{\partial y}\right) - \left(\nu.\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)\right]$$

where,

$$D = \frac{E.h^3}{12.(1-\nu^2)}$$

and,

$$G = \frac{E}{2.(1+\nu)}$$

The values of N_x and N_y above is the corresponding half of shell thickness, while the integration through the thickness valued as zero, because in the mid-plane z = 0 there is no initial mid-plane yet, likewise N_y:

$$N_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x\ dz = 0 \quad N_y = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_y\ dz = 0$$

The shear forces of HOSDT are not zero, but on the contrary with previous, its maximum value occurs in the support region of mid-plane:

$$Q_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{xz}\ dz = \frac{2}{3}Gh.\left(\frac{\partial w}{\partial x} + (1+\tfrac{1}{h})\psi_x\right)$$

$$Q_y = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{yz}\ dz = \frac{2}{3}Gh.\left(\frac{\partial w}{\partial y} + (1+\tfrac{1}{h})\psi_y\right) \quad (7)$$

The strain of HOSDT model can be declared as:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ \gamma_{xz}^o \\ \gamma_{yz}^o \end{Bmatrix} + \begin{Bmatrix} f_1(z).\frac{\partial \psi_x}{\partial x} - f_2(z)\frac{\partial^2 w}{\partial x^2} \\ f_1(z).\frac{\partial \psi_y}{\partial y} - f_2(z)\frac{\partial^2 w}{\partial y^2} \\ f_1(z).\left(\frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x}\right) - 2.f_2(z)\frac{\partial^2 w}{\partial x \partial y} \\ f_3(z).\psi_x + f_4(z).\frac{\partial w}{\partial x} \\ f_3(z).\psi_y + f_4(z).\frac{\partial w}{\partial y} \end{Bmatrix} \quad (8)$$

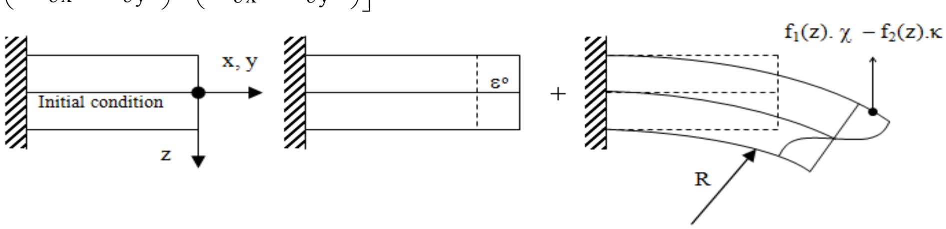

Fig. 5: Total strain of HOSDT model

where, ε^o shows mid-plane strain at $z = 0$ while κ is mid-plane curvature 1/R such shown in Fig. 5.

In order to ease mentioned aims, the following notation and symbols are introduced:

$$\varepsilon_x^o = \frac{\partial u_{ox}}{\partial x} \quad \varepsilon_y^o = \frac{\partial v_{oy}}{\partial y}$$
$$\gamma_{xy}^o = \frac{\partial u_{ox}}{\partial y} + \frac{\partial v_{oy}}{\partial x} \quad \gamma_{xz}^o = \frac{\partial w_o}{\partial x} + \frac{\partial u_{ox}}{\partial z}$$
$$\gamma_{yz}^o = \frac{\partial w_o}{\partial y} + \frac{\partial v_{oy}}{\partial z}$$
$$\chi_x = \frac{\partial \psi_x}{\partial x} \quad \chi_y = \frac{\partial \psi_y}{\partial y} \quad \chi_{xy} = \frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x}$$
$$\kappa_x = \frac{\partial^2 w}{\partial x^2} \quad \kappa_y = \frac{\partial^2 w}{\partial y^2} \quad \kappa_{xy} = 2\frac{\partial^2 w}{\partial x \partial y}$$
$$\mu_x = \frac{\partial w}{\partial x} \quad \mu_y = \frac{\partial w}{\partial y} \tag{9}$$

Implementation procedure for composite laminated material:

Transversely isotropic material: Transversely isotropic composite material consist of one-way longitudinal fibers material as reinforcement that sticked together in a resin matrix. The direction of transversely isotropic composites material that called lamina shown in Fig. 6.

The following Eq. (2-55) describe the integration each lamina into laminated homogenization:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} dz = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} dz$$
$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} z\, dz = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} z\, dz$$
$$\begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix} dz = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} \begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix} dz \tag{10}$$

where, the components of N_x, N_y, N_{xy} and M_x, M_y, M_{xy} and also Q_x, Q_y is the corresponding components in Eq. (2-52a) dan (2-52b) for composite laminated material.

Consider a composite laminated element in Fig. 7, where n is the number of lamina and h_k is the corresponding lamina thickness k^{th} and h_{k-1} is the previous lamina thickness (k-1).

Based on the lamina stress-strain relationship, hence the corresponding stress tensor of HOSDT for the new notation in Eq. (2-56) given as:

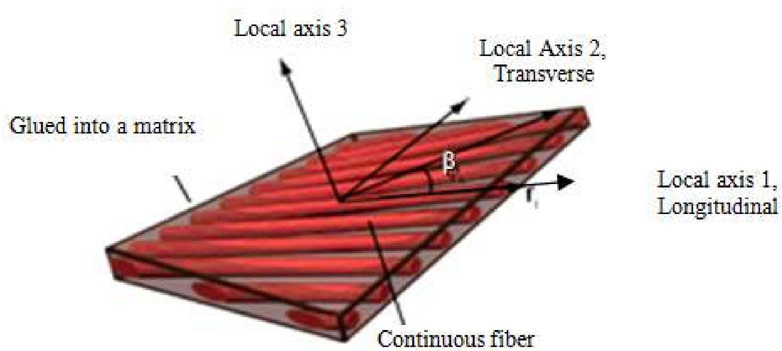

Fig. 6: Lamina scheme (Cugnoni, 2004)

Fig. 7: Laminate nomenclature used in ABD matrix (Hyer, 1998)

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{14} & 0 & 0 \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{24} & 0 & 0 \\ \overline{Q}_{14} & \overline{Q}_{24} & \overline{Q}_{44} & 0 & 0 \\ 0 & 0 & 0 & \overline{Q}_{55} & \overline{Q}_{56} \\ 0 & 0 & 0 & \overline{Q}_{56} & \overline{Q}_{66} \end{bmatrix} \cdot \begin{Bmatrix} \varepsilon_x^o + f_1(z).\chi_x - f_2(z).\kappa_x \\ \varepsilon_y^o + f_1(z).\chi_y - f_2(z).\kappa_y \\ \gamma_{xy}^o + f_1(z).\chi_{xy} - f_2(z).\kappa_{xy} \\ \gamma_{xz}^o + f_3(z).\psi_x + f_4(z).\mu_x \\ \gamma_{yz}^o + f_3(z).\psi_y + f_4(z).\mu_y \end{Bmatrix} \tag{11}$$

Multiplying both matrices in the right term in Eq. (9), thus stress equation can be written as:

$$\begin{aligned}
\sigma_x &= \overline{Q}_{11}.\varepsilon_x^o + \overline{Q}_{12}.\varepsilon_y^o + \overline{Q}_{14}.\gamma_{xy}^o + z.\left(1+\tfrac{1}{h}\right).\left[\overline{Q}_{11}.\chi_x + \overline{Q}_{12}.\chi_y + \overline{Q}_{14}.\chi_{xy}\right] \\
&\quad - \frac{4.z^3}{3.h^2}\left[\overline{Q}_{11}.\kappa_x + \overline{Q}_{12}.\kappa_y + \overline{Q}_{14}.\kappa_{xy}\right] - \frac{4.z^4}{3.h^2}\left(1+\tfrac{1}{h}\right).\left[\overline{Q}_{11}.\chi_x + \overline{Q}_{12}.\chi_y + \overline{Q}_{14}.\chi_{xy}\right] \\
\sigma_x &= \overline{Q}_{12}.\varepsilon_x^o + \overline{Q}_{22}.\varepsilon_y^o + \overline{Q}_{24}.\gamma_{xy}^o + z.\left(1+\tfrac{1}{h}\right).\left[\overline{Q}_{12}.\chi_x + \overline{Q}_{22}.\chi_y + \overline{Q}_{24}.\chi_{xy}\right] \\
&\quad - \frac{4.z^3}{3.h^2}\left[\overline{Q}_{12}.\kappa_x + \overline{Q}_{22}.\kappa_y + \overline{Q}_{24}.\kappa_{xy}\right] - \frac{4.z^4}{3.h^2}\left(1+\tfrac{1}{h}\right).\left[\overline{Q}_{12}.\chi_x + \overline{Q}_{22}.\chi_y + \overline{Q}_{24}.\chi_{xy}\right] \\
\tau_{xy} &= \overline{Q}_{14}.\varepsilon_x^o + \overline{Q}_{24}.\varepsilon_y^o + \overline{Q}_{44}.\gamma_{xy}^o + z.\left(1+\tfrac{1}{h}\right).\left[\overline{Q}_{14}.\chi_x + \overline{Q}_{24}.\chi_y + \overline{Q}_{44}.\chi_{xy}\right] \\
&\quad - \frac{4.z^3}{3.h^2}\left[\overline{Q}_{14}.\kappa_x + \overline{Q}_{24}.\kappa_y + \overline{Q}_{44}.\kappa_{xy}\right] - \frac{4.z^4}{3.h^2}\left(1+\tfrac{1}{h}\right).\left[\overline{Q}_{14}.\chi_x + \overline{Q}_{24}.\chi_y + \overline{Q}_{44}.\chi_{xy}\right] \\
\tau_{xz} &= \overline{Q}_{55}.\gamma_{xz}^o + \overline{Q}_{56}.\gamma_{yz}^o + \overline{Q}_{55}.\mu_x + \overline{Q}_{56}.\mu_y + z.\left(1+\tfrac{1}{h}\right).\left[\overline{Q}_{55}.\psi_x + \overline{Q}_{56}.\psi_y\right] \\
&\quad - \frac{4.z^2}{h^2}\left[\overline{Q}_{55}.\mu_x + \overline{Q}_{56}.\mu_y\right] - \frac{4.z^3}{h^2}\left(1+\tfrac{1}{h}\right).\left[\overline{Q}_{55}.\psi_x + \overline{Q}_{56}.\psi_y\right] \\
\tau_{yz} &= \overline{Q}_{56}.\gamma_{xz}^o + \overline{Q}_{66}.\gamma_{yz}^o + \overline{Q}_{56}.\mu_x + \overline{Q}_{66}.\mu_y + z.\left(1+\tfrac{1}{h}\right).\left[\overline{Q}_{56}.\psi_x + \overline{Q}_{66}.\psi_y\right] \\
&\quad - \frac{4.z^2}{h^2}\left[\overline{Q}_{56}.\mu_x + \overline{Q}_{66}.\mu_y\right] - \frac{4.z^3}{h^2}\left(1+\tfrac{1}{h}\right).\left[\overline{Q}_{56}.\psi_x + \overline{Q}_{66}.\psi_y\right]
\end{aligned} \tag{12}$$

Substituting Eq. (2-58) into (2-56), the components of N_x, M_x dan Q_x can be described as:

$$\begin{aligned}
N_x &= \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} \{\overline{Q}_{11}.\varepsilon_x^o + \overline{Q}_{12}.\varepsilon_y^o + \overline{Q}_{14}.\gamma_{xy}^o + z.\left(1+\tfrac{1}{h}\right).\left[\overline{Q}_{11}.\chi_x + \overline{Q}_{12}.\chi_y + \overline{Q}_{14}.\chi_{xy}\right] \\
&- \frac{4.z^3}{3.h^2}\left[\overline{Q}_{11}.\kappa_x + \overline{Q}_{12}.\kappa_y + \overline{Q}_{14}.\kappa_{xy}\right] - \frac{4.z^4}{3.h^2}\left(1+\tfrac{1}{h}\right).\left[\overline{Q}_{11}.\chi_x + \overline{Q}_{12}.\chi_y + \overline{Q}_{14}.\chi_{xy}\right]\}\, dz \\
M_x &= \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} \{\overline{Q}_{11}.\varepsilon_x^o + \overline{Q}_{12}.\varepsilon_y^o + \overline{Q}_{14}.\gamma_{xy}^o + z.\left(1+\tfrac{1}{h}\right).\left[\overline{Q}_{11}.\chi_x + \overline{Q}_{12}.\chi_y + \overline{Q}_{14}.\chi_{xy}\right] \\
&- \frac{4.z^3}{3.h^2}\left[\overline{Q}_{11}.\kappa_x + \overline{Q}_{12}.\kappa_y + \overline{Q}_{14}.\kappa_{xy}\right] - \frac{4.z^4}{3.h^2}\left(1+\tfrac{1}{h}\right).\left[\overline{Q}_{11}.\chi_x + \overline{Q}_{12}.\chi_y + \overline{Q}_{14}.\chi_{xy}\right]\}\, zdz \\
Q_x &= \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} \{\overline{Q}_{55}.\gamma_{xz}^o + \overline{Q}_{56}.\gamma_{yz}^o + \overline{Q}_{55}.\mu_x + \overline{Q}_{56}.\mu_y + z.\left(1+\tfrac{1}{h}\right).\left[\overline{Q}_{55}.\psi_x + \overline{Q}_{56}.\psi_y\right] \\
&- \frac{4.z^2}{h^2}\left[\overline{Q}_{55}.\mu_x + \overline{Q}_{56}.\mu_y\right] - \frac{4.z^3}{h^2}\left(1+\tfrac{1}{h}\right).\left[\overline{Q}_{55}.\psi_x + \overline{Q}_{56}.\psi_y\right]\}\, dz
\end{aligned} \tag{13}$$

In order to simplify matrices operations, hence A_{ij}, B_{ij} and D_{ij} coefficient are introduced as primary normal and moment components instead of Q_{ij} integration such as usual CLT tipical coefficients as ABD matrix. While E_{ij}, F_{ij}, H_{ij}, V_{ij} coefficients are higher order normal, moment and shear components. Therefore, Eq. (12) becomes:

$$N_x = A_{11}.\varepsilon_x^o + A_{12}.\varepsilon_y^o + A_{14}.\gamma_{xy}^o + B_{11}.\chi_x + B_{12}.\chi_y + B_{14}.\chi_{xy} + E_{11}\kappa_x + E_{12}.\kappa_y + E_{14}\kappa_{xy}$$

$$M_x = B_{11}.\varepsilon_x^o + B_{12}.\varepsilon_y^o + B_{14}.\gamma_{xy}^o + B_{11}.\chi_x + B_{12}.\chi_y + B_{14}.\chi_{xy} + E_{11}\kappa_x + E_{12}.\kappa_y + E_{14}\kappa_{xy}$$

and etc., if they are written in the matrix form gives:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ P_x \\ P_y \\ P_{xy} \\ Q_x \\ Q_y \\ V_x \\ V_y \\ R_x \\ R_y \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{14} & B_{11} & B_{12} & B_{14} & E_{11} & E_{12} & E_{14} & 0 & 0 & 0 & 0 & 0 & 0 \\ A_{12} & A_{22} & A_{24} & B_{12} & B_{22} & B_{24} & E_{12} & E_{22} & E_{24} & 0 & 0 & 0 & 0 & 0 & 0 \\ A_{14} & A_{24} & A_{44} & B_{14} & B_{24} & B_{44} & E_{14} & E_{24} & E_{44} & 0 & 0 & 0 & 0 & 0 & 0 \\ B_{11} & B_{12} & B_{14} & D_{11} & D_{12} & D_{14} & F_{11} & F_{12} & F_{14} & 0 & 0 & 0 & 0 & 0 & 0 \\ B_{12} & B_{22} & B_{24} & D_{12} & D_{22} & D_{24} & F_{12} & F_{22} & F_{24} & 0 & 0 & 0 & 0 & 0 & 0 \\ B_{14} & B_{24} & B_{44} & D_{14} & D_{24} & D_{44} & F_{14} & F_{24} & F_{44} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & F_{11} & F_{12} & F_{14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & F_{12} & F_{22} & F_{24} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & F_{14} & F_{24} & F_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_{55} & A_{56} & A_{55} & A_{56} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_{56} & A_{66} & A_{56} & A_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & B_{55} & B_{56} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & B_{56} & B_{6s6} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & D_{55} & D_{56} & E_{55} & E_{56} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & D_{56} & D_{66} & E_{56} & E_{66} \end{bmatrix} . \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ \chi_x \\ \chi_y \\ \chi_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \\ \gamma_{xz}^o \\ \gamma_{yz}^o \\ \mu_x \\ \mu_y \\ \psi_x \\ \psi_y \end{Bmatrix} \tag{14}$$

where,

$$A_{ij} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \overline{Q}_{ij}^{(k)} dz = \sum_{k=1}^{n} \overline{Q}_{ij}^{(k)}.(h_k - h_{k-1}) \text{ where } 1, j = 1, 2, 4, 5, 6$$

$$B_{ij} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \overline{Q}_{ij}^{(k)} z.\left(1+\tfrac{1}{h}\right) dz = \frac{1}{2}.\left(1+\tfrac{1}{h}\right)\sum_{k=1}^{n} \overline{Q}_{ij}^{(k)}.\left(h_k^{\ 2} - h_{k-1}^{\ 2}\right) \text{ where } i, j = 1, 2, 4, 5, 6$$

$$D_{ij} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \overline{Q}_{ij}^{(k)} z^2 \left(1+\tfrac{1}{h}\right) dz = \frac{1}{3}.\left(1+\tfrac{1}{h}\right)\sum_{k=1}^{n} \overline{Q}_{ij}^{(k)}.\left(h_k^{\ 3} - h_{k-1}^{\ 3}\right) \text{ where } i, j = 1, 2, 4$$

$$D_{rs} = -\int_{-\frac{h}{2}}^{\frac{h}{2}} \overline{Q}_{ij}^{(k)} \frac{4}{h^2} z^2 \ dz = -\frac{4}{3.h^2}.\sum_{k=1}^{n} \overline{Q}_{ij}^{(k)}.\left(h_k^{\ 3} - h_{k-1}^{\ 3}\right) \text{ where } r, s = 5, 6$$

$$E_{ij} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \overline{Q}_{ij}^{(k)} z^3 \ dz = -\frac{1}{3.h^2}.\sum_{k=1}^{n} \overline{Q}_{ij}^{(k)}.\left(h_k^{\ 4} - h_{k-1}^{\ 4}\right) \text{ where } i, j = 1, 2, 4$$

$$E_{rs} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \overline{Q}_{ij}^{(k)} \frac{4.z^3}{3.h^2}\left(1+\tfrac{1}{h}\right) dz = -\frac{1}{3.h^2}.\left(1+\tfrac{1}{h}\right)\sum_{k=1}^{n} \overline{Q}_{ij}^{(k)}.\left(h_k^{\ 4} - h_{k-1}^{\ 4}\right) \text{ where } r, s = 5, 6$$

$$F_{ij} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \overline{Q}_{ij}^{(k)} \frac{4.z^4}{3.h^2}.\left(1+\tfrac{1}{h}\right) dz = -\frac{4}{15.h^2}.\left(1+\tfrac{1}{h}\right)\sum_{k=1}^{n} \overline{Q}_{ij}^{(k)}.\left(h_k^{\ 5} - h_{k-1}^{\ 5}\right) \text{ where } i, j = 1, 2, 4$$

CONCLUSION

The mathematical stress-strain relationship for composite laminated material have been and derived using Higher Order Lamination Theory (HOLT) adopted from HOSDT. A ‘tricky’ asymptotic nonlinear effect is present instead of complicated incremental algorithm. Several higher order internal forces components also developed. Hence new procedure for stress-strain relationship of composite laminated structures has been proposed.

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