

Research Article

Time Truncated Testing Strategy using Multiple Testers: Lognormal Distributed Lifetime

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Abstract: In this study, group acceptance sampling plan proposed by Aslam *et al.* (2011) is reconsidered when the lifetime variant of the test item follows lognormal distribution. The optimal plan parameters are obtained by considering various pre-specified parameters. The plan parameters are obtained using the non-linear optimization solution using two points approach. The advantage of the proposed plan is discussed over the existing plan using the single point approach and the proposed plan is more efficient than the existing plan.

Keywords: Group acceptance sampling plan, lognormal distribution, producer's risk and consumer's risk, testers

INTRODUCTION

Acceptance sampling is one of the methods of quality improvement in the quality-assurance. The main purpose of acceptance sampling is to minimize the time and cost of the experiment. Acceptance sampling plans help the producer to enhance the quality of the product. For the finished product, it may not possible to inspect/test each and every item at the time of the inspection. Then, a random sample is selected with the help of acceptance sampling scheme for the inspection of the lot. The items selected in the sample are put on the test. If the numbers of failures are less than the specified number of failure, the lot is accepted. In these schemes, as the decision is made on the basis of sample information, therefore two risks are always attached with sampling plans. The probability that a good lot is rejected on the basis of sample information is known as producer's risk and the probability of accepting a bad lot is called the consumer's risk. Several authors proposed acceptance sampling plans based on truncated life test for various distributions, for example, Goode and Kao (1961), Gupta and Groll (1961), Gupta (1962), Kantam *et al.* (2001), Tsai and Wu (2006), Balakrishnan *et al.* (2007) and Rosaiah *et al.* (2007).

In usual acceptance sampling plan, a single item is installed on a single tester. In practice, testers are available in the laboratories that can accommodate more than one item. In this situation, for the testing/inspection of the product the experimenters can use the Group Acceptance Sampling Plan (GASP). The GASP is less costly than the usual acceptance sampling plan. The more details about acceptance sampling plans can be seen in Aslam and Jun (2009a, b), Aslam *et al.* (2011), Stephens (2001), Aslam *et al.* (2012) and Khan and Islam (2013).

Kundu and Manglick (2004), discriminate between the two distributions named Weibull and lognormal to the two real data sets. They showed the preference of lognormal distribution over the Weibull distribution using the two real data sets. Khazaei *et al.* (2008) worked on mass and dimensional properties of seeds and kernels of two sunflower varieties grown in Iran using normal, lognormal and the Weibull densities for the prediction of mass and size distributions. According to Khazaei *et al.* (2008) "lognormal distribution is the best fitted as compared to other distributions for modeling the size and mass distributions of sunflower seeds and kernels".

As in practice the Weibull and lognormal distributions widely used for fitting failure time data and to the best of our knowledge still nobody provides the GASP for lognormal distribution.

The objective of this study is to obtain the optimal plan parameters (g and c) by considering various pre-specified parameters, of group acceptance sampling plan, when the lifetime variate follows lognormal distribution.

METHODOLOGY

Designing of proposed plan: In this study, group acceptance sampling plan proposed by Aslam *et al.* (2011) is reconsidered when the lifetime variant of the test item follows lognormal distribution. Aslam *et al.* (2011) proposed the following group acceptance sampling plan:

- Selecting the number of groups g and allocate predefined r items to each of the group so that the sample size for a lot will be $n = rg$.

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- Selecting the acceptance number c for a group and the experiment time t_0 .
- To carry out the experiment for the g groups simultaneously and record the number of failures for each of the group.
- Accept the lot if at most c failures occurs in all of the groups.
- Truncate the experiment, if more than c failures occur and reject the lot or, at time t_0 .

The stated plan is based on two plan parameters g and c . The proposed plan reduces to the ordinary acceptance sampling plan when $r = 1$.

Aslam *et al.* (2011) derived the Operating Characteristic (OC) function of the plan which is given as follows:

$$L(p) = \sum_{i=0}^c \binom{rg}{i} p^i (1-p)^{rg-i} \quad (1)$$

where, p is the probability of the failure of an item before the termination time t_0 .

The probability density function (pdf) and the cumulative distribution function (cdf) of a lognormal distribution are as follows, respectively:

$$f(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma x} \exp\left(-\frac{(\ln x - \mu)^2}{2\sigma^2}\right), \quad x > 0, \sigma > 0, \mu > -\infty \quad (2)$$

where, $\ln x$ denotes the logarithm of x to the base e . The parameters μ and σ^2 are the mean and variance of $\ln x$ which is normally distributed:

$$F(x; \mu, \sigma) = \Phi\left(\frac{\ln(x) - \mu}{\sigma}\right) \quad (3)$$

Under the lognormal distribution, we have $p = \int_{-\infty}^z \frac{\exp(-x^2/2)}{\sqrt{2\pi}} dx = \Phi(z)$, where $z = (\ln t - e^{\mu_0})/\sigma$. Therefore, the OC function given in Eq. (1) can be rewritten as:

$$L(p) = \sum_{i=0}^c \binom{rg}{i} \{\Phi^i(z)\} \{1 - \Phi^i(z)\}^{rg-i} \quad (4)$$

Following Gupta (Aslam and Jun, 2009b), $z = \xi_p - \mu_0/\sigma$, then, $p = \Phi(\xi_p - \mu_0/\sigma)$, also, $z = (t - \xi_p - \sigma\Phi_{(p)}^{-1})/\sigma$ or $t = \xi_p/\sigma + \Phi_{(p)}^{-1}$. Finally, the OC function of the proposed plan using the lognormal distribution is given as follows:

$$L(p) = \sum_{i=0}^c \binom{rg}{i} \left\{ \Phi^i \left(\ln t - \xi_p^0/\sigma + \Phi_{(p)}^{-1} \right) \right\} \left\{ 1 - \ln t - \xi_p^0\sigma + \Phi_p - 1rg - i \right\} \quad (5)$$

The interest is to find the two plan parameters g and c for other specified plan parameters using the

Table 1: Plan parameters for lognormal distribution

a = 0.5, r = 5 and sigma = 2				
β	e^μ/e^{μ_0}	g_i	c	OC values
0.25	2	17	27	0.95259
	4	6	8	0.97304
	6	5	5	0.98453
	8	2	2	0.95620
	10	2	2	0.97466
0.10	2	27	41	0.95388
	4	8	10	0.97107
	6	5	5	0.95545
	8	4	4	0.97896
	10	4	3	0.95871
0.05	2	33	49	0.95023
	4	10	12	0.97104
	6	5	6	0.96417
	8	4	5	0.96601
	10	4	3	0.95871
0.01	2	47	68	0.95248
	4	15	16	0.95168
	6	9	8	0.95351
	8	4	6	0.95583
	10	4	4	0.95251

optimization procedure so that the probability of acceptance is larger than the producer's confidence level $(1 - \alpha)$ at various ratio of true median life to the specified median life (e^μ/e^{μ_0}) and the lot acceptance probability should be less than the consumer's risk β at $e^\mu = e^{\mu_0}$. Let p_1 be the probability of the failure of a product before t_0 using various values of e^μ/e^{μ_0} and p_2 be the probability of failure of an item before t_0 when $e^\mu = e^{\mu_0}$ (Aslam *et al.*, 2011). Therefore the purpose is to find the two plan parameters so that the two inequalities should be satisfied simultaneously using the following optimization procedure:

$$\sum_{i=0}^c \binom{rg}{i} \left\{ \Phi^i \left(\ln t - \xi_{p_1}^0/\sigma + \Phi_{(p_1)}^{-1} \right) \right\} \left\{ 1 - \ln t - \xi_p^0\sigma + \Phi_p - 1rg - i \geq 1 - \alpha \right\} \quad (6)$$

$$\sum_{i=0}^c \binom{rg}{i} \left\{ \Phi^i \left(\ln t - \xi_{p_2}^0/\sigma + \Phi_{(p_2)}^{-1} \right) \right\} \left\{ 1 - \ln t - \xi_p^0\sigma + \Phi_p - 1rg - i \leq \beta \right\} \quad (7)$$

We determined the plan parameters of the proposed plan using the optimization procedure given in Eq. (6)-(7). It is convenient to determine the termination time t_0 as a multiple of the specified median life e^{μ_0} . Sowe will consider $t_0 = ae^{\mu_0}$ for a constant 'a'. For example, $a = 0.5$ means that the experiment time is just half of the specified median life (Aslam and Jun, 2009b). The plan parameters are determined for 50th percentiles ($\Phi_{(p)}^{-1} = 0$) and placed in Table 1.

RESULTS AND DISCUSSION

In the forthcoming paragraphs we will briefly discuss the optimal plan parameters which have been

Table 2: Comparison of the proposed plan (lognormal distribution) with existing plan (weibull distribution) when shape parameter = 2 and using one point approach

β	Proposed plan a = 0.5, r = 5		Existing plan a = 0.5, r = 5		Proposed plan a = 0.5, r = 10		Existing plan a = 0.5, r = 10	
	g_l	c	g_w	c	g_l	c	g_w	c
0.25	1	0	2	0	1	0	1	0
	2	1	4	1	1	1	2	1
	2	2	5	2	1	2	3	2
	3	3	7	3	2	3	4	3
	4	4	8	4	2	4	4	4
0.10	4	5	10	5	2	5	5	5
	2	0	3	0	1	0	2	0
	2	1	5	1	1	1	3	1
	3	2	7	2	2	2	4	2
	4	3	9	3	2	3	5	3
0.05	4	4	10	4	2	4	5	4
	5	5	12	5	3	5	6	5
	2	0	4	0	1	0	2	0
	3	1	6	1	2	1	3	1
	3	2	8	2	2	2	4	2
0.01	4	3	10	3	2	3	5	3
	5	4	11	4	3	4	6	4
	6	5	13	5	3	5	7	5
	3	0	6	0	2	0	3	0
	4	1	8	1	2	1	4	1
	4	2	10	2	2	2	5	2
	5	3	12	3	3	3	6	3
	6	4	14	4	3	4	7	4
	7	5	16	5	4	5	8	5

obtained through simulations. The minimum group sizes and acceptance numbers placed in Table 1, whereas in Table 2, the efficiency of the proposed plan compared with the existing plan in term of sample size. The same values of specified parameters have been considered for the both sampling plans. The plan parameters of both plans also placed in Table 2.

In Table 1, the optimal plan parameters are obtained by considering different values of β , e^μ/e^{μ_0} , $a = 0.5$, $\sigma = 2$ and number of testers $r = 5$. From Table 1, we note that the decreasing trend of plan parameters appears when both of the value of r and β are fixed.

From Table 2, it can be easily observed that the proposed plan, using lognormal distribution, provides the less number of groups than the existing plan. For example, when $\beta = 0.01$, $r = 10$, $c = 2$ and $a = 0.5$, the proposed plan provides $g = 2$ while existing plan provides $g = 5$ which indicates that a sample of size 20 is required by using proposed plan and a sample of size 50 is required for existing plan. Therefore the proposed plan is more efficient than the existing plan.

Application in the industry: The practical application of the proposed plan in the industry for the testing of the products whose lifetime follows the lognormal distribution will be clearer with the help of the following hypothetical example.

Suppose that the experimenter wants to test the quality of electrical cart for 1000 h. Further, in the laboratory he has the facility to install 5 electrical carts on a single tester. Let $\beta = 0.05$, $\alpha = 0.05$ and $e^\mu/e^{\mu_0} = 6$. Then from Table 1, $g = 5$ and $c = 6$. He needs to test the 25 items (by putting 5 items to one tester) for 1000 h.

He needs to accept the product if he notes less than or equal to 6 failures of electrical carts during the 1000 h.

CONCLUSION

The GASP is presented for the lognormal distribution in this study. The tables of plan parameters are provided for practical use. The comparison of the proposed plan over the existing plan is discussed. The proposed plan is more efficient than the existing plan in terms of sample size. The proposed plan can be used in the industries for the inspection of the products. The proposed can be designed using cost model as a future research.

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