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Research Article

New Approach of (G'/G)-expansion Method for RLW Equation

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Abstract: In this study, new extension of the (G'/G)-expansion method with nonlinear ODE has been introduced to investigate the generalized regularized long wave equation. Therefore, many new travelling wave solutions with many parameters are generated. The obtained solutions are more general and solutions include the solutions, the hyperbolic function, the trigonometric function and the rational forms. In addition, some of our solutions are identical with already published results, which validate newly generated solutions and others solutions have not been reported in the previous literature.

Keywords: New (G'/G)-expansion method, nonlinear auxiliary equation, RLW equation, travelling wave solutions

INTRODUCTION

Nonlinear Partial Differential Equations (PDEs) are widely used to depict many important phenomena in various fields of science, such as, mathematical physics, engineering mathematics, chemistry, biology and many others (Wang *et al*., 2006; Elhanbaly and Abdou, 2007; Bekir and Uygun, 2012). As a result, a diverse group of scientists spent huge efforts to investigate analytical solutions of many nonlinear physical models in order to consolidate the current understandings of a vast range of physical matters and their possible applications. Therefore, they introduced many powerful and direct methods. Among these are the Cole-Hopf transformation method (Cole, 1951; Hopf, 1950), the Bäcklund transformation method (Lamb, 1974; Rogers and Shadwick, 1982), the inverse scattering method (Ablowitz and Segur, 1981), the weirstrass elliptic function method (Kudryashov, 1990), the Riccati equation method (Yan and Zhang, 2001; Naher and Abdullah, 2012a, b; Naher *et al*., 2013b), the tanh method (Malfliet, 1992; Wazwaz, 2007), the F-expansion method (Zhou *et al*., 2003; Abdou, 2007), the Exp-function method (He and Wu, 2006; Mohyud-Din *et al*., 2010; Ma and Zhu, 2012; Naher *et al*., 2012a) and so on.

Every method mentioned above can have some limitations in its applications to handle all nonlinear PDEs. Recently, another important method firstly proposed by Wang *et al*. (2008) to generate travelling wave solutions and called the (G'/G)-expansion method. Following Wang *et al*. (2008) many researchers investigated many nonlinear PDEs to construct travelling wave solutions (Zayed and Gepreel, 2009; Feng *et al*., 2011; Naher *et al*., 2011; Abazari and Abazari, 2011; Naher and Abdullah, 2012c; Akbar and Ali, 2013).

Generally, various extensions and improvement of the (G'/G)-expansion method show the effectiveness and reliability of the method. For example, Zhang *et al*. (2010) extended the (G'/G)-expansion method which is called the improved (G'/G)-expansion method. Subsequently, a group of scientists studied various nonlinear PDEs to construct many travelling wave solutions (Hamad *et al*., 2011; Naher and Abdullah, 2012d to f; Naher *et al*., 2012b).

Guo and Zhou (2010) introduced the extended (G'/G)-expansion method for obtaining travelling wave solutions of the Whitham-Broer-Kaup-like equation and couple Hirota-Satsuma KdV equations. Consequently, many researchers implemented this method to obtain analytical solutions of various nonlinear partial differential equations (Zayed and Al-Joudi, 2010; Zayed and El-Malky, 2011).

More recently, Akbar *et al*. (2012) presented the generalized and improved (G'/G)-expansion method with additional parameter. Afterwards, Naher *et al*. (2013a) studied the (3+1)-dimensional nonlinear PDE to generate travelling wave solutions.

Very recently, Naher and Abdullah (2013) further developed this idea and introduced it as much more lucid and straightforward for a class of nonlinear PDEs

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to construct more general and many new travelling wave solutions including many parameters. Every solution has its own physically significant rich structure to describe more insight of real life problems.

In this study, we apply new approach of the (G'/G) -expansion method with many parameters which is the extension of the (G'/G)-expansion method. For illustration of the method the generalized Regularized Long Wave (RLW) equation has been investigated for constructing many new travelling wave solutions.

METHODOLOGY

Description of new approach of (G'/G)-expansion method: Let us consider a general nonlinear PDE:

$$Q(u, u_t, u_x, u_{tt}, u_{xt}u_{xx}, \ldots) = 0, \tag{1}$$

where, $u = u(x, t)$ is an unknown function, Q is a polynomial in $u(x, t)$ and its partial derivatives in which the highest order partial derivatives and nonlinear terms are involved.

Step 1: We suppose that the combination of real variables x and t by a complex variable ξ:

$$u(x,t) = u(\xi), \quad \xi = x - wt, \tag{2}$$

where, w is the speed of the travelling wave. Now using Eq. (2) (1) is converted into an ordinary differential equation for $u = u(\xi)$:

$$P(u, u', u'', u''', \ldots) = 0, \tag{3}$$

where, the superscripts indicate the ordinary derivatives with respect to ξ.

Step 2: According to possibility, Eq. (3) can be integrated term by term one or more times, yields constant (s) of integration. The integral constant may be zero, for simplicity.

Step 3: Suppose that the travelling wave solution of Eq. (3) can be expressed (Naher and Abdullah, 2013):

$$u(\xi) = \sum_{i=0}^{l} a_i F^i(\xi) + \sum_{i=1}^{l} b_i F^{-i}(\xi), \tag{4}$$

where, either a_l or b_l may be zero, but both a_l and b_l cannot be zero at a time, a_i (i = 0, 1, 2, …, l) and b_i (i = 1, 2, …, l) are arbitrary constants to be determined later and $F(\xi)$ is:

$$F(\xi) = \frac{G'(\xi)}{G(\xi)} \tag{5}$$

where, $G = G(\xi)$ satisfies the following auxiliary nonlinear Ordinary Differential Equation (ODE):

$$GG'' = AG^2 + BGG' + C(G')^2, \tag{6}$$

where, prime denotes the derivative with respect to ξ. A, B and C are real parameters.

Step 4: To determine the positive integer l taking the homogeneous balance between the highest order nonlinear terms and the highest order derivatives appearing in Eq. (3).

Step 5: Substituting Eq. (4) and (6) including Eq. (5) into Eq. (3) with the value of l obtained in Step 4 and we obtain polynomials in F^l (l = 0, 1, 2, …) and F^{-l} (l = 1, 2, 3,…). Then, we collect each coefficient of the resulted polynomials to zero, yields a set of algebraic equations for a_i (i = 0, 1, 2, …, l), b_i (i = 1, 2, …, l) and w

Step 6: Suppose that the value of the constants a_i (i = 0, 1, 2, …, l), b_i (i = 1, 2, …, l) and w can be found by solving the algebraic equations which are obtained in step 5. Since the general solution of Eq. (6) is well known to us, substituting the values of constants into Eq. (4), we can obtain more general type and new exact travelling wave solutions of the nonlinear partial differential Eq. (1).

Using the general solution of Eq. (6), we have the following solutions of Eq. (5).

Family 1: Hyperbolic function solution: When $B \neq 0$, $\Psi = 1 - C$ and $\Delta = B^2 + 4A - 4AC > 0$:

$$F(\xi) = \frac{B}{2\Psi} + \frac{\sqrt{\Delta}}{2\Psi} \frac{C_1 \sinh\left(\frac{\sqrt{\Delta}}{2\Psi}\xi\right) + C_2 \cosh\left(\frac{\sqrt{\Delta}}{2\Psi}\xi\right)}{C_1 \cosh\left(\frac{\sqrt{\Delta}}{2\Psi}\xi\right) + C_2 \sinh\left(\frac{\sqrt{\Delta}}{2\Psi}\xi\right)} \tag{7}$$

Family 2: Trigonometric function solution: When $B \neq 0$, $\Psi = 1 - C$ and $\Delta = B^2 + 4A - 4AC < 0$:

$$F(\xi) = \frac{B}{2\Psi} + \frac{\sqrt{-\Delta}}{2\Psi} \frac{-C_1 \sin\left(\frac{\sqrt{-\Delta}}{2\Psi}\xi\right) + C_2 \cos\left(\frac{\sqrt{-\Delta}}{2\Psi}\xi\right)}{C_1 \cos\left(\frac{\sqrt{-\Delta}}{2\Psi}\xi\right) + C_2 \sin\left(\frac{\sqrt{-\Delta}}{2\Psi}\xi\right)} \tag{8}$$

Family 3: Rational form solution: When $B \neq 0$, $\Psi = 1 - C$ and $\Delta = B^2 + 4A - 4AC = 0$:

$$H(\xi)=\left(\frac{G'}{G}\right)=\frac{B}{2\Psi}+\frac{C_2}{C_1+C_2\xi} \quad (9)$$

Family 4: Hyperbolic function solution: When $B = 0$, $\Psi = 1 - C$ and $\Phi = A(1 - C) = A\Psi>0$:

$$F(\xi)=\frac{\sqrt{\Phi}}{\Psi}\frac{C_1\sinh\left(\frac{\sqrt{\Phi}}{\Psi}\xi\right)+C_2\cosh\left(\frac{\sqrt{\Phi}}{\Psi}\xi\right)}{C_1\cosh\left(\frac{\sqrt{\Phi}}{\Psi}\xi\right)+C_2\sinh\left(\frac{\sqrt{\Phi}}{\Psi}\xi\right)} \quad (10)$$

Family 5: Trigonometric function solution: When $B = 0$, $\Psi = 1 - C$ and $\Phi = A(1 - C) = A\Psi<0$:

$$F(\xi)=\frac{\sqrt{-\Phi}}{\Psi}\frac{-C_1\sin\left(\frac{\sqrt{-\Phi}}{\Psi}\xi\right)+C_2\cos\left(\frac{\sqrt{-\Phi}}{\Psi}\xi\right)}{C_1\cos\left(\frac{\sqrt{-\Phi}}{\Psi}\xi\right)+C_2\sin\left(\frac{\sqrt{-\Phi}}{\Psi}\xi\right)} \quad (11)$$

Application of method: In this section, new approach of the (G'/G)-expansion method has been implemented for the generalized RLW equation.

Generalized RLW equation: Now let us consider the generalized RLW equation followed by Abazari (2010):

$$u_t+u_x+a\left(u^2\right)_x-bu_{xxt}=0, \quad (12)$$

where, a and b are positive constants.

Now, we use the wave transformation Eq. (2) into the (12), which yields:

$$-wu'+u'+2auu'+bwu'''=0. \quad (13)$$

Equation (13) is integral, therefore, integrating with respect ξ once yields:

$$-wu+u+au^2+bwu''=0, \quad (14)$$

Taking the homogeneous balance between u^2 and u'' in Eq. (14), we obtain $l = 2$.
Therefore, the solution of Eq. (14) is of the form:

$$u(\xi)=a_0+a_1F(\xi)+a_2F^2(\xi)+b_1F^{-1}(\xi)+b_2F^{-2}(\xi), \quad (15)$$

where, a_0, a_1, a_2, b_1 and b_2 are arbitrary constants to be determined.

Substituting Eq. (15) together with Eq. (5) and (6) into Eq. (14), the left-hand side is converted into polynomials in F^l ($l = 0, 1, 2, \ldots$) and F^{-l} ($l = 1, 2, 3,\ldots$). We collect each coefficient of these resulted polynomials to zero, yields a set of simultaneous algebraic equations (for simplicity, which are not presented) for a_0, a_1, a_2, b_1, b_2 and w. Solving these algebraic equations with the help of algebraic software Maple, we obtain following.

Case 1:

$$w=\frac{-1}{a\left(4bAC-1-4bA-bB^2\right)}, a_0=\frac{b\left(2AC+B^2-2A\right)}{a\left(4bAC-1-4bA-bB^2\right)}, b_1=0,$$
$$a_1=\frac{6Bb(C-1)}{a\left(4bAC-1-4bA-bB^2\right)}, a_2=\frac{6b\left(C^2-2C+1\right)}{a\left(4bAC-1-4bA-bB^2\right)}, b_2=0, \quad (16)$$

where A, B and C are free parameters.

Case 2:

$$w=\frac{1}{a\left(4bAC+1-4bA-bB^2\right)}, a_0=\frac{-6bA(C-1)}{a\left(4bAC+1-4bA-bB^2\right)}, b_1=0,$$
$$a_1=\frac{-6Bb(C-1)}{a\left(4bAC+1-4bA-bB^2\right)}, a_2=\frac{-6b\left(C^2-2C+1\right)}{a\left(4bAC+1-4bA-bB^2\right)}, b_2=0, \quad (17)$$

where A, B and C are free parameters.

Case 3:

$$w=\frac{-1}{a\left(4bAC-1-4bA-bB^2\right)}, a_0=\frac{b\left(2AC+B^2-2A\right)}{a\left(4bAC-1-4bA-bB^2\right)}, a_1=0,$$
$$b_1=\frac{6bAB}{a\left(4bAC-1-4bA-bB^2\right)}, b_2=\frac{6bA^2}{a\left(4bAC-1-4bA-bB^2\right)}, a_2=0, \quad (18)$$

where A, B and C are free parameters.

Case 4:

$$w=\frac{1}{a\left(4bAC+1-4bA-bB^2\right)}, a_0=\frac{-6bA(C-1)}{a\left(4bAC+1-4bA-bB^2\right)}, a_1=0,$$
$$b_1=\frac{-6bAB}{a\left(4bAC+1-4bA-bB^2\right)}, b_2=\frac{-6bA^2}{a\left(4bAC+1-4bA-bB^2\right)}, a_2=0, \quad (19)$$

where, A, B and C are free parameters.

Family 1: Substituting Eq. (16) to (19) together with Eq. (7) into (15) and simplifying, yield following travelling wave solutions (if $C_1 = 0$ but $C_2 \neq 0$) respectively:

$$u1_1(x,t) = \frac{b}{2a\left\{1+b\left(4A\Psi+B^2\right)\right\}}\left\{B^2+4A\Psi-3\Delta\coth^2\left(\frac{\sqrt{\Delta}}{2\Psi}\xi\right)\right\},$$

where, $\Psi = 1-C$ and $\xi = x - \frac{1}{a\left\{1+b\left(4A\Psi+B^2\right)\right\}}t.$

$$u2_1(x,t) = \frac{3b}{2a\left\{1-b\left(4A\Psi+B^2\right)\right\}}\left\{B^2+4A\Psi-\Delta\coth^2\left(\frac{\sqrt{\Delta}}{2\Psi}\xi\right)\right\},$$

where, $\Psi = 1-C$ and $\xi = x - \frac{1}{a\left\{1-b\left(4A\Psi+B^2\right)\right\}}t.$

$$u3_1(\xi) = \frac{(b/a)}{\left(4bAC-1-4bA-bB^2\right)}\left\{\left(B^2-2A\Psi\right)+\left\{\frac{6AB}{\left(\frac{B}{2\Psi}+\frac{\sqrt{\Delta}}{2\Psi}\coth\left(\frac{\sqrt{\Delta}}{2\Psi}\xi\right)\right)}+\frac{6A^2}{\left(\frac{B}{2\Psi}+\frac{\sqrt{\Delta}}{2\Psi}\coth\left(\frac{\sqrt{\Delta}}{2\Psi}\xi\right)\right)^2}\right\}\right\},$$

where $\Psi = 1-C$ and $\xi = x - \frac{1}{a\left\{1+b\left(4A\Psi+B^2\right)\right\}}t.$

$$u4_1(\xi) = \frac{-6bA\Psi}{a\left(4bAC+1-4bA-bB^2\right)}\left\{\frac{2B\left(B+\sqrt{\Delta}\coth\left(\left(\sqrt{\Delta}/2\Psi\right)\xi\right)\right)+4A\Psi-\left(B+\sqrt{\Delta}\coth\left(\left(\sqrt{\Delta}/2\Psi\right)\xi\right)\right)^2}{\left(B+\sqrt{\Delta}\coth\left(\left(\sqrt{\Delta}/2\Psi\right)\xi\right)\right)^2}\right\},$$

where, $\xi = x - \frac{1}{a\left\{1-b\left(4A\Psi+B^2\right)\right\}}t.$

Again, substituting Eq. (16) to (19) together with Eq. (7) into (15) and simplifying, our obtained solutions become ($C_2 = 0$ but $C_1 \neq 0$) respectively:

$$u1_2(x,t) = \frac{b}{2a\left\{1+b\left(4A\Psi+B^2\right)\right\}}\left\{B^2+4A\Psi-3\Delta\tanh^2\left(\frac{\sqrt{\Delta}}{2\Psi}\xi\right)\right\},$$

$$u2_2(x,t) = \frac{3b}{2a\left\{1-b\left(4A\Psi+B^2\right)\right\}}\left\{B^2+4A\Psi-\Delta\tanh^2\left(\frac{\sqrt{\Delta}}{2\Psi}\xi\right)\right\},$$

$$u3_2(\xi) = b\left\{\frac{\left(B^2-2A\Psi\right)\left(B+\sqrt{\Delta}\tanh\left(\left(\sqrt{\Delta}/2\Psi\right)\xi\right)\right)^2+12A\Psi\left\{B\left(B+\sqrt{\Delta}\tanh\left(\left(\sqrt{\Delta}/2\Psi\right)\xi\right)\right)+2A\Psi\right\}}{a\left(4bAC-1-4bA-bB^2\right)\left(B+\sqrt{\Delta}\tanh\left(\left(\sqrt{\Delta}/2\Psi\right)\xi\right)\right)^2}\right\},$$

$$u4_2(\xi) = \frac{-6bA\Psi}{a\left(4bAC+1-4bA-bB^2\right)}\left\{\frac{2B\left(B+\sqrt{\Delta}\tanh\left(\left(\sqrt{\Delta}/2\Psi\right)\xi\right)\right)+4A\Psi-\left(B+\sqrt{\Delta}\tanh\left(\left(\sqrt{\Delta}/2\Psi\right)\xi\right)\right)^2}{\left(B+\sqrt{\Delta}\tanh\left(\left(\sqrt{\Delta}/2\Psi\right)\xi\right)\right)^2}\right\},$$

Family 2: Substituting Eq. (16) to (19) together with Eq. (8) into (15) and simplifying, the travelling wave solutions become (if $C_1 = 0$ but $C_2 \neq 0$) respectively:

$$u1_3(x,t) = \frac{b}{2a\left\{1+b\left(4A\Psi+B^2\right)\right\}}\left\{B^2+4A\Psi+3\Delta\cot^2\left(\frac{\sqrt{-\Delta}}{2\Psi}\xi\right)\right\},$$

$$u2_3(x,t) = \frac{3b}{2a\left\{1-b\left(4A\Psi+B^2\right)\right\}}\left\{B^2+4A\Psi+\Delta\cot^2\left(\frac{\sqrt{-\Delta}}{2\Psi}\xi\right)\right\},$$

$$u3_3(\xi) = \frac{(b/a)}{\left(4bAC-1-4bA-bB^2\right)}\left\{\left(B^2-2A\Psi\right)+\left\{\frac{6AB}{\left(\frac{B}{2\Psi}+\frac{i\sqrt{\Delta}}{2\Psi}\cot\left(\frac{\sqrt{-\Delta}}{2\Psi}\xi\right)\right)}+\frac{6A^2}{\left(\frac{B}{2\Psi}+\frac{i\sqrt{\Delta}}{2\Psi}\cot\left(\frac{\sqrt{-\Delta}}{2\Psi}\xi\right)\right)^2}\right\}\right\},$$

$$u4_3(\xi) = \frac{-12bA\Psi}{a\left(4bAC+1-4bA-bB^2\right)}\left\{\frac{B}{B+i\sqrt{\Delta}\cot\left(\left(\sqrt{-\Delta}/2\Psi\right)\xi\right)}+\frac{2A\Psi}{\left(B+i\sqrt{\Delta}\cot\left(\left(\sqrt{-\Delta}/2\Psi\right)\xi\right)\right)^2}-\frac{1}{2}\right\},$$

Further, substituting Eq. (16) to (19) together with Eq. (8) into Eq. (15) and simplifying, yield following travelling wave solutions (if $C_2 = 0$ but $C_1 \neq 0$) respectively:

$$u1_4(x,t) = \frac{b}{2a\left\{1+b\left(4A\Psi+B^2\right)\right\}}\left\{B^2+4A\Psi+3\Delta\tan^2\left(\frac{\sqrt{-\Delta}}{2\Psi}\xi\right)\right\},$$

$$u2_4(x,t) = \frac{3b}{2a\left\{1-b\left(4A\Psi+B^2\right)\right\}}\left\{B^2+4A\Psi+\Delta\tan^2\left(\frac{\sqrt{-\Delta}}{2\Psi}\xi\right)\right\},$$

$$u3_4(\xi) = \frac{(b/a)}{\left(4bAC-1-4bA-bB^2\right)}\left\{\left(B^2-2A\Psi\right)+6A\left(B\left(\frac{B}{2\Psi}-\frac{i\sqrt{\Delta}}{2\Psi}\tan\left(\frac{\sqrt{-\Delta}}{2\Psi}\xi\right)\right)^{-1}+A\left(\frac{B}{2\Psi}-\frac{i\sqrt{\Delta}}{2\Psi}\tan\left(\frac{\sqrt{-\Delta}}{2\Psi}\xi\right)\right)^{-2}\right)\right\},$$

$$u4_4(\xi) = \frac{-12bA\Psi}{a\left(4bAC+1-4bA-bB^2\right)}\left\{\frac{B}{B-i\sqrt{\Delta}\tan\left(\left(\sqrt{-\Delta}/2\Psi\right)\xi\right)}+\frac{2A\Psi}{\left(B-i\sqrt{\Delta}\tan\left(\left(\sqrt{-\Delta}/2\Psi\right)\xi\right)\right)^2}-\frac{1}{2}\right\},$$

Family 3: Substituting Eq. (16) to (19) together with Eq. (9) into (15) and simplifying, we construct following travelling wave solutions:

$$u1_5(x,t) = \frac{-6b}{a}\left(\frac{\Psi C_2}{C_1+C_2\xi}\right)^2,$$

where, $\xi = x-\left(\frac{1}{a}\right)t,$ C_1 and C_2 are arbitrary constants. Solutions $u1_5$ and $u2_5$ are identical. Therefore, we have not display the solution $u2_5$:

$$u3_5(\xi) = \frac{-b}{a}\left\{\left(B^2-2A\Psi\right)+6A\left\{B\left(\frac{B}{2\Psi}+\frac{C_2}{C_1+C_2\xi}\right)^{-1}+A\left(\frac{B}{2\Psi}+\frac{C_2}{C_1+C_2\xi}\right)^{-2}\right\}\right\},$$

$$u4_5(\xi)=\frac{-6bA}{a}\left\{B\left(\frac{B}{2\Psi}+\frac{C_2}{C_1+C_2\xi}\right)^{-1}+A\left(\frac{B}{2\Psi}+\frac{C_2}{C_1+C_2\xi}\right)^{-2}-\Psi\right\},$$

Family 4: Substituting Eq. (16) to (19) together with Eq. (10) into (15) and simplifying, yield following travelling wave solutions (C_1 = 0 but $C_2 \neq 0$) respectively:

$$u1_6(x,t)=\frac{2bA\Psi}{a(1+4bA\Psi)}\left\{1-3\coth^2\left(\frac{\sqrt{\Phi}}{\Psi}\xi\right)\right\},$$

where, $\xi = x-\frac{1}{a(1+4bA\Psi)}t.$

$$u2_6(x,t)=\frac{6bA\Psi}{a(1-4bA\Psi)}\left\{1-\coth^2\left(\frac{\sqrt{\Phi}}{\Psi}\xi\right)\right\},$$

where, $\xi = x-\frac{1}{a(1-4bA\Psi)}t.$

$$u3_6(x,t)=\frac{2bA\Psi}{a(1+4bA\Psi)}\left\{1-3\tanh^2\left(\frac{\sqrt{\Phi}}{\Psi}\xi\right)\right\},$$

where, $\xi = x-\frac{1}{a(1+4bA\Psi)}t.$

$$u4_6(x,t)=\frac{6bA\Psi}{a(1-4bA\Psi)}\left\{1-\tanh^2\left(\frac{\sqrt{\Phi}}{\Psi}\xi\right)\right\},$$

where $\xi = x-\frac{1}{a(1-4bA\Psi)}t.$

Moreover, substituting Eq. (16) to (19) together with Eq. (10) into (15) and simplifying, yield following travelling wave solutions (C_2 = 0 but $C_1 \neq 0$) respectively, our solutions $u1_7$ and $u3_6$; $u2_7$ and $u4_6$; $u3_7$ and $u1_6$; $u4_7$ and $u2_6$ are identical. Therefore, the solutions $u1_7$, $u2_7$, $u3_7$ and $u4_7$ have not been shown.

Family 5: Substituting Eq. (16) to (19) together with Eq. (11) into (15) and simplifying, our wave solutions become (if $C_1 = 0$ but $C_2 \neq 0$) respectively:

$$u1_8(x,t)=\frac{2bA\Psi}{a(1+4bA\Psi)}\left\{1+3\cot^2\left(\frac{\sqrt{-\Phi}}{\Psi}\xi\right)\right\},$$

$$u2_8(x,t)=\frac{6bA\Psi}{a(1-4bA\Psi)}\left\{1-\cot^2\left(\frac{\sqrt{-\Phi}}{\Psi}\xi\right)\right\},$$

$$u3_8(x,t)=\frac{2bA\Psi}{a(1+4bA\Psi)}\left\{1+3\tan^2\left(\frac{\sqrt{-\Phi}}{\Psi}\xi\right)\right\},$$

$$u4_8(x,t)=\frac{6bA\Psi}{a(1-4bA\Psi)}\left\{1+\tan^2\left(\frac{\sqrt{-\Phi}}{\Psi}\xi\right)\right\}.$$

Furthermore, substituting Eq. (16) to (19) together with Eq. (11) into (15) and simplifying (if $C_2 = 0$ but $C_1 \neq 0$), the solutions $u1_9$ and $u3_8$; $u2_9$ and $u4_8$; $u3_9$ $u1_7$ and $u1_8$; $u4_9$ and $u2_8$ have been coincided. Hence, the solutions $u1_9$, $u2_9$, $u3_9$ and $u4_9$ are not presented.

DISCUSSION

The advantage and validity of used method over the basic (G'/G)-expansion method has been discussed as follows.

Advantages: The vital advantage of used method over the basic (G'/G)-expansion method is that this method provides more general and many new travelling wave solutions including many parameters. The travelling wave solutions of PDEs have its vital significant to reveal more insight of the complex physical phenomena.

Validity: Abazari (2010) used the basic (G'/G)-expansion method with Linear Ordinary Differential Equation (LODE) as auxiliary equation and the presentation of the solution is:

$$u(\xi)=\sum_{i=0}^{m}a_i(G'/G)^i$$

where $a_m \neq 0$.

It is important to point out that if A takes $-\mu$; B replaced by $-\lambda$ and C = 0, some of our solutions coincide with the solution of Abazari (2010):

- $u1_1$ and $u1_2$ matched with Eq. (31a)
- $u1_3$ and $u1_4$ identical with Eq. (32a)
- $u2_1$ and $u2_2$ coincided with Eq. (31a)
- $u2_3$ and $u2_4$ identical with Eq. (35a)
- $u1_5$ and $u2_5$ matched with Eq. (36)

In addition, the obtained solutions $u1_6$, $u1_8$, $u2_6$, $u2_8$, $u3_1$ to $u3_6$, $u3_8$, $u4_1$ to $u4_6$ and $u4_8$ are new and have not presented in the earlier literature.

CONCLUSION

In this study, we have studied RLW equation via new approach of (G'/G)-expansion method. Some benefits are obtainable for this method, in contrast to other (G'/G)-expansion method. We have successfully constructed a rich class of new travelling wave solutions with free parameters of distinct physical structures and solutions presented through the hyperbolic function, the trigonometric function and the rational forms solutions. Moreover, some of our solutions are coincided with earlier literature, if parameters take specific values and others have not been reported in previous literature.

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