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**Research Article**

# Number Theory for the Selection of an Investment Portfolio: An Application to the Mexican Stock Exchange

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**Abstract:** The calculation of variance is defined as a objective and increasing function, this definition allows establish the hypotheses to calculate diversified investment portfolios from the dominion of the function. In order to apply these hypotheses our mathematical multi-objective linear model is modified. Diversified portfolios are selected from the stocks of the Prices and Quotations Index of the Mexican Stock Exchange. It is shown with a statistical test using the coefficient of variation that the selected portfolio yields a higher profit at a lower risk.

**Keywords:** Investment portfolio, Mexican stock exchange, number theory

## INTRODUCTION

In the selection of an investment portfolio it is sought the highest profit at the lowest possible risk. For which diverse mathematical models have been elaborated, like the one of Markowitz (1952) and the Capital Asset Pricing Model (CAPM) (Fama and French, 2004). With base to these models others have been elaborated, (Love, 1979; Subbu *et al*., 2005; Deng *et al*., 2010a, b) and like ours, that when assumes the equal correlation to one between each pair of shares is applied (Zavala-Díaz *et al*., 2011a, b).

Of the suppositions of the models of Markowitz and CAPM, it is revealed that if investment portfolio is in the area of feasible portfolios in the return-risk plane, then the same portfolio will be in the region of feasible portfolios in the return-variance plane. In addition, when portfolio is not feasible in a model, in the other it is not it either. The relation between both regions is given by the function of the calculation of the variance.

This study proposes to use the relationship between the elements of the feasible regions between the two models, in order to select the investment portfolios with base to the theory of numbers, particularly from the definition of a function. The calculation of the variance is defined by means of the bijective function (Rosen Kenneth, 2007). In addition, the definition of the increasing function is used to establish the hypotheses for the selection of investment portfolio from the dominion of the function.

In order to apply these hypotheses our mathematical model is modified (Zavala-Díaz *et al*., 2011a, b). A factor that forces the model to obtain diversified portfolios is introduced. The modification is necessary because the model can determine an optimal solution with one or two titles. With the modification of the model another optimal solution looks for, but with diversified portfolio. With the modified model an investment portfolio is selected with the shares of the Index of Prices and Quotations (IPQ) of the Mexican Stock-Market (MSM).

The foundations of the models are shown in the second part of this study; additionally, the function is defined. In the third part, the hypotheses are presented by a model that uses only the elements of the domain; the multi-objective linear model is presented and the solution process is briefly described. The description of the process has the objective to indicate in what point is made the modification. In the fourth part, the model is applied to select an investment portfolio with stocks that comprise the IPQ of the MSM. In the fifth part, the statistical test is presented. Finally, in the last section, the conclusions of this study are presented.

## FOUNDATIONS

Since the investigation is based on the principles of the models of Markowitz (1952) and CAPM (Fama and French, 2004), these briefly are described next.

**Markowitz model:** In the Markowitz model, a portfolio is efficient if it has the lowest possible risk for a certain level of profitability. The set of efficient portfolios is estimated by the following parametric quadratic problem (Markowitz, 1952):

$$\min \sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij} \tag{1}$$

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Subject to:

$$\sum_{i=1}^{n} x_i R_i = V^* \tag{2}$$

$$\sum_{i=1}^{n} x_i = 1, \quad 0 \le x_i \le 1 \quad (i = 1,2,...,n) \tag{3}$$

where,

$$\sigma_{ij} = \sigma_i \sigma_j \rho_{ij}, \ \sigma_i = \sqrt{\frac{\sum_{j=1}^{m} (r_{ij} - R_i)^2}{m-1}}, \ R_i = \frac{\sum_{j=1}^{m} r_{ij}}{m},$$

$$\rho_{12} = \frac{\sum_{j=1}^{m} r_{1j} r_{2j} - \frac{\left(\sum_{j=1}^{m} r_{1j}\right)\left(\sum_{j=1}^{m} r_{2j}\right)}{m}}{\sqrt{\sum_{j=1}^{m} (r_{1j} - R_1)^2 \sum_{j=1}^{m} (r_{2j} - R_2)^2}}$$

where,

- $x_i$ = The matter of the problem and the financial portion of the financial asset *I*
- $\sigma_p^2$ = The variance of portfolio *p*
- $\sigma_{ij}$ = The covariance between the profits of the shares $x_i$ and $x_j$
- $\rho_{ij}$ = The correlation between the profits of the shares $x_i$ and $x_j$
- $R_p$ = The profits of the portfolio *p* and equal to $V^*$
- $V^*$ = A parameter that varies to minimize the risk of the portfolio and obtain the set of proportions $x_i$
- $R_i$ = The average profit of the share *I*
- $r_{ij}$ = The incoming of each share *i* in each period *j*
- $m$ = The number of periods considered
- $n$ = The number of titles

Figure 1 shows a diagram of the Pareto frontier, built with efficient portfolios obtained using the Markowitz model. Figure 1 is the solution of the Markowitz's problem, which presents the dominant, or efficient, portfolios. The feasible solutions of the optimization problem are located above this frontier (Coello Coello *et al*., 2007). Different multi-objective algorithms solve this optimization problem-mainly the so-called evolutionary algorithms (Branke *et al*., 2009).

**Capital Asset Pricing Model (CAPM):** The CAPM relies on two statistics that describe financial assets, one of position and another of dispersion (Fama and French, 2004). The position measure is the average of profits, which provides the profitability of assets during a given period of time. The dispersion measure is the standard deviation of the average earnings of different titles or shares; it measures the risk of financial assets. In this model, the profitability and risk of a portfolio investment are determined by the following equations.

Return:

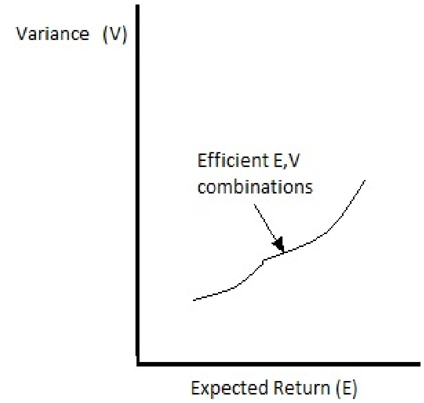


Fig. 1: Pareto frontier of efficient portfolios (Markowitz, 1952)

$$R_p = \sum_{i=1}^{n} x_i R_i \tag{4}$$

Risk:

$$\sigma_P = \sqrt{\sum_{i=1}^{n} \left(x_i^2 \sigma_i^2 + x_j^2 \sigma_j^2\right) + 2\sum_{i=1}^{n} \left(x_i x_j \sigma_i \sigma_j \rho_{ij}\right)} \tag{5}$$

Equation (4) is the same as Eq. (2) of the Markowitz Model, indicating that profitability is determined in the same way. It is also observed that Eq. (1) is the square of Eq. (5). Consequently, because Eq. (1) uses only the positive root of Eq. (5), it is possible set a function between both sets of solutions, i.e., the calculation of the variance. In the CAPM, the sum of financial assets must be equal to 100%, similar to restriction (3) of the Markowitz Model.

In the CAPM, the investment portfolio is formed by allocating percentages of investment to financial assets $x_i$ in Eq. (4) and (5). Figure 2 shows the graph of the model solutions.

In Fig. 2, the set of efficient portfolios is on the curve among the Point a, Point b and Point c. The efficient portfolios that do not integrate risk-free assets are located at Point b and the efficient portfolios that integrate risk-free assets are located at Point T. The portfolios of the problem where the risk-return difference is at the minimum are located at Point b. As a result, it is possible to obtain two optimal solutions in this model: Point T and Point b. The feasible solutions of the problem area are bounded by the abc curve (Ruppert, 2011).

**Establishment of the bijective function:** This fact implies that there is a correspondence from one plane to the other, i.e., from profit-variance to profit-risk and vice versa. Therefore, it is possible to define the existence of a function between these two sets of elements. Each of the points on the variance-profit plane has a corresponding point in the risk-profit plane.

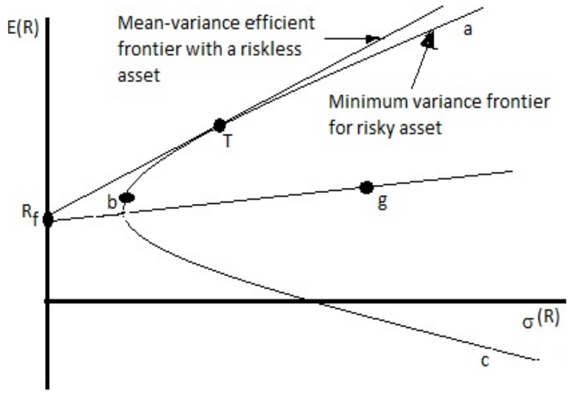


Fig. 2: Efficient portfolios obtained with the CAPM (Fama and French, 2004)

That is, each of the ordered pairs ($R_i$, $\sigma_i^2$) is related to a pair ($R_i$, $\sigma_i$). In the two ordered pairs, the first element is the return and it is calculated in the same way. However, the second element of the ordered pairs is related to the function that calculates the variance with standard deviation.

If the ordered pairs of the feasible region of the CAPM is the domain and ordered pairs of the feasible region of the Markowitz model is the codomain, then the function is denoted by $f((R_i, \sigma_i)) = (R_i, \sigma_i^2)$. The function definition is.
Let A, B and C sets:

$$A = \{R_i | R_i \in R \text{ and it is the profit of the share } i\}$$
$$B\left\{\sigma_i \middle| \begin{matrix}\sigma_i \in R \text{ and it is the standard} \\ \text{deviation of the share } i\end{matrix}\right\}$$
$$C\left\{\sigma_i^2 \middle| \begin{matrix}\sigma_i^2 \in R \text{ and it is the variance} \\ \text{of the share } i\end{matrix}\right\}$$
$$f: AXB \rightarrow AXC, \text{ of } ((R_i, \sigma_i)) = (R_i, \sigma_i^2) = (R_i, g(\sigma_i))$$

The function $f$ is bijective, because for each point in the plane risk-profit will be a point in the plane variance-profit and vice-versa. Also, in $f$ function $g$ is included to calculate the variance, its dominion is the joint B and its co-domain is set C.

**Hypothesis:** The assumptions of this study are as follows:

- The profit of portfolio belongs to set ($R_p \in A$) and its value is in the range given by the magnitude of the profits of the titles ($R_{imin} \leq R_P \leq R_{imax}$).
- The standard deviation of the portfolio belongs to Set $B$ ($\sigma_P \in B$) and its value is in the range given by the magnitude of the standard deviations of the shares ($\sigma_{imin} \leq \sigma_P \leq \sigma_{imax}$).
- The variance of the portfolio belongs to Set $C$ ($\sigma_P^2 \in C$) and its value is in the range given by the magnitude of the variances of the shares ($\sigma_{imin}^2 \leq \sigma_P^2 \leq \sigma_{imax}^2$).
- The calculation of the variance function is an increasing function, $g$ is increasing in $a$ if and only if there is a setting of $a$ such that for any $x$ belonging to the environment of $a$ the following is met: $x > a \Rightarrow g(x) \geq g(a)$ and $x < a \Longrightarrow g(x) \leq g(a)$ (Rosen Kenneth, 2007).

The hypotheses state that the risk of the investment portfolio is one more element of the set of standard deviations and its magnitude is given by the maximum and minimum intervals of the values of the shares.

Hypothesis 3 claims that the $g$ function is an increasing function. This claim implies that if the minimum of the domain is determined, the minimum in the co-domain will also be determined. As a result, for portfolio performance $R_p$ the problem can be solved with the values of the domain of $g$. This claim is contrary to that of other models that seek to optimize the function using different values of the domain to obtain the minimum value in the co-domain.

In arriving at a solution to the problem, the domain perspective has the following advantage: the search space in Set $B$ is linear. For example, movement from the real number 4 to the real number 5 is linear, contrary to the not linear search space of Set $C$. For example, movement from the real number $4^2$ to the real number $5^2$ is not linear. The linearity of search space for Set $B$ can be used to raise the problem as one of linear programming.

In order to take advantage of the linearity the problem our model is modified, where the approach of the linear programming problem is multi-objective, maximizes profit and minimizes risk. The approach of this model is shown (Zavala-Díaz *et al*., 2011a) as follows:

$$minize\ z = \sum_{i=1}^{n} x_i(\sigma_i - R_i) \tag{6}$$

Subject to:

$$\sum_{i=1}^{n} x_i(\sigma_i - \sigma_{min}) = \lambda_1(\sigma_{max} - \sigma_{min}) \tag{7}$$

$$\sum_{i=1}^{n} x_i(R_i - R_{min}) = \lambda_2(R_{max} - R_{min}) \tag{8}$$

$$\sum_{i=1}^{n} x_i Pv_i \geq Pv_{min} \tag{9}$$

$$\sum_{i=1}^{n} x_i = 1 \tag{10}$$

$$0 \leq x_i \leq 1 \tag{11}$$

The used nomenclature in Eq. (6) to (11) is the same as used in the models described above. The term $Pv_i$ is the sale price of the title *i* at the close, $Pv_{min}$ is the minimum sale price of the titles considered for the selection of investment portfolio.

The approach of the multi-objective linear model seeks to obtain Point b in the risk-return plane. The variables $\lambda_1$ and $\lambda_2$, where $0 \leq \lambda_1$, $\lambda_2 \leq 1$, allow explore feasible solution sets until the optimal solution is reached.

The resolution process is iterative. In each one of the iterations, a linear programming problem is resolved for each new value of $\lambda_1$ and $\lambda_2$ until the optimal solution is reached (Zavala-Díaz *et al*., 2011a). This model determines the Point b portfolio, which can be composed of one, two, or more assets (Zavala-Díaz *et al*., 2011b).

Restriction (11) is modified to force the portfolio to be formed by a larger number of shares. The modification consists of changing the value of the upper limits of the variable $x_i$, which is given by the following:

$$0 \leq x_i \leq \lambda_3 \tag{12}$$

The $\lambda_3$ variable begins at a maximum value of 1.0 and its minimum value is determined experimentally. The computational experimentation is necessary for the following reason: For example, if a portfolio consists of 10 assets, the value minimum cannot be 0.01 or 0.1. The first value of all assets would be 0.1 and would not be the 1.0 value of restriction (10); therefore, it is not possible to obtain feasible solutions. For the second value, in the best of cases, all shares will have an asset of 0.1 and the sum of all values will comply with the restriction (10). However, the procedure cannot select a percentage larger or smaller than 0.1 for a share because it ceases to comply with this restriction. This circumstance implies that an optimal solution is not sought. Therefore, to determine the minimum value, it is necessary to carry out computational experiments.

## SELECTION OF THE INVESTMENT PORTFOLIO

The Index of Prices and Quotes (IPQ) of the Mexican Stock Market (MSM) is an indicator that expresses the performance of the stock market according to the price changes of a sample that represents the set of stocks listed in the MSM. The IPQ shows the performance and dynamism of the Mexican stock market since 1978; it is reviewed every year and in mid-January, its assets are known for the next twelve months. Table 1 shows the companies and securities that will comprise the IPQ from February 2013 to January 2014[1].

Two of the assets do not have historical public information for the analyzed period, from December 14, 2012 to March 15, 2013. These two shares are indicated in Table 1 in bold text, without the number of shares indicated. These shares are TV AZTECA CPO (AZTECACPO.MX) and Industrias Peñoles, S.A.B. of C.V. (PE and OLES.MX). Therefore, the analysis is carried out by the 33 shares listed in Table 1.

The study considers the 62 days of the analyzed period. Table 2 shows the percentages of the performance and risk of 33 shares obtained with the daily closing price.

**Brief description of the algorithm used:** The algorithm used to solve the linear multi-objective problem is an iterative process. In each one of the iterations, a linear programming problem with the new values of the magnitude of restrictions (7) and (8) is resolved. The magnitude of the variables $\lambda_1$ and $\lambda_2$ is determined by Zavala-Díaz *et al*. (2011b) the following:

$$\lambda_{1_{i+1}} = \lambda_{1_i} \pm \Delta\lambda_{1_i} \tag{13}$$

$$\lambda_{2_{i+1}} = \lambda_{2_i} \pm \Delta\lambda_{2_i} \tag{14}$$

At the beginning of the process, the magnitude of $\Delta\lambda_1$ and $\Delta\lambda_2$ are equal to one-tenth. This increase is first used to reach the region of feasible solutions and subsequently used to reach the upper border of the efficient portfolios. The next step is to determine Point *b* of the CAPM model, the point at which the portfolios with the optimal risk-return profiles are located. The determination of the optimal point is made by crossing the border to determine the points highest (for the highest return) and farthest to the left (for the lowest risk). These movements in the solution space are made by refining the magnitude of $\Delta\lambda_1$ and $\Delta\lambda_2$. The refinement consists of dividing the increase by two whenever a change comes in the direction of the searching of feasible solutions. Dividing the increase by two allows make fine approaches to the point of interest. Up to 20 divisions are used in this study, which gives an approximation of: $\frac{1}{2^{20}} = \frac{1}{1048576} = 9.53674X10^{-7}$.

In each one of the iterations, a linear programming problem is resolved with the new values of $\lambda_1$ and $\lambda_2$; this is solved using the SIMPLEX method (Taha, 2003). The optimal solution is obtained when there is no significant variance between two consecutive iterations.

**Experimentation:** Considering the values of Table 2 and the refinement of $\lambda_1$ and $\lambda_2$, the investment portfolios are calculated and shown in Fig. 3 and Table 3 for different values of $\lambda_3$. It is important to mention that for each value of $\lambda_3$ the optimal solution is obtained.

As is shown in Table 3 and Fig. 3, with the modification of the algorithm is possible to increase the number of titles of the investment portfolios, two to

Table 1: Assets that will constitute the IPQ from February 2013 to January 2014[2]

| $x_i$ | Company (share) | $x_i$ | Company (Share) |
|---|---|---|---|
| 1 | Arca Continental, S.A.B. de C.V. (AC.MX) | 18 | Grupo Mexico, S.A.B. de C.V. (GMEXICOB.MX) |
| 2 | Alfa, S.A.B. de C.V. (ALFAA.MX) | 19 | Grupo Modelo, S.A.B. de C.V. (GMODELOC.MX) |
| 3 | ALPEK-A (ALPEKA.MX) | 20 | Gruma, S.A.B. de C.V. (GRUMAB.MX) |
| 4 | Alsea SAB de CV (ALSEA.MX) | 21 | Desarrolladora Homex SAB de CV. (HOMEX.MX) |
| 5 | America Movil, S.A.B. de C.V. (AMXL.MX) | 22 | Empresas ICA, S.A.B. de C.V. (ICA.MX) |
| 6 | Grupo Aeroportuario del Sureste, S.A.B. de C.V. (ASURB.MX) | 23 | Industrias Ch, S.A.B. de C.V. (ICHB.MX) |
| | TV AZTECA CPO (AZTECACPO.MX) | 24 | Kimberly-Clark de Mexico S.A.B. de C.V. (KIMBERA.MX) |
| 7 | Grupo Bimbo SAB de CV (BIMBOA.MX) | 25 | Coca-Cola Femsa, S.A.B. de C.V. (KOFL.MX) |
| 8 | Bolsa Mexicana de Valores SAB de CV (BOLSAA.MX) | 26 | Genomma Lab Internacional, S.A.B. de C.V. (LABB.MX) |
| 9 | Cemex, S.A.B. de C.V. (CEMEXCPO.MX) | 27 | El Puerto de Liverpool, S.A.B. de C.V. (LIVEPOLC-1.MX) |
| 10 | Grupo Comercial Chedraui, S.A.B. de C.V. (CHDRAUIB.MX) | 28 | Mexichem, S.A.B. de C.V. (MEXCHEM.MX) |
| 11 | COMPARTAMOS (COMPARC.MX) | 29 | MINERAS FRISCO-A-1 (MFRISCOA-1.MX) |
| 12 | Grupo Elektra, S.A. de C.V. (ELEKTRA.MX) | 30 | OHL MEXICO (OHLMEX.MX) |
| 13 | Fomento Economico Mexicano SAB de CV (FEMSAUBD.MX) | | Industrias Peñoles, S.A.B. de C. V. (PE&OLES.MX) |
| 14 | Grupo Aeroportuario del Pacifico SAB de CV(GAPB.MX) | 31 | Grupo Televisa, S.A. (TLEVISACPO.MX) |
| 15 | Corporacion Geo, S.A.B. de C.V. (GEOB.MX) | 32 | Urbi Desarrollos Urbanos, S.A.B. de C.V. (URBI.MX) |
| 16 | Grupo Financiero Inbursa, S.A.B. de C.V. (GFINBURO.MX) | 33 | Wal-Mart de Mexico, S.A.B. de C.V. (WALMEXV.MX) |
| 17 | Grupo Financiero Banorte SAB de CV (GFNORTEO.MX) | - | - |

Table 2: Profitability and risk of 33 shares of the IPQ of the MSM, December 14, 2012 to March 15, 2013[1]

| $x_i$ | Profit (%) | Risk (%) | Sale price | $x_i$ | Profit (%) | Risk (%) | Sale price |
|---|---|---|---|---|---|---|---|
| 1 | -0.04119130 | 0.7961129 | 92.96 | 18 | 0.22024890 | 1.4093297 | 51.55 |
| 2 | 0.13174110 | 2.0995924 | 30.64 | 19 | -0.00550670 | 1.2646505 | 111.63 |
| 3 | -0.23007130 | 1.5957292 | 30.74 | 20 | 0.50264720 | 1.5597941 | 52.67 |
| 4 | 0.44206020 | 1.7213649 | 32.30 | 21 | -0.04187750 | 3.0848198 | 24.60 |
| 5 | -0.41030860 | 2.0468815 | 11.60 | 22 | 0.41323940 | 1.6659463 | 40.78 |
| 6 | 0.20315260 | 1.3086237 | 163.67 | 23 | 0.27437100 | 1.9374036 | 104.62 |
| 7 | 0.13299560 | 1.1937644 | 34.49 | 24 | 0.37190390 | 1.5610787 | 39.32 |
| 8 | 0.01976160 | 1.5465463 | 32.28 | 25 | 0.09660910 | 1.0375112 | 198.87 |
| 9 | 0.45918850 | 1.6863209 | 15.45 | 26 | 0.14823320 | 1.8356327 | 29.07 |
| 10 | -0.01155880 | 1.5307634 | 40.84 | 27 | 0.23354540 | 1.0278827 | 148.54 |
| 11 | 0.17599880 | 2.0680374 | 21.28 | 28 | -0.19085240 | 1.8709866 | 63.97 |
| 12 | -0.15605340 | 2.1515588 | 497.16 | 29 | -0.03861010 | 0.5800393 | 52.67 |
| 13 | 0.16539930 | 0.9893488 | 139.19 | 30 | 0.62835440 | 2.0689859 | 35.19 |
| 14 | -0.00275034 | 1.5160343 | 68.73 | 31 | 0.00241862 | 1.2699857 | 65.07 |
| 15 | -0.86454610 | 3.7617265 | 7.96 | 32 | -1.43374170 | 4.7537258 | 3.14 |
| 16 | -0.14161400 | 1.4934510 | 34.55 | 33 | -0.19314340 | 1.3094721 | 38.09 |
| 17 | 0.32320290 | 1.7127118 | 98.67 | - | - | - | - |

Table 3: Profit-risk of the portfolio for different values of $\lambda_3$

| | | Share number | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\lambda_3$ (%) | Attribute | 1 | 9 | 13 | 20 | 24 | 25 | 27 | 29 | Total |
| 15 | Profit | -0.0008 | 0.0689 | 0.0248 | 0.0754 | 0.0297 | 0.0145 | 0.0350 | -0.0058 | 0.2417 |
| | Risk | 0.0160 | 0.2529 | 0.1484 | 0.2340 | 0.1247 | 0.1556 | 0.1542 | 0.0870 | 1.1729 |
| | Contribution | 0.0201 | 0.1500 | 0.1500 | 0.1500 | 0.0799 | 0.1500 | 0.1500 | 0.1500 | 1.0000 |
| 30 | Profit | -0.0035 | - | 0.0496 | 0.0081 | - | - | 0.0701 | -0.0116 | 0.1128 |
| | Risk | 0.0667 | - | 0.2968 | 0.0253 | - | - | 0.3084 | 0.1740 | 0.8712 |
| | Contribution | 0.0838 | - | 0.3000 | 0.0162 | - | - | 0.3000 | 0.3000 | 1.0000 |
| 45 | Profit | - | - | - | 0.0508 | - | - | 0.1051 | -0.0173 | 0.1386 |
| | Risk | - | - | - | 0.1577 | - | - | 0.4625 | 0.2604 | 0.8806 |
| | Contribution | - | - | - | 0.1011 | - | - | 0.4500 | 0.4489 | 1.0000 |
| 60 | Profit | - | - | - | 0.1146 | - | - | 0.1401 | -0.0066 | 0.2481 |
| | Risk | - | - | - | 0.3557 | - | - | 0.6167 | 0.0997 | 1.0722 |
| | Contribution | - | - | - | 0.2281 | - | - | 0.6000 | 0.1719 | 1.0000 |
| 75 | Profit | - | - | - | 0.0947 | - | - | 0.1751 | -0.0024 | 0.2675 |
| | Risk | - | - | - | 0.2940 | - | - | 0.7708 | 0.0358 | 1.1005 |
| | Contribution | - | - | - | 0.1885 | - | - | 0.7499 | 0.0616 | 1.0000 |
| 90 | Profit | - | - | - | 0.0633 | - | - | 0.2042 | - | 0.2674 |
| | Risk | - | - | - | 0.1963 | - | - | 0.8985 | - | 1.0948 |
| | Contribution | - | - | - | 0.1258 | - | - | 0.8742 | - | 1.0000 |
| 100 | Profit | - | - | - | 0.0633 | - | - | 0.2042 | - | 0.2674 |
| | Risk | - | - | - | 0.1963 | - | - | 0.8985 | - | 1.0948 |
| | Contribution | - | - | - | 0.1258 | - | - | 0.8742 | - | 1.0000 |

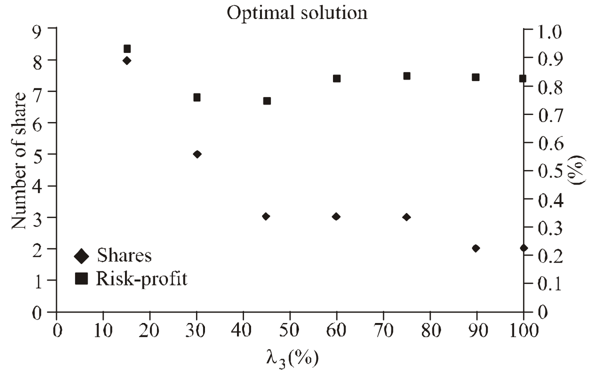


Fig. 3: Number of shares and risk-profits of investment portfolios for different values of $\lambda_3$

eight. It is observed that the best investment portfolio, from the objective function min (Risk-Profits), is when $\lambda_3$ has a value of 45%. But this portfolio has the lowest performance and the lowest risk. However, the portfolio obtained with $\lambda_3$ = 75% has the highest performance and the highest risk for all optimal solutions. Both portfolios are formed by such three titles; it indicates that when varying the value of $\lambda_3$ are forced to obtain another optimal solution with different contents from the titles that form it.

When $\lambda_3$ = 15% a portfolio with a slightly lower profit and a slightly higher risk than those obtained with $\lambda_3$ = 75% is obtained. These risks and returns are closer to those obtained portfolios with fewer titles.

These portfolios determine a key dominant share that is the best of all of the IPQ stocks; in this experiment, the share was the 27$^{th}$. When the number of diversified shares is increased, the shares that are included generate a lower risk or higher profit than the dominant share. None of the included assets break this rule, as can be seen in Table 1 and 3.

Importantly, it was not possible to obtain investment portfolios with values of $\lambda_3$ lesser than 15% because no feasible solutions are generated.

**Hypothesis testing:** The Coefficient of Variation (CV) is used to verify that the selected portfolios generate higher profits at a lower risk. CV allows compare different datasets with the two statistical parameters that interest, the media (profit) and the standard deviation (risk).

The best portfolios are those that have the lowest CVs because the average is the denominator and the standard deviation is the numerator. Therefore, if the

Table 4: Coefficient of variation during the analyzed period, the 33 assets in the IPQ and investment portfolios

| $x_i$ | CV | $x_i$ | CV | Portfolio $\lambda_3$ (%) | CV |
|---|---|---|---|---|---|
| 1 | 19.32720180 | 18 | 6.39880396 | 15 | 4.8527 |
| 2 | 15.93726470 | 19 | 229.65674900 | 30 | 7.7234 |
| 3 | 6.93580378 | 20 | 3.10315905 | 45 | 6.3535 |
| 4 | 3.89396072 | 21 | 73.66288340 | 60 | 4.3216 |
| 5 | 4.98863921 | 22 | 4.03143145 | 75 | 4.1140 |
| 6 | 6.44158073 | 23 | 7.06125402 | 90 | 4.0942 |
| 7 | 8.97597180 | 24 | 4.19753294 | 100 | 4.0942 |
| 8 | 78.25999190 | 25 | 10.73927070 | - | - |
| 9 | 3.67239346 | 26 | 12.38340820 | - | - |
| 10 | 132.43271000 | 27 | 4.40121199 | - | - |
| 11 | 11.75029410 | 28 | 9.80331442 | - | - |
| 12 | 13.78732470 | 29 | 15.02298990 | - | - |
| 13 | 5.98157621 | 30 | 3.29270553 | - | - |
| 14 | 551.21706800 | 31 | 525.08669900 | - | - |
| 15 | 4.35109977 | 32 | 3.31560834 | - | - |
| 16 | 10.54592790 | 33 | 6.77979359 | - | - |
| 17 | 5.29918385 | - | - | - | - |

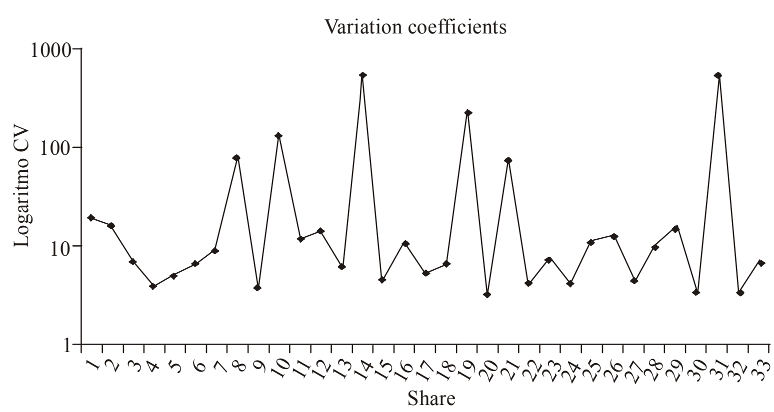


Fig. 4: Coefficient of variation of 33 assets in the IPQ during the analyzed period

denominator is increased and the numerator is decreased, the CV would tend to generate lower values. Table 4 shows the CV of the shares that comprise the IPQ and the portfolios obtained during the analyzed period.

Table 4 shows that portfolios with higher CV are those with the smallest difference-Profit Risk and the portfolios with the lowest CV are the ones with the greatest differences. Therefore, proportionally speaking, this coefficient indicates that the best portfolios are those with the lowest risk for a given performance and these are the ones with the lowest CV. Considering the above, then the diversified portfolio is attractive. This finding is consistent with the premise of the other models that diversification produces higher profit and a lower risk. Figure 4 shows the CV of the assets and of the portfolios; this figure proves that the CV of the portfolios is closer to the lower values.

The profit and risk of all of the shares are calculated with the same formula used to compute the return and risk of the 33 assets. Their values are 0.03632 and 1.9372%, respectively. With these values, a CV of the total population of 53.3370 is obtained.

## CONCLUSION

We conclude the following from the obtained results.

The approach of the problem of the selection of the portfolio of investment like a function between both models, CAPM y Markowitz, lets resolve it from the domain of the function with a linear multi-objective model.

The introduction of the increasing function allows establish the hypotheses for the calculation of investment portfolio with the elements of the dominion of the function *g*.

Limiting the percentage of titles makes it possible to calculate optimal solutions with other diversified portfolios.

Performance and risk diversified portfolios are near portfolios that have the best CV.

The coefficient of variation is a good indicator with which to compare different investment portfolios because it is obtained using the two main statistics that define the profitability of an investment portfolio.

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**End note:**

1 Financial web page (2013). http://mx.finanzas.yahoo. com/2013.

2 Mexican stock-market (2013). http:// bolsamexicanadevalores.com.mx/ipc-bolsa-mexicana/