

## Research Article

### Note on ‘Combining an Improved Multi-delivery Policy into a Single-producer Multi-retailer Integrated Inventory System with Scrap in Production’

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**Abstract:** In a recent study, Chiu *et al.* (2014) employed a mathematical modeling and conventional optimization technique to determine the optimal production-shipment policy for a single-producer multi-retailer integrated inventory system with scrap and an improved product distribution policy. This study replaces their optimization process of using differential calculus with an algebraic derivation. Such a simplified approach enables practitioners, who may have insufficient knowledge of calculus, to manage with ease the real world supply-chain systems.

**Keywords:** Algebraic approach, cost reducing delivery policy, multiple retailers, optimization, production-shipment policy, supply chains management, synchronous deliveries

## INTRODUCTION

The most common method for solving the optimal replenishment lot-size problem is the mathematical modeling approach along with differential calculus as an optimization procedure (Tersine, 1994; Hillier and Lieberman, 2001; Nahmias, 2009). Recently, Grubbström and Erdem (1999) introduced an algebraic derivation rather than calculus to determine the Economic Order Quantity (EOQ) model with backlogging over a decade ago. Their algebraic approach successfully found the optimal order quantity for the EOQ model without reference to the first-order derivatives.

Similar methodologies have since been adopted to resolve different aspects of Economic Production Quantity (EPQ) models and various kinds of supply chain systems (Wu and Ouyang, 2003; Wee and Chung, 2006; Francis Leung, 2008; Lin *et al.*, 2008; Chen *et al.*, 2012). This study extends such an algebraic approach to a specific intra-supply chain system studied by Chiu *et al.* (2014) and demonstrates that the optimal replenishment lot-size and shipment policy along with a simplified formula for the total system cost can be derived without derivatives.

**Problem description and formulation:** Reconsider the production-shipment problem for a single-producer multi-retailer integrated inventory system with scrap and an improved product distribution policy as studied by Chiu *et al.* (2014). In their production-shipment model, annual production rate for a product manufactured by a single production unit is  $P$  and manufacturing cost is  $C$  per item. An  $x$  portion of scrap

items may be randomly produced at a rate  $d$  during the production process through a 100% quality screening. By not allowing shortages, it is assumed  $(P - d - \lambda) > 0$ , where  $\lambda$  is the sum of the demands of all  $m$  retailers (i.e., the sum of  $\lambda_i$  where  $i = 1, 2, \dots, m$ ) and  $d$  can be expressed as  $d = Px$ . Cost related parameters used in the cost analysis include the following: unit disposal cost  $C_S$ ; set-up cost per production cycle  $K$ ; fixed delivery cost  $K_{i_1}$  per shipment delivered to retailer location  $i$ ; unit holding cost  $h$  for item retained by the production unit; unit holding cost  $h_{2i}$  for items retained by retailer  $i$ ; and unit shipping cost  $C_{T_i}$  for items shipped to retailer location  $i$ . To ease the readability, the following same notations are also adopted:

- $Q$  = Production lot size per cycle-decision variable
- $n$  = Number of fixed quantity installments of the remaining lot to be delivered to retailers-the other decision variable
- $H_1$  = Level of on-hand inventory in units for satisfying product demands during production unit's production uptime  $t_1$
- $H$  = Maximum level of on-hand inventory in units when regular production process ends
- $t_1$  = The production uptime
- $t_2$  = Time required for delivering all remaining finished items in a lot to retailers
- $D_i$  = Number of fixed quantity finished items distributed to retailer  $i$  per delivery
- $I_i$  = Number of left over items per delivery after the depletion during  $t_{ni}$  for retailer  $i$
- $T$  = Production cycle length
- $t_n$  = A fixed interval of time between each installment of finished products delivered during  $t_2$

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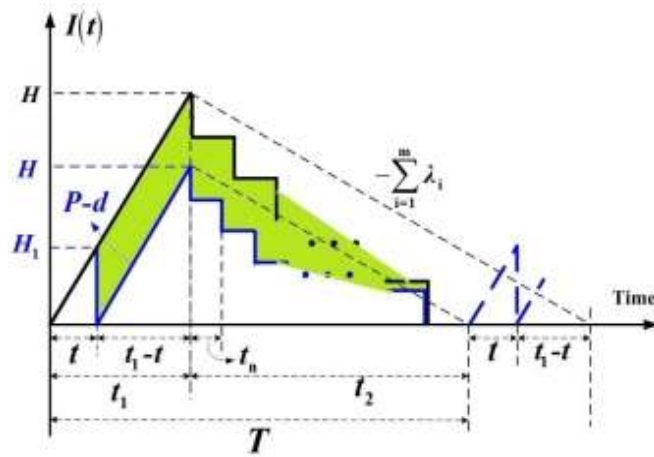


Fig. 1: Expected reduction in the production unit's holding costs (in green shaded area) of the proposed model (in blue) in comparison with that of Chiu *et al.* (2013) model (in black) (Chiu *et al.*, 2014)

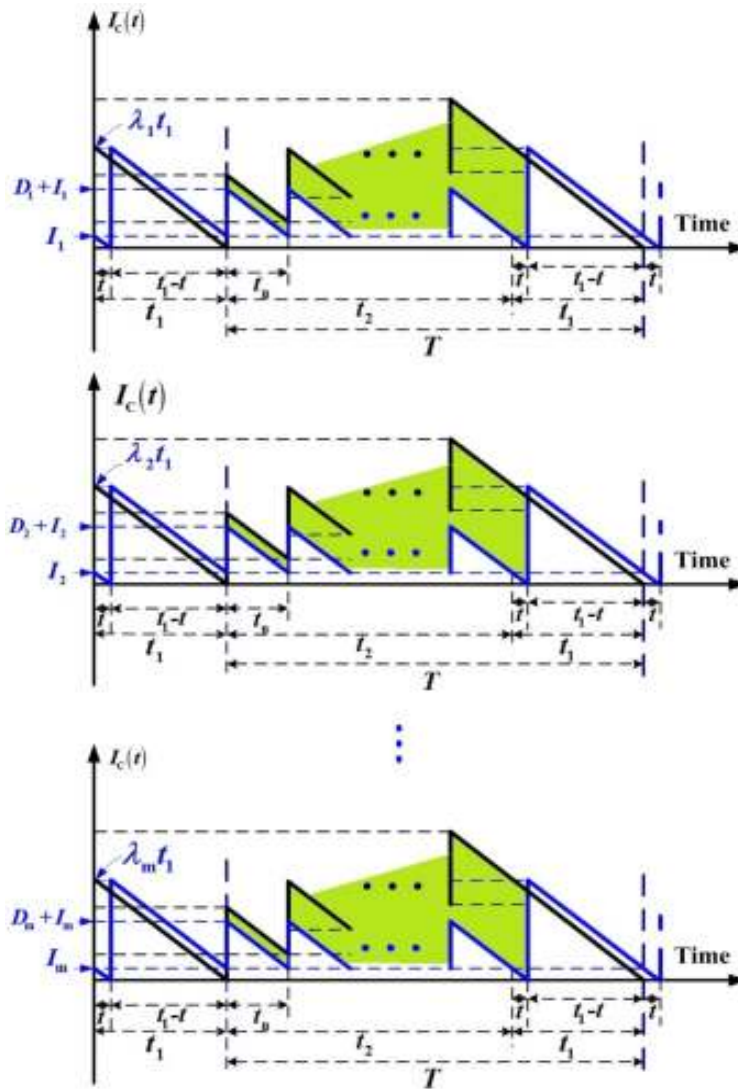


Fig. 2: Expected reduction in retailers' inventory holding costs (in green shaded area) of the proposed model (in blue) in comparison with that of Chiu *et al.* (2013) model (in black) (Chiu *et al.*, 2014)

- $m$  = Number of retailers
- $I(t)$  = The level of on-hand inventory of perfect quality items at time  $t$
- $I_c(t)$  = The level of retailers' on-hand inventory at time  $t$
- $TC(Q, n+1)$  = Total production-inventory-delivery costs per cycle for the proposed  $n+1$  delivery model
- $E[TCU(Q, n+1)]$  = The expected total production-inventory-delivery costs per unit time for the proposed  $n+1$  delivery model

Under an improved  $n+1$  product distribution policy, one initial delivery of finished products is distributed multiple retailers to meet demand during the production unit's uptime (Fig. 1). Upon the remaining production lot is produced and screened,  $n$  fixed quantity installments of the finished products are distributed retailers, at a fixed time interval. Figure 1 illustrates the expected reduction in the production unit's holding costs (in green shaded area) of the proposed model (in blue) in comparison with that of Chiu *et al.* (2013) model (in black).

Figure 2 depicts the expected reduction in retailers' inventory holding costs (in green shaded area) of the proposed model (in blue) in comparison with that of Chiu *et al.* (2013) model (in black).

Total production-inventory-delivery costs per replenishment cycle for the proposed model,  $TC(Q, n+1)$  consists of the setup cost, variable manufacturing cost, disposal cost, the fixed and variable shipping costs, inventory holding cost incurred in the production unit and stock holding cost incurred in the retailers' locations as follows (Chiu *et al.*, 2014):

$$\begin{aligned}
 TC(Q, n+1) = & K + CQ + C_s[xQ] + (n+1) \sum_{i=1}^m K_{li} + \sum_{i=1}^m C_{Ti} [Q(1-x)] \\
 & + h \left[ \frac{H_1}{2}(t) + \frac{H}{2}(t_1 - t) + \frac{dt_1}{2}(t_1) + \left( \frac{n-1}{2n} \right) Ht_2 \right] \\
 & + \sum_{i=1}^m h_{2i} \left[ \frac{\lambda_i(t_i)^2}{2} + n \left( \frac{D_i + 2I_i}{2} \right) t_n \right]
 \end{aligned} \tag{1}$$

Taking randomness of scrap rate into account by applying the expected values of  $x$  and with further derivations,  $E[TCU(Q, n+1)]$  becomes (Chiu *et al.*, 2014):

$$\begin{aligned}
 E[TCU(Q, n+1)] = & C\lambda E_0 + \frac{K\lambda}{Q} E_0 + C_s \lambda E_2 + \sum_{i=1}^m C_{Ti} \lambda_i + (n+1) \frac{\lambda}{Q} E_0 \sum_{i=1}^m K_{li} \\
 & + \frac{hQ\lambda}{2} \left\{ \frac{2\lambda^2}{P^3} E_1 - \frac{(1-2E[x])}{P} E_0 - \frac{\lambda}{P^2} E_0 + \frac{1}{\lambda E_0} \left[ \frac{1}{n} \left( \frac{1}{\lambda E_0} - \frac{2}{P} + \frac{\lambda}{P^2} E_0 \right) \right] \right\} \\
 & + \sum_{i=1}^m h_{2i} Q \lambda_i \left\{ \frac{\lambda}{2P^2} E_0 + \frac{1}{2n\lambda E_0} - \frac{1}{nP} + \frac{\lambda}{P^2} E_0 + \frac{\lambda}{2nP^2} E_0 - \frac{\lambda^2}{P^3} E_1 \right\}
 \end{aligned} \tag{2}$$

where  $E_i$  denotes the following:

$$\begin{aligned}
 E_0 = & \frac{1}{1-E[x]}; E_1 = E_3 E_0; E_2 = E[x] E_0; \\
 E_3 = & E \left( \frac{1}{1-x} \right); \lambda = \sum_{i=1}^m \lambda_i
 \end{aligned} \tag{3}$$

Unlike conventional solution procedure which uses the differential calculus on the cost function  $E[TCU(Q, n+1)]$  to derive the optimal production-shipment operating policy (Chiu *et al.*, 2014), this study proposes an alternative two-phase algebraic approach as follows.

### METHODOLOGY

**Two-phase algebraic approach:** In the phase 1, we first derive the optimal number of distribution. It can be seen that Eq. (2) has two decision variables, namely  $Q$  and  $n$ . In Eq. (2) there are different forms of decision variables in its Right-Hand Side (RHS), namely  $Q$ ,  $Q^{-1}$ ,  $Qn^{-1}$  and  $nQ^{-1}$ . We first let  $v_0, v_1, v_2, v_3$  and  $v_4$  denote the following:

$$v_0 = C\lambda E_0 + C_s \lambda E_2 + \sum_{i=1}^m C_{Ti} \lambda_i \tag{4}$$

$$v_1 = \left[ K + \lambda \sum_{i=1}^m K_{li} \right] E_0 \tag{5}$$

$$v_2 = \lambda E_0 \sum_{i=1}^m K_{li} \tag{6}$$

$$\begin{aligned}
 v_3 = & \frac{h\lambda}{2} \left[ \frac{2\lambda^2}{P^3} E_1 - \frac{(1-2E[x])}{P} E_0 - \frac{\lambda}{P^2} E_0 + \frac{1}{\lambda E_0} \right] \\
 & + \sum_{i=1}^m h_{2i} \lambda_i \left\{ \frac{\lambda}{2P^2} E_0 + \frac{\lambda}{P^2} E_0 - \frac{\lambda^2}{P^3} E_1 \right\}
 \end{aligned} \tag{7}$$

$$\begin{aligned}
 v_4 = & -\frac{h\lambda}{2} \left\{ \left( \frac{1}{n} \right) \left[ \frac{1}{\lambda E_0} - \frac{2}{P} + \frac{\lambda}{P^2} E_0 \right] \right\} \\
 & + \sum_{i=1}^m h_{2i} \lambda_i \left\{ \frac{1}{2n\lambda E_0} - \frac{1}{nP} + \frac{\lambda}{2nP^2} E_0 \right\}
 \end{aligned} \tag{8}$$

Then, Eq. (2) can be rearranged as:

$$E[TCU(Q, n+1)] = v_0 + v_1 Q^{-1} + v_3 Q + v_2 (nQ^{-1}) + v_4 (Qn^{-1}) \tag{9}$$

Further rearrange Eq. (9) as:

$$\begin{aligned}
 E[TCU(Q, n+1)] = & v_0 + Q^{-1} [v_1 + v_3 Q^2] \\
 & + (nQ^{-1}) [v_2 + v_4 (Qn^{-1})^2]
 \end{aligned} \tag{10}$$

or,

$$E[TCU(Q, n+1)] = v_0 + Q^{-1} [\sqrt{v_1} - \sqrt{v_3}Q]^2 + 2\sqrt{v_1v_3} + (nQ^{-1}) [\sqrt{v_2} - \sqrt{v_4}(Qn^{-1})]^2 + 2\sqrt{v_2v_4} \quad (11)$$

One notes that if the second and fourth terms in RHS of Eq. (11) equal zero, then E [TCU (Q, n+1)] can be minimized. That is:

$$Q = \sqrt{\frac{v_1}{v_3}} \text{ and } n = \sqrt{\frac{v_4}{v_2}}(Q) \quad (12)$$

or,

$$n^* = \sqrt{\frac{v_4}{v_2}} \cdot \sqrt{\frac{v_1}{v_3}} \quad (13)$$

By substituting Eq. (5) to (8) in Eq. (13), one has the optimal number of distribution as:

$$n^* = \sqrt{\frac{\left(\sum_{i=1}^m K_{1i} + K\right) \left(h\lambda - \sum_{i=1}^m h_{2i}\lambda_i\right) \left[\frac{1}{2\lambda E_0^2} - \frac{1}{PE_0} + \frac{\lambda}{2P^2}\right]}{\left(-\sum_{i=1}^m K_i\right) \left\{ h\lambda \left\{ \frac{\lambda^2 E_3}{P^3} - \frac{1}{2P} + \frac{E[x]}{P} - \frac{\lambda}{2P^2} + \frac{1}{2\lambda E_0^2} \right\} + \sum_{i=1}^m h_{2i}\lambda_i \left\{ \frac{\lambda}{2P^2} + \frac{\lambda}{P^2} - \frac{\lambda^2 E_3}{P^3} \right\} \right\}}}} \quad (14)$$

It is noted that Eq. (14) is identical to that obtained in Chiu *et al.* (2014).

In real-life production-shipment applications, the number of deliveries  $n$  can only be an integer. Let  $n^+$  represent the smallest integer greater than or equal to  $n$  (i.e., the computational result of Eq. (15)) and  $n^-$  be the largest integer less than or equal to  $n$ . Then, it is obviously that the optimal  $n^*$  is either  $n^+$  or  $n^-$ .

In the phase 2, we then derive the optimal replenishment lot-size  $Q^*$ . By rearranging Eq. (2) as a single variable  $Q$  cost function as follows:

$$E[TCU(Q, n+1)] = v_0 + Q^{-1}(v_1 + v_2n) + Q(v_3 + v_4n^{-1}) \quad (15)$$

or,

$$E[TCU(Q, n+1)] = v_0 + Q^{-1}\omega_1 + Q\omega_2 \quad (16)$$

where,

$$\omega_1 = v_1 + v_2n; \quad \omega_2 = v_3 + v_4n^{-1} \quad (17)$$

Equation (16) can be rearranged as:

$$E[TCU(Q, n+1)] = v_0 + Q^{-1} [\sqrt{\omega_1} - \sqrt{\omega_2}Q]^2 + 2\sqrt{\omega_1\omega_2} \quad (18)$$

It can be seen that if the second term in RHS of Eq. (18) equals zero, then E [TCU (Q, n+1)] can be minimized. That is:

$$Q^* = \sqrt{\frac{\omega_1}{\omega_2}} \quad (19)$$

Substituting Eq. (5) to (8) in Eq. (15) and then in Eq. (17), one obtains optimal replenishment lot-size  $Q^*$  as:

$$Q^* = \sqrt{\frac{2 \left[ K + (n+1) \sum_{i=1}^m K_{1i} \right] \lambda}{h\lambda \left\{ \frac{1}{\lambda E_0^2} - \frac{1}{P} + \frac{2E[x]}{P} \right\} + \sum_{i=1}^m h_{2i}\lambda_i \left( \frac{2}{P^2} \right) + \left( h\lambda - \sum_{i=1}^m h_{2i}\lambda_i \right) \left\{ \frac{2\lambda^2 E_3}{P^3} - \frac{\lambda}{P^2} - \left( \frac{1}{n} \right) \left[ \frac{1}{\lambda E_0^2} - \frac{2}{PE_0} + \frac{\lambda}{P^2} \right] \right\}}}} \quad (20)$$

It is noted that both Eq. (14) and (20) are identical to that obtained in Chiu *et al.* (2014) where the conventional differential calculus was used. Moreover, from Eq. (18) one obtains a simplified form for the optimal long-run average cost function E [TCU (Q\*, n\*+1)] as:

$$E[TCU(Q, n+1)] = v_0 + 2\sqrt{\omega_1\omega_2} \quad (21)$$

## NUMERICAL EXAMPLE

To demonstrate applicability of our research results and to ease readers' comparison efforts, we adopt the same numerical example as in Chiu *et al.* (2014). The variables are given as follows:

- $P$  = 60,000 units per year
- $\lambda_i$  = 400, 500, 600, 700 and 800 units, respectively (i.e., total  $\lambda$  = 3000 units per year)
- $x$  = Random defective rate that follows a uniform distribution over the interval (0, 0.3)
- $C$  = \$100 per item
- $C_S$  = \$20, disposal cost per scrap item
- $K$  = \$35,000 per production run
- $h$  = \$25 per item per year at the producer side
- $K_{1i}$  = \$100, \$200, \$300, \$400 and \$500 per shipment for retailer  $i$ , respectively
- $h_{2i}$  = \$75, \$70, \$65, \$60 and \$55, per item retained by retailer  $i$ , respectively
- $C_{Ti}$  = \$0.5, \$0.4, \$0.3, \$0.2 and \$0.1 per unit shipped to retailer  $i$ , respectively

Applying Eq. (14), we have  $n = 5.83$ . Because  $n$  can only be an integer, by examining two adjacent integers to  $n$  and plugging them in Eq. (20), respectively, we

have  $(Q, n^+ + 1) = (3426, 6)$  and  $(Q, n^+ + 1) = (3548, 7)$ . Applying these policies in Eq. (2), respectively and select the one that gives minimum system cost, we derive the optimal production-shipment policy as  $(Q^*, n^* + 1) = (3548, 7)$ .

Finally, applying Eq. (21) the long-run average cost for the system is obtained as \$454,840. These results are identical to that obtained in Chiu *et al.* (2014).

## CONCLUSION

This study presents an algebraic derivation to replace (Chiu *et al.*, 2014) differential calculus method in their optimization procedure for solving a production-shipment problem for a single-producer multi-retailer integrated system with scrap and an improved product distribution policy. Such a straightforward simplified algebraic approach can help practitioners, who may have insufficient knowledge of calculus, to manage with ease the real world supply-chain systems.

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