

## Research Article

### Stabilizations of Strange Attractors by Feedback Linearization

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**Abstract:** This study deals with the control of the Lorenz, Chen and Lu chaotic system. Feedback linearization was successfully implemented on these chaotic systems. Feedback linearization was successful to transform the three attractor systems to a general system that simplify the linear control analysis. Hence, a linear controller is designed for the feedback linearized general system. Furthermore, some numerical simulations were carried out for the closed loop systems. These simulations show that the developed controller design method is effective in stabilizing and regulating the response.

**Keywords:** Chen attractor, feedback linearization, Lorenz attractor, Lu attractor, nonlinear control

#### INTRODUCTION

Recently, the control of Lorenz attractor is an active research area. Edward Lorenz demonstrated that the thermal convection loop can be characterized by a three ordinary nonlinear differential equations. Then he showed that the system is chaotic for a certain conditions. Just after Lorenz have found the first chaotic attractor (Lorenz, 1963). Many researchers have been encouraged to look for new chaotic attractors. Recently, some new chaotic systems were found, such as Chen attractor (Chen and Ueta, 1999) and Lu attractor (Lu and Chen, 2002). In the recent years, a variety of nonlinear control techniques have been implemented for a Lorenz, Chen and Lu attractors, such as back-stepping control (Ge *et al.*, 2000; Li, 2006), sliding mode control (Nazzal and Natsheh, 2007; Chiang *et al.*, 2007), feedback linearization control (Alvarez-Gallegos, 1994; Pishkenari *et al.*, 2007). In this study, Feedback linearization is employed to control the three attractors. In order to simplify the control analysis for the three systems, the method of feedback linearization is used to transform the three attractor systems to a general system. Then, a linear control law is proposed for the feedback linearized system.

**Dynamical descriptions of the attractors:** Three dynamical systems are considered here; The Lorenz attractor, Chen attractor and Lu attractor. In the next subsections, descriptions of the three chaotic systems, their chaotic attractors and their equilibrium points will be presented.

#### Lorenz attractor:

$$\begin{aligned}\dot{x} &= -ax + ay \\ \dot{y} &= -y - xz + cx \\ \dot{z} &= -bz + xy\end{aligned}\quad (1)$$

The parameters of the chaotic Lorenz attractor is typically set to  $a = 10$ ,  $b = 8/3$ ,  $c = 28$ . The Lorenz attractor has three equilibrium points located at  $(0, 0, 0)$ ;  $((\pm\sqrt{b(c-1)}), (\pm\sqrt{b(c-1)}, c-1))$ . The Lorenz attractor is depicted in Fig. 1a and b.

#### Chen attractor:

$$\begin{aligned}\dot{x} &= -ax + ay \\ \dot{y} &= cy - xz + (c-a)x \\ \dot{z} &= -bz + xy\end{aligned}\quad (2)$$

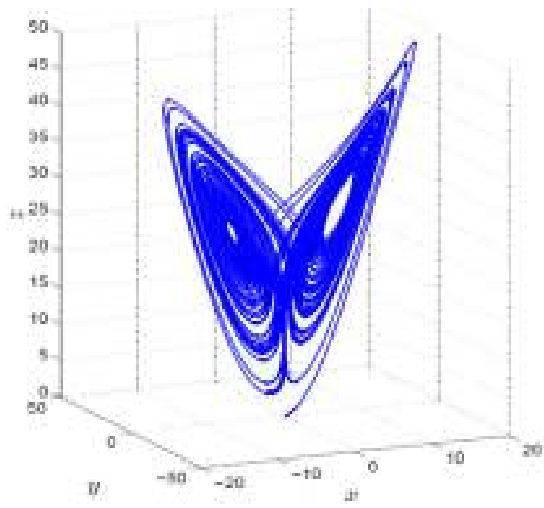
The parameters of the chaotic Chen attractor is typically set to  $a = 36$ ,  $b = 3$ ,  $c = 20$ . The Chen attractor has three equilibrium points located at  $(0, 0, 0)$ ;  $((\pm\sqrt{b(2c-a)}), (\pm\sqrt{b(2c-a)}, 2c-a))$ . The Chen attractor is depicted in Fig. 1c and d.

#### Lu attractor:

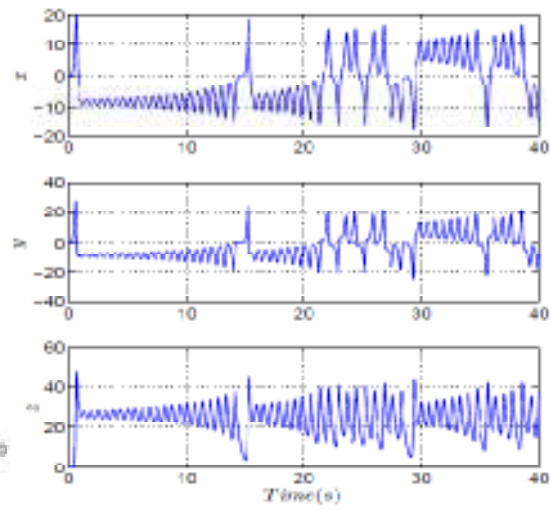
$$\begin{aligned}\dot{x} &= -ax + ay \\ \dot{y} &= cy - xz \\ \dot{z} &= -bz + xy\end{aligned}\quad (3)$$

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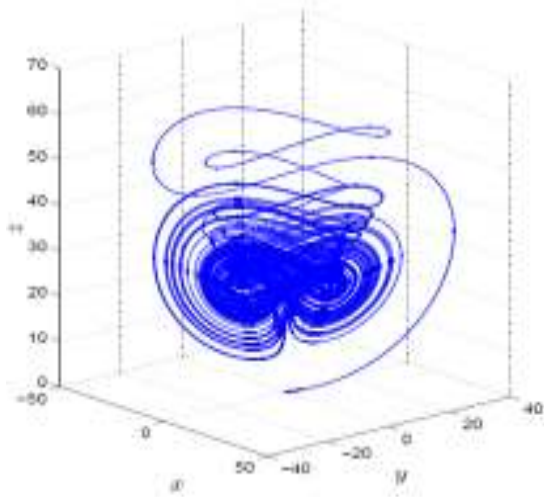
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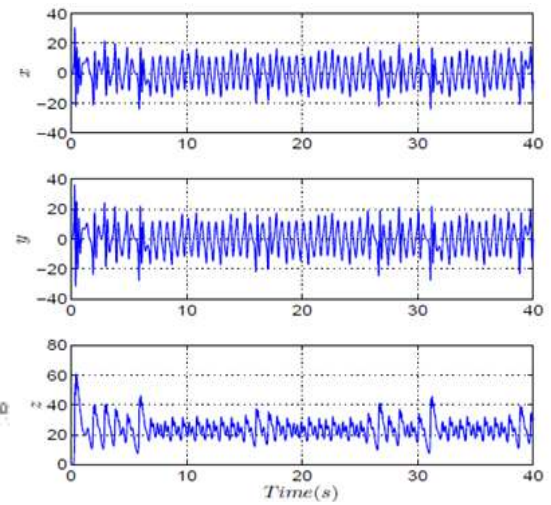
(a) The chaotic Lorenz attractor



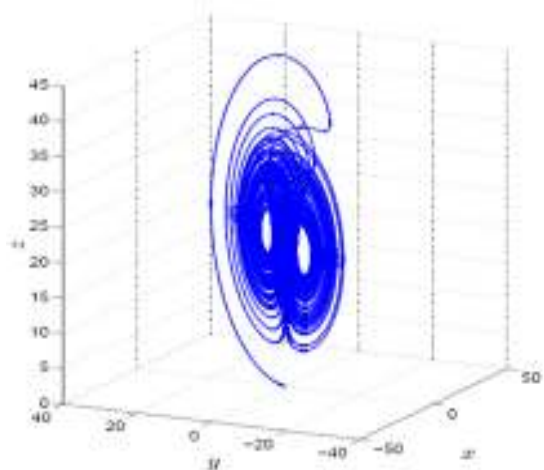
(b) Open loop response for controlled Lorenz system



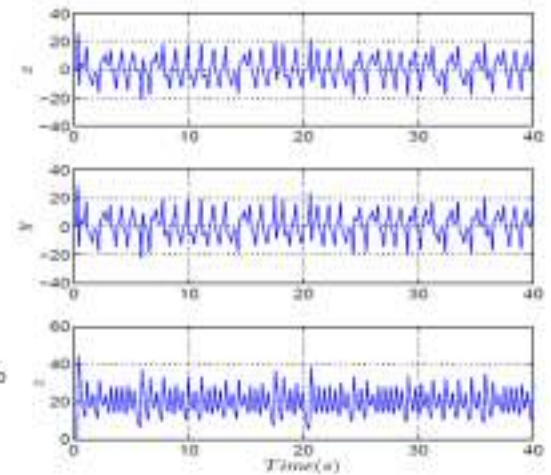
(c) The chaotic Chen attractor



(d) Open loop response for controlled Chen system



(e) The chaotic Lu attractor



(f) Open loop response for controlled Lu system

Fig. 1: Phase plot and open loop response of the attractors

The parameters of the chaotic Lu attractor is typically set to  $a = 36$ ,  $b = 3$ ,  $c = 20$ . The Lu attractor has three equilibrium points located at  $(0, 0, 0)$ ;  $((\mp\sqrt{bc}), (\mp\sqrt{bc}), c)$ . The Lu attractor is depicted in Fig. 1e and f.

### METHODOLOGY

**Control strategy:** In this section, the controlled dynamical systems for the Lorenz, Chen and Lu attractors are presented. The method of feedback linearization is applied on the attractors. Then, a steady state analysis is presented to facilitate the reference tracking problem for any of the three states  $(x, y, z)$ .

**The controlled dynamical systems:** The controlled Lorenz attractor:

$$\begin{aligned}\dot{x} &= -ax + ay \\ \dot{y} &= -y - xz + cx + u \\ \dot{z} &= -bz + xy\end{aligned}\quad (4)$$

The controlled Chen attractor:

$$\begin{aligned}\dot{x} &= -ax + ay \\ \dot{y} &= cy - xz + (c - a)x + u \\ \dot{z} &= -bz + xy\end{aligned}\quad (5)$$

The controlled Lu attractor:

$$\begin{aligned}\dot{x} &= -ax + ay \\ \dot{y} &= cy - xz + u \\ \dot{z} &= -bz + xy\end{aligned}\quad (6)$$

where,  $u$  is the nonlinear control term.

**Feedback linearization:** The method of feedback linearization transforms the nonlinear system into a linear system under certain situations. Hence, most of linear control theory can be applied to the feedback linearized system. All the systems (4, 5 and 6) can be transformed easily to the system 7 by selecting the appropriate  $u$  as depicted in Table 1:

$$\begin{aligned}\dot{x} &= -ax + ay \\ \dot{y} &= -y + v \\ \dot{z} &= -bz + xy\end{aligned}\quad (7)$$

where,  $v$  now is a linear control term, thus, the second state  $y$  can be controlled easily and also it can be used to derive the other two states, by considering the term  $(ay)$  in the first equation and term  $(xy)$  in the second equation as an imaginary control terms.

Table 1: Nonlinear control laws of the attractors

Attractor	Control law
Lorenz system	$u = -cx + xz + v$
Chen system	$u = (a - c)x + xz - (c + 1)y + v$
Lu system	$u = xz - (c + 1)y + v$

The system 7 can be transformed to:

$$\begin{aligned}\dot{x} &= -ax + v_1 \\ \dot{y} &= -y + v \\ \dot{z} &= -bz + v_2\end{aligned}\quad (8)$$

where,  $v_1 = f(x)$  and  $v_2 = f(x, y)$  we notice that the second state equation in system 7 is linear and totally uncoupled from the other two states, moreover, the other two states are highly dependent on the second state ( $y$ ). We may select  $v = -ky$  in the second state equation. Where,  $k$  is the state feedback gain.

Then the closed loop system becomes:

$$\dot{y} = -(k + 1)y\quad (9)$$

In order to have a stable closed loop system, the term  $-(k + 1)$  must be negative, then  $k > -1$ .

**Steady state analysis:** In the steady state system 7 becomes:

$$\begin{aligned}0 &= -ax_{ss} + ay_{ss} \\ 0 &= -y_{ss} + v \\ 0 &= -bz_{ss} + x_{ss}y_{ss}\end{aligned}\quad (10)$$

Which means that  $y_{ss} = v$ ,  $x_{ss} = y_{ss}$  and consequently  $Z_{ss} = (x_{ss}y_{ss})/b$ . If  $v = -ky + r$  where  $r$  is the reference. Then:

$$y_{ss} = \frac{r}{(k + 1)}\quad (11)$$

The Eq. (11) is well known as reference scaling in the state feedback control design. These relationships will facilitate the set point tracking for any of the three states by a single controller and without the need for designing a new controller for tracking each state of interest as done in Pishkenari *et al.* (2007).

### RESULTS AND DISCUSSION

Each of the chaotic systems studied here are to be controlled using the method of the feedback linearization. The closed loop systems of the Lorenz, Chen and Lu attractor and the feedback linearization controllers were simulated using MATLAB SIMULINK. The state feedback gain was selected to be  $k = 5$ , which makes the closed loop system stable and also gives good performance characteristics. The

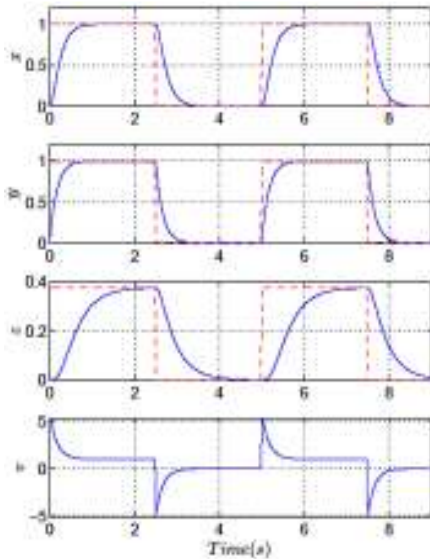


Fig. 2: Closed loop response of the three states of Lorenz attractor and the control action

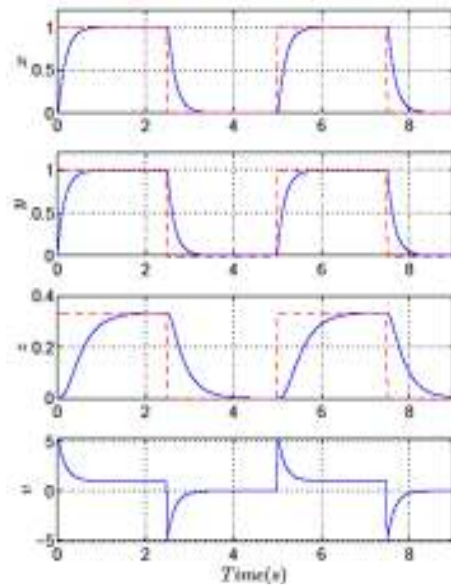


Fig. 4: Closed loop response of the three states of Lu attractor and the control action

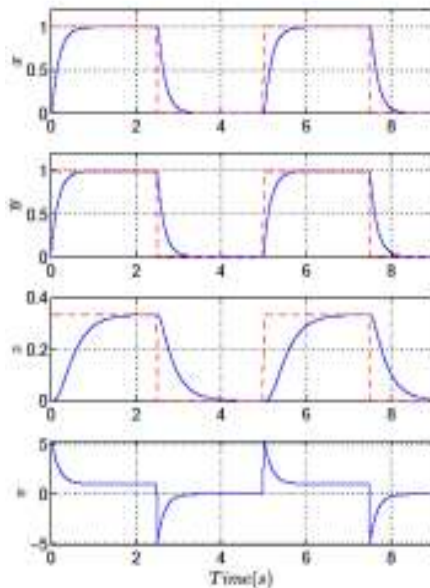


Fig. 3: Closed loop response of the three states of Chen attractor and the control action

Figure 2 to 4 show the closed loop response of the three states ( $x$ ,  $y$ ,  $z$ ) and the control action ( $v$ ) of the Lorenz, Chen and Lu systems respectively. The figures almost show identical response for all the systems except for the third state  $z$  that has different steady state value. That agrees with equation  $z_{ss} = (x_{ss} y_{ss}) / b$  where  $b = 8/3$  for Lorenz system and  $b = 3$  for Chen and Lu systems. The Figures show efficient closed loop performance and reference tracking. The settling time is 0.53, 0.5 and 1.33 sec for the states  $x$ ,  $y$  and  $z$ , respectively. Where the second state  $y$  has the fastest settling time even though it has the slowest pole, but this is because the other two states  $x$ ;  $z$  are driven by the second state  $y$ , which totally agrees with Eq. (8).

## CONCLUSION

The method of feedback linearization is implemented on three chaotic attractors. The chaotic oscillations are completely suppressed and the feedback linearized system behaves like a first order system. A linear controller is designed and the closed loop systems show an efficient regulation and reference tracking.

## ACKNOWLEDGMENT

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