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Research Article

Simulation of a Ball on a Beam Model Using a Fuzzy-dynamic and a Fuzzy-static Sliding-mode Controller

Muawia A. Magzoub, Nordin B. Saad and Rosdiazli B. Ibrahim
Department of Electrical and Electronic Engineering, Universiti Teknologi PETRONAS,
Bandar Sri Iskandar, 31750 Tronoh, Perak, Malaysia

Abstract: This study presents the design of a Fuzzy Static (FS) and a Fuzzy Dynamic (FD) Sliding-Mode Controllers (SMC) for both basic and complete ball on beam system. At first, the FSSMC was designed for the simplified and the complete models. Then, the FDSMC was designed on the simplified and the comprehensive models of the system in which the ball is placed on a beam as well. In addition, the lyapunov stability and linearization were used to check the stability of the system. There is an in-built issue of chattering with (FSSMC). However, (FDSMC) counter it well. Also, FDSMC is effective with respect to matched disturbance rejection. It has been found out from this research study that the designs of the models which utilize a FDSMC with a comprehensive model of the system were more efficient than the designs that utilize the basic system's prototype. Lastly, a comprehensive comparative analysis is provided and MATLAB/SIMULINK outcomes confirm the dominance of FDSMC.

Keywords: Fuzzy-dynamic, fuzzy-static, Sliding Mode Control (SMC), under triggered system of a ball on a beam

INTRODUCTION

The issue of the positioning of the ball on a system in which the ball is positioned on a beam has been the matter of great interest in many previous researches. The consideration of the nonlinear system of a ball on a beam has created an issue. For example, it was supposed that a system of a ball on a beam is not considered as an input-output linearizable because the degree related to this is not clearly explained (Marton and Lantos, 2006; Tomlin and Sastry, 1997). To overcome this problem, a simplified nonlinear model (i.e., by neglecting some terms in designing a controller) was utilized to estimate the real model of a system in which the ball is placed on a beam. Applying this technique (a simplified nonlinear prototype) of the system in which the ball is placed on a beam (Manan Khan *et al*., 2012; Almutairi and Zribi, 2010), most of the researchers have presented a variety of controllers; as an example, look to Chen and Balance (2002) and Li *et al*. (2003). From the point of view of previously carried out researches, the "the linear feedback is approximated in the condition if the system is near to singularities. On the contrary, an approximate sliding mode is used for the controller by changing it to the "exact feedback-linearization controller." The changing of the denominator of the control law is carried out in order to overcome the issue of singularity. Because of this alteration in the control mechanism, the resulting bounded sliding surface causes an overall efficiency to decrease.

In 1965 Fuzzy Logic was presented as a new kind of mathematical set method by Zadah (Jaradat *et al*., 2012), which consists of the fuzzy set theory that was proved to be the foundation theory of fuzzy logic. The fuzzy control system is basically established on the fuzzy logic principle that mainly comprises of three phases; fuzzification, inference engine and defuzzification. The first phase converts the inputs into fuzzy sets. While, the inference engine describes the fuzzy rules in the second phase, which is related to the outputs via specific rules using the sets of inputs. The last phase pools the outcomes of the fuzzy rules and infers the decision, which is then transformed from fuzzy sets to a sharp value (Magzoub *et al*., 2013).

Ball and beam are a practical example which is used in many laboratories because it is easy to understand. It is an open loop system which is not stable due to the unrestricted movement of the ball on the beam. Many applications (Chang *et al*., 2012; Hung and Chung, 2006; Nagarale and Patre, 2013a, b) show that FLC is superior to conventional controls in terms of control performance. It can be developed and implemented relatively easily and is capable of providing adequate control. The elimination of the need for a detailed modeling work is another advantage. Moreover, less sensitive, more tolerant to process state deviations and also to process parameter changes are

Corresponding Author: Muawia A. Magzoub, Department of Electrical and Electronic Engineering, Universiti Teknologi PETRONAS, Bandar Sri Iskandar, 31750 Tronoh, Perak, Malaysia, Tel.: +60-11-16480787

presented. However, the limitation of the FLC is the lack of a sound design methodology. The feedback control is required to hold the ball in the actual position on the beam. In this research study, a fuzzy-stationary and fuzzy-dynamic sliding mode control of the model will be applied by considering both simplified and complete modes. Moreover, the study of the system input, output linearization will be carried out and the stability will be monitored by using the lyapunov stability theory. Controller designs and the simulation results will be shown using a MATLAB/SIMULINK program. Figure 1 shows a real system in which the ball is positioned on a beam.

METHODOLOGY

Modeling of the system in which the ball is placed on the beam: Considering the Fig. 1, which shows a system of a ball on beam, there is a ball on a beam which is allowed to move freely over the length of the beam with one degree of freedom. One end of the beam is fixed with a lever arm and the other side with a servo gear. By changing the servo gear with an angle (θ), the beam angle is altered by the lever with an angle (α). By altering the angle horizontally, the ball is rolled over on the beam due to gravity. Mathematically, the system comprises of the dynamic prototype of the system and the dynamics of DC servomotor in which the ball is positioned on a beam. The equations from are described as equations of motion which describe the system of the ball placed on a beam are presented by Manan Khan *et al*. (2012) and Almutairi and Zribi (2010) (Fig. 2):

$$(mr^2+k_1)\ddot{\alpha}+(2mr\dot{r}+k_2)\dot{\alpha}+(mgr+\frac{L}{2}Mg)\cos\alpha=u \quad (1)$$
$$k_4\ddot{r}-r\dot{\alpha}^2+g\sin\alpha=0 \quad (1)$$

where,
α (t): Angle of the beam
r (t) : Position of the ball
θ (t): Angle of servo gear
g : Gravitational constant
m : Mass occupied by ball
M : Mass occupied by beam
L : Beam length

The system's parameters are:
R_m : Resistance of an armature of the motor
J_m : Operative inertial moment
K_m : Torque constant of motor
K_g : Ratio of gear
d : Level arm offset
K_b : Back EMF constant
J_1 : Moment of inertia of the beam

The factors k_1, k_2, k_3 and k_4 are the functions of the system parameter as below:

$$k_1=\frac{R_mJ_m}{K_mK_g}\frac{L}{d}+\mathrm{J}_1,\ k_2=\frac{L}{d}\left(\frac{K_mK_b}{R_m}+K_b+\frac{R_mB_m}{K_mK_g}\right),$$
$$k_3=1+\frac{K_m}{R_m},\ k_4=7/5$$

v_{in} : Voltage given as input of motor
u (t) = K_3v_{in} (t) : Control input to the ball on a beam system

Modeling of the system by utilizing the simplified prototype: This section presents the designing of dynamic mode controllers and the stationary for the system of a ball on the beam by utilizing the simplified method (i.e., neglect the term $r\dot{\alpha}^2$). The Eq. (1) and (2) can be revised as:

$$\ddot{\alpha}=\frac{1}{(mr^2+k_1)}\left[u-(2mr\dot{r}+k_2)\dot{\alpha}-\left(mgr+\frac{L}{2}Mg\right)\cos\alpha\right] \quad (2)$$

$$\ddot{r}=\frac{1}{k_4}\left(r\dot{\alpha}^2-g\sin\alpha\right) \quad (3)$$

State space model of the system: After the term $r\dot{\alpha}^2$ is ignored in Eq. (4), now the condition of the system can be presented as:

$$x_1=\alpha,\ x_2=\dot{\alpha},\ x_3=r \text{ and } x_4=\dot{r}$$

A model after the simplification of the system will be redrafted as:

Fig. 1: Real ball on a beam system

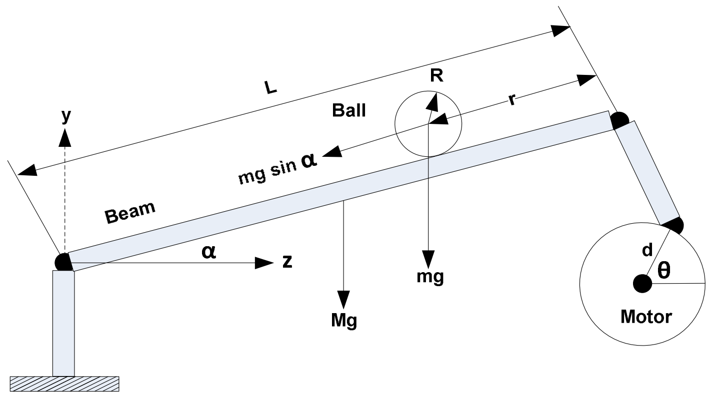

Fig. 2: Schematic diagram of the ball on a beam system

$$\left.\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{1}{(mx_3^{\ 2}+k_1)}\begin{bmatrix} u-(2mx_3x_4+k_2)x_2 - \\ (mgx_3+\frac{L}{2}Mg)\cos x_1 \end{bmatrix} \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= -\frac{g}{k_4}\sin x_1 \end{aligned}\right\} \tag{4}$$

And suppose that the system's output is presented as:

$$y = x_3 \tag{5}$$

Linearization of the system model: Jacobian method has been used to get the linear model around the origin $X_e = [0\ 0\ 0\ 0]^T$, which is the stable point of the system. So the linear model will be as the following:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -{k_2}/{k_1} & -{mg}/{k_1} & 0 \\ 0 & 0 & 0 & 1 \\ -{g}/{k_4} & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ {1}/{k_1} \\ 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$$

After the stability is checked around this point (local stability), the system is found locally unstable then globally will be unstable.

DESIGN OF A FSSMC AND FDSMC BY UTILIZING THE SIMPLE MODEL

The general design procedure of FSSMC and FDSMC is discussed in this section. The basic block diagram for a ball on a beam with FSSMC and FDSMC are illustrated in Fig. 3. These proposed controllers are approximates the discontinuous control and minimizes the chattering problem of SMC. And they are an extension of an SMC with the boundary layer as shown in Fig. 4. This extension will guarantee asymptotic stability of fuzzy controllers, which they lack in them.

Fuzzy logic controller: The system has a Single-input-Single-output. The linguistic variables for the components of input membership functions s_i are developed in Table 1. The output membership function is a singleton function, whose fuzzy sets are described on the normalized universe of discourse as:

$$\mathrm{u} = \mathrm{u_e} \pm k$$

where, u_e is an equivalent control derived using equivalent control technique and k is the gain of the control, the output and input membership functions are revealed in Fig. 5. The singleton membership function is given as:

$$\mu_H(u) = \begin{cases} 1, & for\ x = u_i \\ 0, & otherwise \end{cases}$$

where, u_i is an element of a universe of discourse. The general fuzzy rule for the FSSMC and FDSMC is defined as:

$$R^i : if\ s_i\ is\ F^n_{s_i}\ then\ u_i\ is\ F^n_{u_i}$$

where, $F^n_{s_i}$ and $F^n_{u_i}$ are fuzzy sets. The compositional rule of inference, which determines the impact of the antecedent measure of the fuzzy rule on ensuing part of the fuzzy rule, is described as:

$$\mu_{s_i} \circ R^i(u_i) = \sup_{x\in s_i}\left[\min(\mu F^i_x(s_i)), \min(\mu F^i_x(s_i), \mu_{u_i}(u_i))\right] \tag{6}$$

The crisp output value is taken out from the fuzzy output via the center of an area defuzzification principle known as:

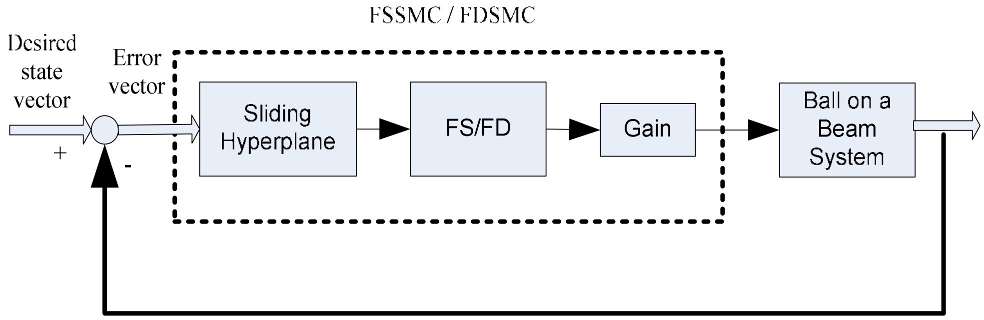

Fig. 3: The block diagram of FSSMC/FDSMC for a ball on a beam system

Table 1: Table of the fuzzy rules

s_i	PB	PM	Z	NM	NB
u_i	Smaller	Small	Medium	Big	Bigger

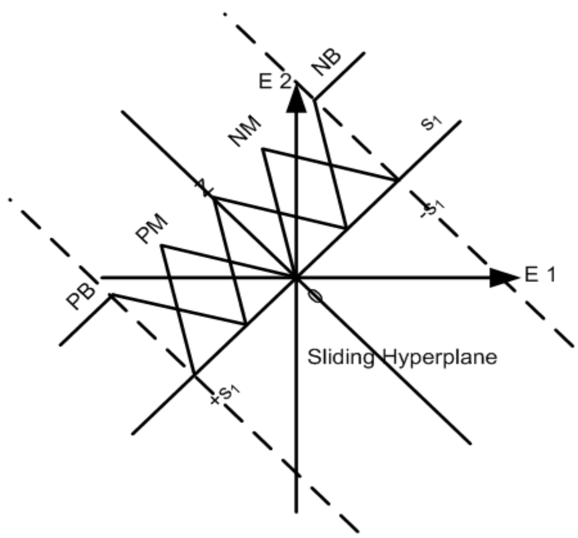

Fig. 4: The main idea of FSSMC and FDSMC schematic diagram

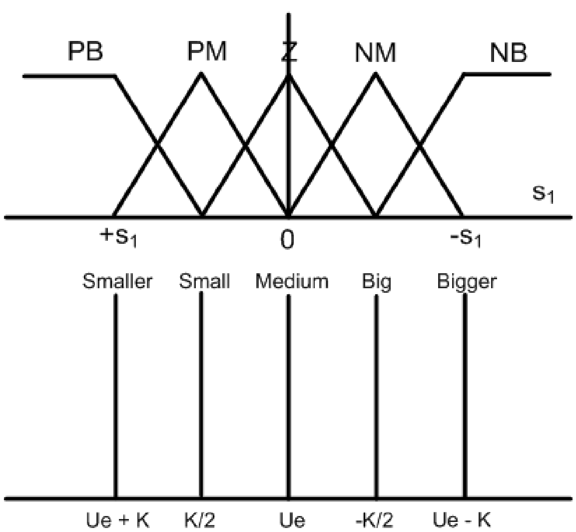

Fig. 5: Membership functions of the fuzzy rules

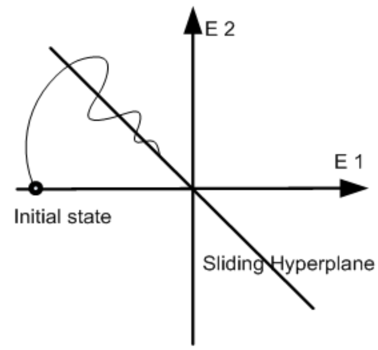

Fig. 6: State trajectory of sliding mode control

$$u_{fn}(s_i, u_i) = \frac{\sum_i u_{fn}\mu(s_i, u_i, u_{fn})}{\sum_i \mu(s_i, u_i, u_{fn})} \quad (7)$$

The outcome of the defuzzified output for fuzzy inputs has the form:

$$u_{fn} = K_f s(x) = FSSMC / FDSMC(s(x)) \quad (8)$$

Fuzzy static sliding mode controller: The SMC theory (Chen and Balance, 2002; Li *et al*., 2003) utilizes uneven control action to stimulate state trajectories toward a particular hyperplane in the state space and to preserve the state trajectories sliding on the particular hyperplane until stable equilibrium state is reached (Fig. 6). This standard offers assistance to design a FSSMC. The first controller with a sliding mode for a system of a ball on a beam will be designed by utilizing the basic prototype of the system which consists of a ball on a beam in Eq. (5). This controller aims to adjust the position of r to the required constant value defined as r_d while causing α to reach to its stability value $\alpha_e = 0$.

For FSSMC let the regulation error to be:

$$e_r = e - r_d \quad (9)$$

Then, a slidcharacterizede can be characterized as:

$$s_1 = x_2 + b_1 x_1 + b_2 x_4 + b_3 (x_3 - r_d) \quad (10)$$

where, b_1, b_2 and b_3 are the sliding hyperplane parameters. In order to design the control input u (t) so that the state trajectories are determined and enticed toward the sliding hyperplane and then keep sliding on it, the following inequality must be satisfied:

$$s_1 \dot{s}_1 \langle \ 0 \quad (11)$$

So that implies:

$$s_1 = -k_1 s_1 - k_2 sign(s_1) \quad (12)$$

where, k_1 and k_2 constants chosen strictly positive. Asymptotically causes the system's output to become stable $y = x_3 = r$ to its required value r_d and it results in stabilizing all the other states of the system. The simulation is carried out to form the controller and the surface into the MATLAB to form s-function and after that, a SIMULINK is used to achieve the required results and it is supposed that α, $\dot{\alpha}$ and $\dot{r}$ are reaching to zero while $t \rightarrow \infty$ otherwise $r \rightarrow r_d$. The designed controller encounters the undesirable and unwanted issue of chattering which is also faced by all the other variable-structure controllers. A schematic of the dynamic sliding-mode controller is suggested which resolves the issue of chattering as follows.

Fuzzy dynamic sliding mode controller: For FDSMC defining the error sliding hyperplane as:

$$s_2 = \dot{x}_2 + a_1 x_2 + a_2 x_1 + a_3 x_4 + a_4 (x_3 - r_d) \quad (13)$$

Then the resultant is:

$$\dot{s}_2 = -k_3 s_2 - k_4 sign(s_2) \quad (14)$$

This is guaranteed that:

$$s_2 \dot{s}_2 \langle \ 0 \quad (15)$$

Hence, it is verified that the controller with a FDSMC ensures the asymptotic convergence of all the conditions of the system to their required values. This causes a condition of the system to become asymptotically stabile to their required states after the application of the system of a ball on a beam.

Design of a FSSMC and FDSMC by utilizing the comprehensive model: A FDSMC and a FSSMC for the system of a ball on a beam, by utilizing the precise model of the system, are presented without ignoring the part $r\dot{\alpha}^2$. Similar to design of FSSMC and FDSMC by utilizing simplified model, the error sliding hyperplane and the sliding hyperplane can be defined. As noticed previously from the earlier discussed section the proposed static controller was encountering the problem of noise in the control signal, which is practically not desirable. So these types of controllers with comprehensive model have been presented in this portion to resolve this issue. So that it causes the states of the system to become steady asymptotically up to the required values, once implemented in the system of a ball on a beam.

SIMULATION RESULTS

The controllers will be designed by using MATLAB/SIMULINK software. Parameters of the system are extracted from the Quanser ball on a beam apparatus (Motion, Rotary and Servo Plant, Year). In the Table 2, the detailed values of the parameters of the system in which the ball is placed on a beam are presented for simulation purpose.

Figure 7 to 9 shows the simulation results of ball position r_d, the output of the system controller and the angle of the beam α output with respect to time respectively, when simplified model has been used. It is clear that the issue of chattering has been resolved up to a greater extent and the desired output ball position is being achieved faster when FDSMC have been used in comprehensive models compare to FSSMC and conventional SMC.

From Fig. 10 to 12, it can be concluded from the above observations that ball was positioned at the positive responses in all the circumstances are satisfactory and the problem of chattering has been overcome in the case of using the FDSMC in a complete model. Also, the system is healthy for the coordinated disturbances under the design control law

Table 2: Values of the parameters of the ball on a beam system

Parameter	Value
m	$0.064 kg$
g	$9.81 m/s^2$
L	$0.43 m$
M	$0.15 kg$
R_m	9Ω
J_m	$7.35*10^{-4} \frac{Nm}{rad/s^2}$
K_m	$0.0075 Nm/A$
K_g	75
d	$0.3 m$
J_1	$0.001 kgm^2$
K_b	$0.5625 \frac{V}{rad/s}$

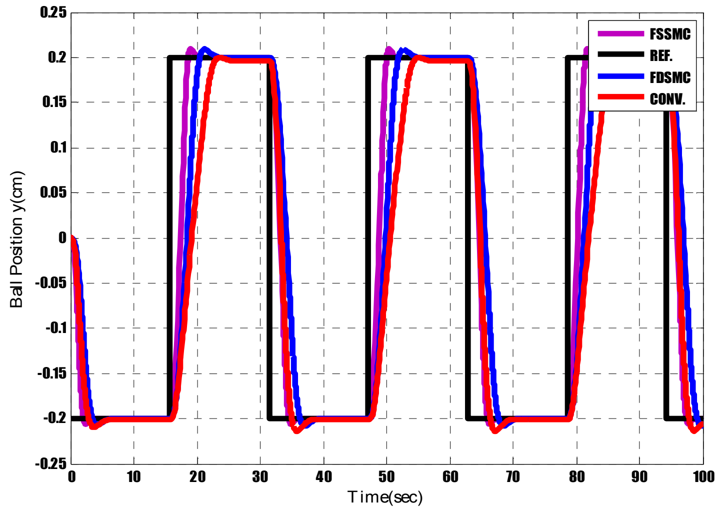

Fig. 7: The response of the system for ball position in a simplified model

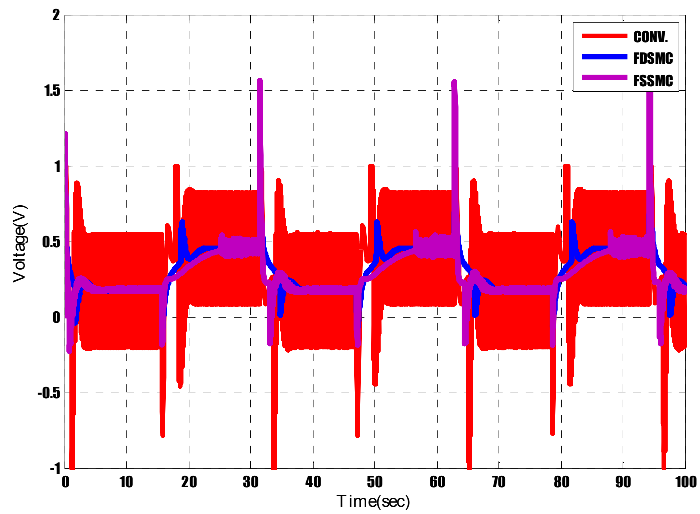

Fig. 8: The output of the system controller in a simplified model

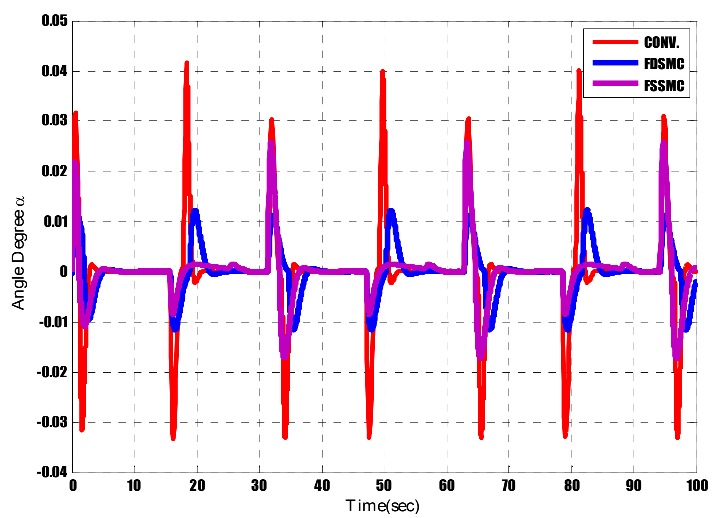

Fig. 9: The angle of the beam output with respect to time in a simplified model

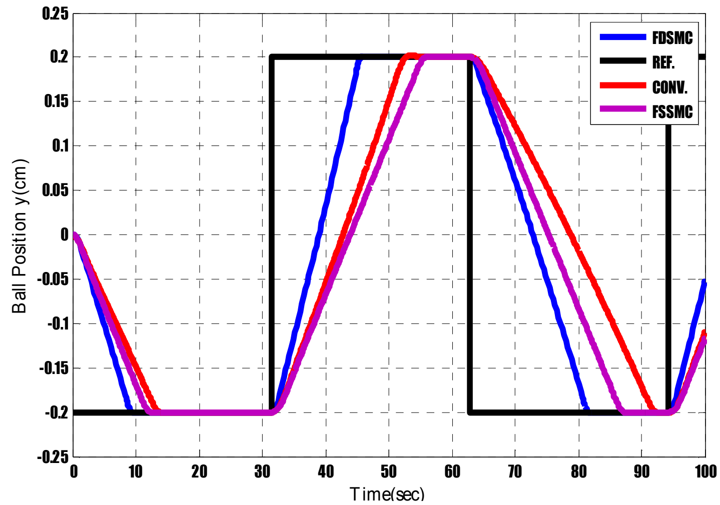

Fig. 10: The response of the system for ball position in a comprehensive model

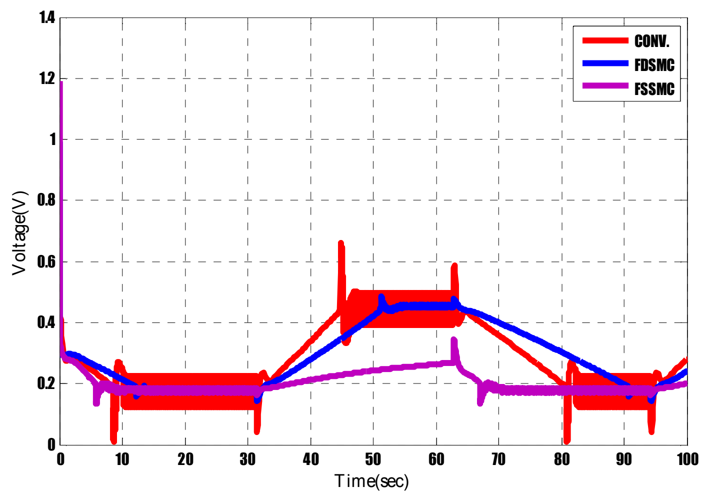

Fig. 11: The output of the system controller in a comprehensive model

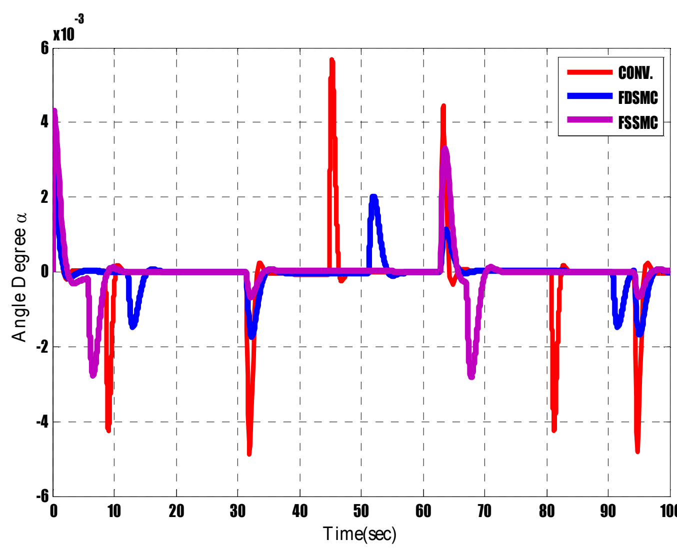

Fig. 12: The angle of the beam output with respect to time in a comprehensive model

and the desired reference is tracked by the ball position well.

CONCLUSION

This study presents a method to create a schematic of a Fuzzy Static (FS) and Fuzzy Dynamic (FD) Sliding-Mode Controllers (SMC) for both a complete and a simplified system of a ball on the beam. Firstly, a FSSMC is designed on the simplified and the complete and model. Then, a FDSMC is designed for the complete and the simplified Ball on a Beam system as well. In addition, the lyapunov stability and linearization have been used to check the system's stability. The result drawn from this research study is that the design which uses the FDSMC in comprehensive prototype of the system is more efficient than the ones that were designed by utilizing the basic conventional prototype of the system. Simulations are shown using MATLAB/SIMULINK program.

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REFERENCES

Almutairi, N.B. and M. Zribi, 2010. On the sliding mode control of a ball on a beam system. Nonlinear Dynam., 59(1-2): 221-238.

Chang, Y.H., C.W. Chang, C.W. Tao, H.W. Lin and J.S. Taur, 2012. Fuzzy sliding-mode control for ball and beam system with fuzzy ant colony optimization. Expert Syst. Appl., 39(3): 3624-3633.

Chen, W.H. and D.J. Ballance, 2002. On a switching control scheme for nonlinear systems with ill-defined relative degree. Syst. Control Lett., 47(2): 159-166.

Hung, L.C. and H.Y. Chung, 2006. Decoupling adaptive fuzzy sliding-mode control with rule reduction for nonlinear system. Proceeding of IEEE Conference on Cybernetics and Intelligent Systems, pp: 1-5.

Jaradat, M.A., M.I. Awad and B.S. El-Khasawneh, 2012. Genetic-fuzzy sliding mode controller for a DC servomotor system. Proceeding of the 8th International Symposium on Mechatronics and its Applications, pp: 1-6.

Li, J.H., T.H. Ou and T.H.S Li, 2003. Design and implementation of fuzzy sliding-mode controller for a wedge balancing system. J. Intell. Robot. Syst., 37(3): 285-306.

Magzoub, M.A., N.B. Saad and R.B. Ibrahim, 2013. An intelligent speed controller for indirect field-oriented controlled induction motor drives. Proceeding of IEEE Conference on Clean Energy and Technology (CEAT, 2013), pp: 327-331.

Manan Khan, A., A. Iqbal Bhatti, Sami-ud-Din and Q. Khan, 2012. Static & dynamic sliding mode control of ball and beam system. Proceeding of 9th International Bhurban Conference on Applied Sciences and Technology (IBCAST, 2012), pp: 32-36.

Marton, L. and B. Lantos, 2006. Stable adaptive ball and beam control. Proceeding of IEEE 3rd International Conference on Mechatronics, pp: 507-512.

Motion, Rotary and Servo Plant, Year. SRV02 Ball and Beam User Manual.

Nagarale, B.M. and R.M. Patre, 2013a. Decoupled neural fuzzy sliding mode control of nonlinear systems. Poceeding of IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), pp: 1-8.

Nagarale, R.M. and B.M. Patre, 2013b. Exponential function based decoupled fuzzy sliding mode control of nonlinear systems. Poceeding of Annual IEEE India Conference (INDICON), pp: 1-6.

Tomlin, C.J. and S.S. Sastry, 1997. Switching through singularities. Proceeding of the 36th Conference on Decision and Control, pp: 1-6.