

Research Article

Convexity-preserving using Rational Cubic Spline Interpolation

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Abstract: This study is a continuation of our previous paper. The rational cubic spline with three parameters has been used to preserve the convexity of the data. The sufficient condition for rational interpolant to be convex on entire subinterval will be developed. The constraint will be on one of the parameter with data dependent meanwhile the other are free parameters and will determine the final shape of the convex curves. Several numerical results will be presented to test the capability of the proposed rational interpolant scheme. Comparisons with the existing scheme also have been done. From all numerical results, the new rational cubic spline interpolant gives satisfactory results.

Keywords: Convexity-preserving, parameters, rational cubic spline, sufficient condition

INTRODUCTION

Shape preserving interpolation is an important task when the user are require to interpolate all finite sets of data which preserves the original geometric characteristics of the data. For example, if the given data is convex, the interpolating curves must be able to produce the convex curves on each subinterval. There are three important geometric features of the data i.e., convexity, monotonicity and positivity. It is the first is of interest of this study. Convexity always happens in many disciplines and applications. For example in nonlinear programming which arises in finance and engineering based problem. Normally the cost function is convex or concave, thus the spline interpolant must be able to retain the convexity of the data.

Since the introduction of the paper by Dodd and Roulier (1983) and Roulier (1987) many researcher have discussed the convexity preserving by using various type of spline interpolant. The spline interpolant can be either polynomial or rational spline. Dougherty *et al.* (1989) have discussed the cubic and quintic polynomial spline for positivity, monotonicity and convexity preserving interpolation. Brodlie and Butt (1991) have study the convexity preserving by using cubic spline interpolation by inserting extra knots in which the convexity of the data is not preserves. Lam (1990) and Schumaker (1983) have used quadratic polynomial spline to preserves the convexity of the data. Their methods also require the introduction of extra knots. Sarfraz (2002) and Sarfraz *et al.* (2001) have discussed the convexity and positivity preserving

by using rational cubic spline (cubic/cubic). They derived the sufficient conditions for the rational interpolant to be convex and positive. Abbas *et al.* (2012) also discuss the convexity preserving by using rational cubic spline (cubic/quadratic) with three shape parameters. Hussain and Hussain (2007) have explored the use of rational cubic spline for preserving convexity of 3D data. Gregory (1986) has study the use of rational cubic spline (cubic/quadratic) with one shape parameter for convexity preserving with C^2 continuity. Hussain *et al.* (2011) have proposed shape preserving for positive and convex data by using rational cubic spline of the form cubic/quadratic with two shape parameters. Tian *et al.* (2005) have discussed the use of rational cubic spline (cubic/quadratic) with two shape parameters. Their rational function slightly is different from the works of Hussain *et al.* (2011). Hussain and Hussain (2008) have utilized rational cubic spline of Tian *et al.* (2005) for convexity problem for surfaces lying on rectangular grids. Delbourgo and Gregory (1985a) also study the use of rational cubic spline for monotone and convexity preserving. Sarfraz *et al.* (2013) have proposed new rational cubic spline of the form cubic/quadratic with two parameters. They derived the sufficient conditions for the rational interpolant to be positive, monotone and convex on entire given interval. Beside the use of rational cubic spline with cubic denominator or quadratic denominator, there exists another rational interpolant schemes that have linear denominator. For examples, Hussain *et al.* (2010) and Zhang *et al.* (2007) have study the convexity preserving by using rational cubic

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spline with linear denominator. Meanwhile Hussain *et al.* (2008) and Karim and Piah (2009) have preserves the convexity data by using rational quartic spline with linear denominator. Recently the convexity preserving by using trigonometric spline also have attract many researchers. Pan and Wang (2007) have used trigonometric spline polynomial with one shape parameter for convexity preserving. Dube and Tiwari (2012, 2013) also discussed the convexity preserving by using C^2 rational trigonometric spline. Zhu *et al.* (2012) preserves the convexity of planar data by using a new quartic trigonometric Bézier polynomial. Delbourgo (1989) also discussed the convexity preserving but with rational spline interpolant of the form quadratic numerator and linear denominator. Greiner (1991) provides a good survey on shape preserving methods. As indicated in the above abstract, this study is continuation of our previous paper in Karim and Kong (2013). The main different is that instead of applying the new rational cubic spline in positivity preserving (Karim and Kong, 2013), in this study the rational cubic spline will be used for convexity preserving of scalar data sets. Thus the main objective of this paper is to use rational cubic spline originally proposed by Karim and Kong (2013) for convexity preserving with C^1 continuity. Furthermore, we identify several features of our rational cubic spline for convexity preserving. It is summarized below:

- In this study the rational cubic spline (cubic/quadratic) with three parameters of Karim and Kong (2013) has been used for convexity preserving while in Sarfraz (2002), Abbas *et al.* (2012) and Hussain and Hussain (2007) the rational cubic (cubic/cubic) have been used for convexity preserving. Whereas, Hussain *et al.* (2011) and Sarfraz *et al.* (2013) the rational cubic spline (cubic/quadratic) with two parameters have been used instead.
- The rational cubic spline reproduces the rational cubic spline of Tian *et al.* (2005) when one the parameter is equal to zero, i.e., $\gamma_i = 0$ Indeed, when $\gamma_i = 0$ our convexity-preserving scheme reduced to the work of Tian *et al.* (2005).
- The degree smoothness attained in this paper is C^1 whereas in Gregory (1986) the degree smoothness attained is C^2 . By having C^2 continuity, some nonlinear equations need to be solved through some iteration methods.
- Numerical comparison between the proposed rational cubic schemes with Tian *et al.* (2005) and Fritch and Carlson (1980) (*pchip*) and Dougherty *et al.* (1989) also has been done. Furthermore no derivative modification require in our rational cubic scheme while to preserves the convexity of the data by using the cubic and quintic spline by using Dougherty *et al.* (1989) ideas, the first derivative must be modified when any shape violation are found.

- Our method likes the works of Sarfraz (2002), Abbas *et al.* (2012), Hussain and Hussain (2007, 2008), Hussain *et al.* (2011) and Sarfraz *et al.* (2013) etc., do not require the inserting knots. Lam (1990) and Schumaker (1983) requires an extra knots in the interval in which the convexity of the data is not preserves. Even though their methods also works well for the tested data, by inserting extra knot, the computation to generate the curves or surfaces will be increased.

METHODOLOGY

Rational cubic spline interpolant: This section will introduce a new rational cubic spline interpolant with three parameters originally proposed by Karim and Kong (2013). Suppose $\{(x_i, f_i), i = 0, 1, \dots, n\}$ is a given set of data points, where $x_0 < x_1 < \dots < x_n$. Let $h_i = x_{i+1} - x_i$,

$$\Delta_i = \frac{(f_{i+1} - f_i)}{h_i} \text{ and } \theta = \frac{(x - x_i)}{h_i} \text{ where } 0 \leq \theta \leq 1.$$

For,

$$x \in [x_i, x_{i+1}], i = 0, 1, 2, \dots, n-1$$

$$s(x) \equiv s_i(x) = \frac{P_i(\theta)}{Q_i(\theta)}, \tag{1}$$

where,

$$P_i(\theta) = A_0(1-\theta)^3 + A_1\theta(1-\theta)^2 + A_2\theta^2(1-\theta) + A_3\theta^3,$$

$$Q_i(\theta) = (1-\theta)^2\alpha_i + \theta(1-\theta)(2\alpha_i\beta_i + \gamma_i) + \theta^2\beta_i.$$

with,

$$A_0 = \alpha_i f_i,$$

$$A_1 = (2\alpha_i\beta_i + \alpha_i + \gamma_i) f_i + \alpha_i h_i d_i,$$

$$A_2 = (2\alpha_i\beta_i + \beta_i + \gamma_i) f_{i+1} - \beta_i h_i d_{i+1},$$

$$A_3 = \beta_i f_{i+1}$$

The rational cubic spline in (1) satisfies the following C^1 conditions:

$$s(x_i) = f_i, \quad s(x_{i+1}) = f_{i+1},$$

$$s_i^{(1)}(x_i) = d_i, \quad s_i^{(1)}(x_{i+1}) = d_{i+1}. \tag{2}$$

where, $s_i^{(1)}(x)$ denotes derivative with respect to x and d_i denotes the derivative value which is given at the k not $x_i, i = 0, 1, 2, \dots, n$. The parameters $\alpha_i, \beta_i, \gamma_i > 0$. The data dependent sufficient conditions on the parameters α_i and β_i will be developed in order to preserves the positivity on the entire interval $[x_i, x_{i+1}], i = 0, 1, 2, \dots, n-1$. When $\alpha_i = \beta_i = 1, \gamma_i = 0$ rational cubic interpolant in (1) is just a standard C^1 cubic Hermite spline given as follow:

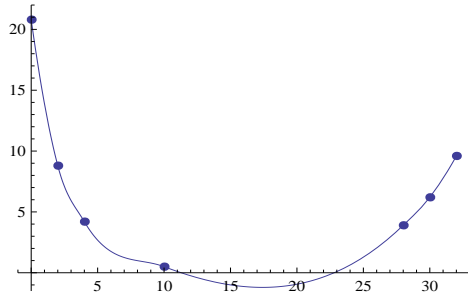


Fig. 1: Default cubic spline polynomial ($\alpha_i = \beta_i = 1, \gamma_i = 0$) for data in Table 1

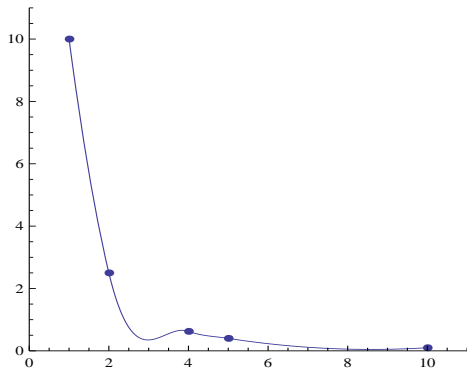


Fig. 2: Default cubic spline polynomial ($\alpha_i = \beta_i = 1, \gamma_i = 0$) for data in Table 2

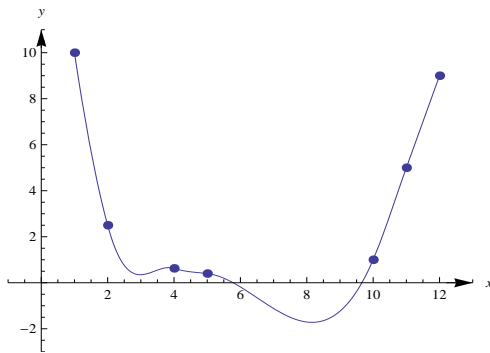


Fig. 3: Default cubic spline polynomial ($\alpha_i = \beta_i = 1, \gamma_i = 0$) for data in Table 3

$$s(x) = (1 - \theta)^2 (1 + 2\theta) f_i + \theta^2 (3 - 2\theta) f_{i+1} + \theta (1 - \theta)^2 d_i - \theta^2 (1 - \theta) d_{i+1} \quad (3)$$

The shape parameters α_i and β_i $i = 0, 1, 2, \dots, n - 1$ are free parameter (independent) while the positivity constrained will be derived from the other parameter γ_i (dependent). The two parameters α_i, β_i can be used to refined and modify the final shape of the interpolating curve. Some observations including shape control analysis were discussed in details by Karim and Kong (2013).

Determination of derivatives: Normally, when the scalar data are being interpolated, the first derivative

parameters d_i must be estimated by using some mathematical formulation. In this study, the Arithmetic Mean Method (AMM) has been used to estimate the first derivative for positivity preserving. The formula for AMM is given as follows (Delbourgo and Gregory, 1985b; Sarfraz *et al.*, 1997).

The first derivatives for $i = 0$ and $i = n$ are given by:

$$d_0 = \Delta_0 + (\Delta_0 - \Delta_1) \left(\frac{h_0}{h_0 + h_1} \right), \quad (4)$$

$$d_n = \Delta_{n-1} + (\Delta_{n-1} - \Delta_{n-2}) \left(\frac{h_{n-1}}{h_{n-1} + h_{n-2}} \right). \quad (5)$$

For, $i = 1, 2, \dots, n - 1$, the values of d_i are given as:

$$d_i = \frac{h_{i-1} \Delta_i + h_i \Delta_{i-1}}{h_{i-1} + h_i}. \quad (6)$$

Convexity-preserving using rational cubic spline interpolant: The rational cubic spline interpolant (cubic/quadratic) originally proposed by Karim and Kong (2013) in Section above cannot preserves the convexity of the convex data. There might be some part of the interpolating curves are not convex. In general, the cubic spline interpolation will be unable to produce the convex interpolating curves for convex data sets. This fact can be observed from Fig. 1 to 3 respectively. As proposed by many authors for instance Sarfraz (2002), Abbas *et al.* (2012) and Tian *et al.* (2005) and others, the automated choices of parameters that will ensure the convexity of the data must be find. This can be achieved by developed the sufficient conditions for convexity of the rational interpolation defined by Eq. (1). This condition is data dependent and it will be derived on one parameter γ_i while the other two free parameters α_i and β_i can be used for the refine the interpolating convex function. We begin the derivation of sufficient condition for convexity by giving the definition of the convex data sets. It started as follows.

Given a strictly convex data set $(x_i, f_i), i = 0, 1, \dots, n$ so that $\Delta_0 < \Delta_1 < \dots < \Delta_{i-1} < \Delta_i < \dots < \Delta_{n-1}, x_0 < x_1 < \dots < x_n$, and:

$$d_0 < \Delta_0 < d_1 < \dots < \Delta_{i-1} < d_i < \dots < \Delta_{n-1} < d_n, \quad (7)$$

Equation (7) is equivalent to:

$$d_i < \Delta_i < d_{i+1}. \quad (8)$$

(work on concavity preserving also can be treated in the same manner).

Now, rational cubic interpolant $s(x)$ is convex if and only if $s^{(2)}(x) \geq 0, x \in [x_i, x_{i+1}], i = 0, 1, \dots, n - 1$. Now, the first and second-order derivative of rational interpolant $s(x)$ defined by Eq. (1) are given as follows:

$$s^{(1)}(x) = \frac{\sum_{j=0}^4 B_{ij} (1-\theta)^{4-j} \theta^j}{[Q_i(\theta)]^2} \tag{9}$$

where,

$$B_{i0} = \alpha_i^2 d_i, B_{i1} = 2\alpha_i \left((1+2\alpha_i)\Delta_i - d_{i+1} \right) \beta_i + \gamma_i \Delta_i, \\ B_{i3} = 2\beta_i \left((1+2\beta_i)\Delta_i - d_i \right) \alpha_i + \gamma_i \Delta_i, B_{i4} = \beta_i^2 d_{i+1}$$

and

$$B_{i2} = 2\alpha_i^2 \beta_i \left((1+2\beta_i)\Delta_i - d_i \right) + \gamma_i \left(\gamma_i \Delta_i - \beta_i d_{i+1} + \beta_i \Delta_i \right) + \alpha_i \left(4\beta_i \left(1 + \gamma_i \right) \Delta_i + \gamma_i \Delta_i - \gamma_i d_i - \beta_i d_{i+1} - \beta_i d_i + 2\beta_i^2 \left(\Delta_i - d_{i+1} \right) \right).$$

$$s^{(2)}(x) = \frac{\sum_{j=0}^3 C_{ij} (1-\theta)^{3-j} \theta^j}{h_i [Q_i(\theta)]^3} \tag{10}$$

where,

$$C_{i0} = 2\alpha_i^2 \left(\gamma_i (\Delta_i - d_i) - \beta_i (d_{i+1} - \Delta_i) - 2\alpha_i \beta_i (d_i - \Delta_i) \right), \\ C_{i1} = 6\alpha_i^2 \beta_i (\Delta_i - d_i), C_{i2} = 6\alpha_i \beta_i^2 (d_{i+1} - \Delta_i), \\ C_{i3} = 2\beta_i^2 \left[\gamma_i (d_{i+1} - \Delta_i) - \alpha_i (\Delta_i - d_i - 2\beta_i (d_{i+1} - \Delta_i)) \right].$$

Now, if $s^{(2)}(x) \geq 0$ then the rational interpolant $s(x)$ will be convex for all $x \in [x_0, x_n]$. For $x \in [x_i, x_{i+1}]$, with $\alpha_i, \beta_i > 0$ and $\gamma_i \geq 0$, the denominator in (10) is always positive i.e. $[Q_i(\theta)]^3 > 0, i = 0, 1, \dots, n-1$. Thus the sufficient condition for convexity is depend to the positivity of numerator in Eq. (10). From calculus, $s^{(2)}(x)$ is non-negative if and only if $C_{ij} > 0, j = 0, 1, 2, 3$. The necessary conditions for convexity are $\Delta_i - d_i \geq 0, d_{i+1} - \Delta_i \geq 0$. The sufficient condition for convexity of rational interpolant in (1) can be determined from the condition: $C_{ij} > 0, j = 0, 1, 2, 3$. It is obvious that $C_{i1} > 0, C_{i2} > 0$. For strictly convex data and from $C_{i0} > 0, C_{i3} > 0$ will gives us the following conditions:

$$C_{i0} > 0 \text{ if} \\ 2\alpha_i^2 \left(\gamma_i (\Delta_i - d_i) - \beta_i (d_{i+1} - \Delta_i) - 2\alpha_i \beta_i (d_i - \Delta_i) \right) > 0, \tag{11}$$

and

$$C_{i3} > 0 \text{ if}$$

$$2\beta_i^2 \left[\gamma_i (d_{i+1} - \Delta_i) - \alpha_i (\Delta_i - d_i - 2\beta_i (d_{i+1} - \Delta_i)) \right] > 0. \tag{12}$$

Equation (11) and (12) provides the following conditions given in Eq. (13) and (14) respectively:

$$\gamma_i > \beta_i \left(\frac{d_{i+1} - \Delta_i}{\Delta_i - d_i} - 2\alpha_i \right), \tag{13}$$

$$\gamma_i > \alpha_i \left(\frac{\Delta_i - d_i}{d_{i+1} - \Delta_i} - 2\beta_i \right). \tag{14}$$

Thus $s^{(2)}(x) > 0$ if Eq. (13) and (14) holds. The following Theorem gives the sufficient condition for convexity preserving by using rational cubic spline (cubic/quadratic) interpolant.

Theorem 1: Given a strictly convex data satisfying (7) or (8), there exists a class of convex rational (of the form cubic/quadratic) interpolating spline $s(x) \in C^1[x_0, x_n]$ involving parameters α_i, β_i and γ_i provided that if it satisfy the following convexity sufficient conditions:

$$\alpha_i > 0, \beta_i > 0, \\ \gamma_i > \text{Max} \left\{ 0, \beta_i \left(\frac{d_{i+1} - \Delta_i}{\Delta_i - d_i} - 2\alpha_i \right), \alpha_i \left(\frac{\Delta_i - d_i}{d_{i+1} - \Delta_i} - 2\beta_i \right) \right\}, i = 0, 1, \dots, n-1. \tag{15}$$

Remark 1: If the data gives $\Delta_i - d_i = 0$ or $d_{i+1} - \Delta_i = 0$, then we may set $d_i = d_{i+1} = \Delta_i$. This will results the rational interpolant in (1) reproduce the linear segment in that interval:

$$s(x) = (1-\theta) f_i + \theta f_{i+1}. \tag{16}$$

Remark 2: The sufficient condition for convexity preserving in (17) can be rewritten as:

$$\gamma_i = \eta_i + \text{Max} \left\{ 0, \beta_i \left(\frac{d_{i+1} - \Delta_i}{\Delta_i - d_i} - 2\alpha_i \right), \alpha_i \left(\frac{\Delta_i - d_i}{d_{i+1} - \Delta_i} - 2\beta_i \right) \right\}. \tag{17}$$

where $\eta_i > 0$.

The following is an algorithm that can be used to generate C^1 convexity-preserving curves using the results in Theorem 1. It is given as follows.

Algorithm for convexity-preserving:

Input: Data points (x_i, f_i) , d_i , shape parameters $\alpha_i > 0, \beta_i > 0$.

Output: γ_i and piecewise convexity interpolating curves:

Table 1: A convex data from Brodlie and Butt (1991)

x_i	0	2	4	10	28	30	32
f_i	20.80	8.80	4.2000	0.5000	3.9000	6.200	9.600
d_i	-7.85	-4.15	-1.8792	-0.4153	1.0539	1.425	1.975

Table 2: A convex data from Sarfraz (2002)

x_i	1	2	4	5	10
f_i	10	2.5000	0.6250	0.4000	0.10000
d_i	-9.6875	-5.3125	-0.4625	-0.1975	-0.00775

Table 3: A convex data from Tian (2010)

x_i	1	2	4	5	10	11	12
f_i	10	2.5000	0.6250	0.4000	1	5	9
d_i	-9.6875	-5.3125	-0.4625	-0.1675	3.3533	4	4

1. For $i = 0, 1, \dots, n$, input data points (x_i, f_i)
2. For $i = 0, 1, \dots, n$, estimate d_i using Arithmetic Mean Method (AMM)
3. For $i = 0, 1, \dots, n - 1$
 - Calculate h_i and Δ_i
 - Choose any suitable values of $\alpha_i > 0, \beta_i > 0$
 - Calculate the shape parameter γ_i using (19) with suitable choices of $\eta_i > 0$
 - Calculate the inner control ordinates A_1 and A_2
4. For $i = 0, 1, \dots, n - 1$

Construct the piecewise convexity interpolating curves using (1).

Remark 3: When $\gamma_i = 0$, the convexity preserving is corresponding to the work by Tian *et al.* (2005). For the purpose of numerical comparison later, we cite the sufficient condition for convexity preserving using rational cubic spline with two parameters originally proposed by Tian *et al.* (2005).

Theorem 2 (Tian *et al.*, 2005): The piecewise rational cubic function in (1) with $\gamma_i = 0$ preserves the convexity of the data if in subinterval, the free parameters α_i, β_i satisfies the following sufficient conditions:

$$\alpha_i = \beta_i = \text{Max} \left\{ 0, \frac{\Delta_i - d_i}{2(d_{i+1} - \Delta_i)}, \frac{d_{i+1} - \Delta_i}{2(\Delta_i - d_{i+1})} \right\}. \quad (18)$$

The conditions in (18) can be obtained by rearrange Eq. (13) and (14) with $\gamma_i = 0$. In the original work of Tian *et al.* (2005), there is no free parameter (s) to refine the final shape of the convex curve; meanwhile in this study, there exists two free parameters α_i and β_i that can be used to changes the shape of convex curves. Section below gives the comparison between our proposed convexity-preserving by using rational cubic spline with the works of Tian *et al.* (2005) and Dougherty *et al.* (1989) respectively.

RESULTS AND DISCUSSION

In order to illustrate the convexity-preserving interpolation by using the proposed rational cubic spline interpolation (cubic/quadratic), three sets of convex data were taken from Brodlie and Butt (1991), Sarfraz (2002) and Tian (2010).

Figure 1 to 3 shows the default cubic spline interpolation for data in Table 1 to 3 respectively. Figure 4 to 6 shows the convexity preserving using (Tian *et al.*, 2005; Hussain and Hussain, 2007) for all tested data sets. Figure 7 to 9 show the convexity preserving using our rational cubic spline for data in Table 1. Figure 10 to 12 show convexity preserving for

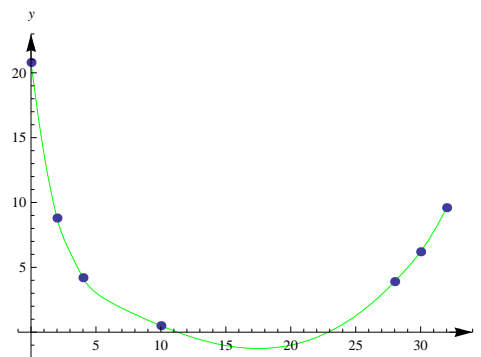


Fig. 4: Shape preserving using Tian *et al.* (2005) for data in Table 1

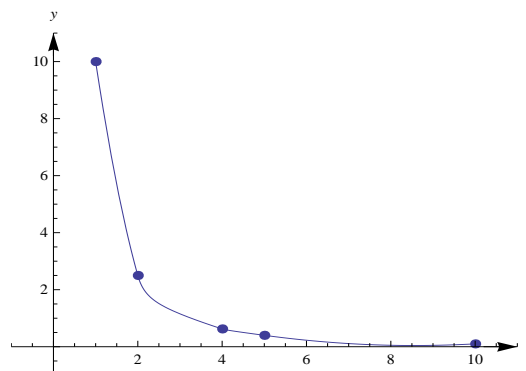


Fig. 5: Shape preserving using Tian *et al.* (2005) for data in Table 2

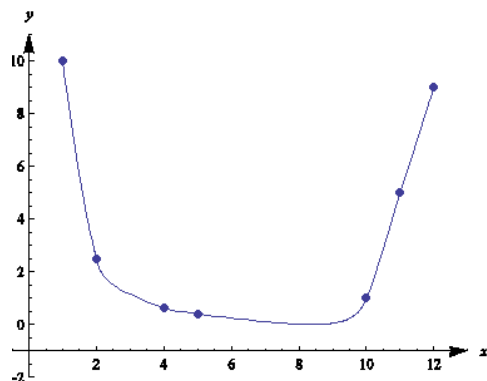


Fig. 6: Shape preserving using Tian *et al.* (2005) for data in Table 3

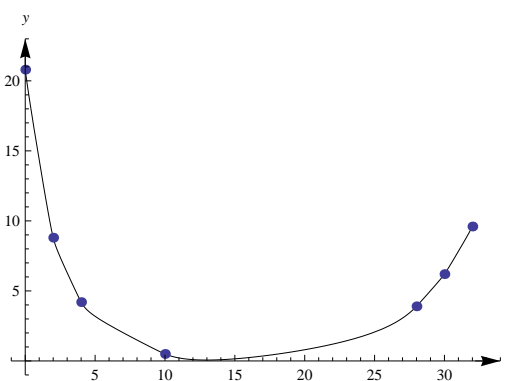


Fig. 9: Shape preserving using our proposed rational spline with $(\alpha_i = \beta_i = 0.05, \eta_i = 0.25)$ for data in Table 1

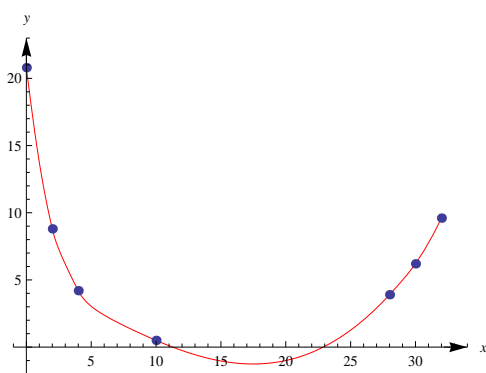


Fig. 7: Shape preserving using our proposed rational spline with $(\alpha_i = \beta_i = 0.5, \eta_i = 0.25)$ for data in Table 1

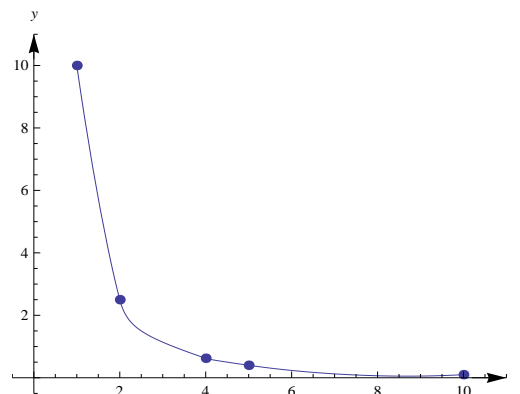


Fig. 10: Shape preserving using our proposed rational spline with $(\alpha_i = \beta_i = 1, \eta_i = 0.25)$ for data in Table 2

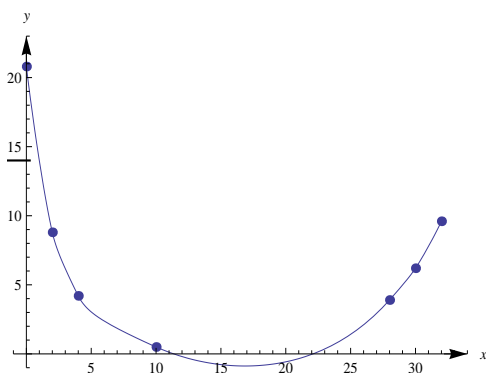


Fig. 8: Shape preserving using our proposed rational spline with $(\alpha_i = \beta_i = 1, \eta_i = 0.25)$ for data in Table 1

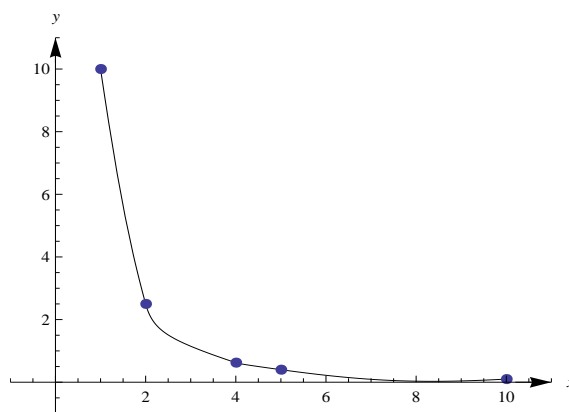


Fig. 11: Shape preserving using our proposed rational spline with $(\alpha_i = \beta_i = 0.5, \eta_i = 0.25)$ for data in Table 2

data in Table 2. Figure 13 to 15 show the convexity preserving using our rational scheme for convex data in Table 3. Figure 16, 17 and 18 show the comparison between our rational scheme and Tian *et al.* (2005) for data in Table 1 to 3 respectively. Figure 19 to 21 show the convexity preserving by using cubic spline interpolation from Dougherty *et al.* (1989). It is implemented through *pchip* function in MATLAB.

Through careful inspection, there are some parts of convex curves by using Dougherty *et al.* (1989)

methods that the curves tend to overshoot and not smooth and not very visual pleasing. For example, from Fig. 21, in the interval (5, 11), the 4th and 5th curve segments are not smooth and not visual pleasing for computer graphics visualization. Whereas from Fig. 14 and 15, the convex curves is smooth and very visual pleasing. One of the main differences between our rational cubic spline with three parameters with the

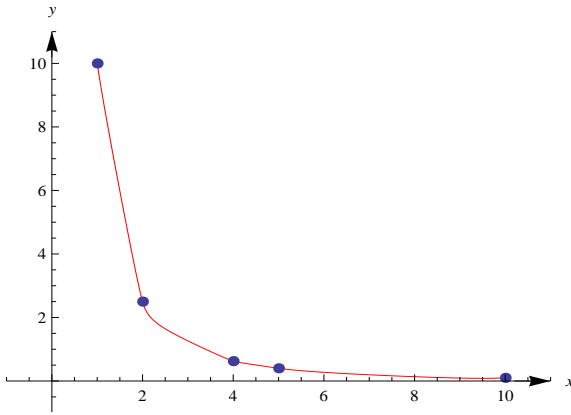


Fig. 12: Shape preserving using our proposed rational spline with $(\alpha_i = \beta_i = 0.05, \eta_i = 0.25)$ for data in Table 2

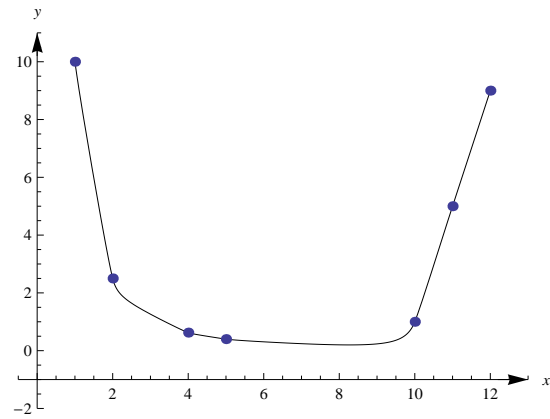


Fig. 15: Shape preserving using our proposed rational spline with $(\alpha_i = \beta_i = 0.05, \eta_i = 0.25)$ for data in Table 3

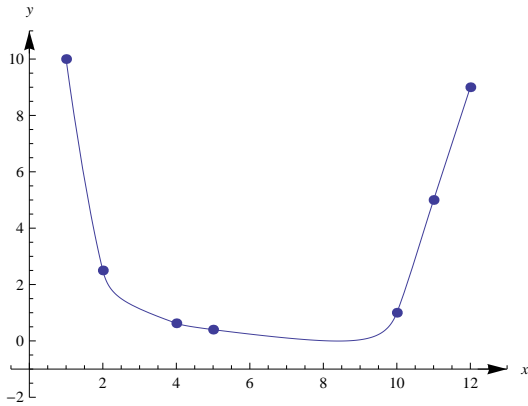


Fig. 13: Shape preserving using our proposed rational spline with $(\alpha_i = \beta_i = 1, \eta_i = 0.25)$ for data in Table 3

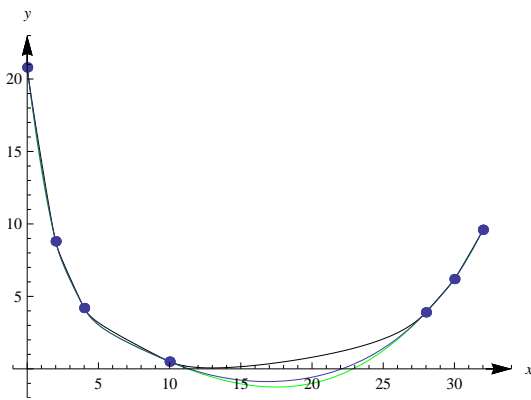


Fig. 16: Shape preserving with $\alpha_i = \beta_i = 0.05, \eta_i = 0.25$ (blue), $\alpha_i = \beta_i = 1, \eta_i = 0.25$ (black) and Tian *et al.* (2005) (green) for data in Table 2

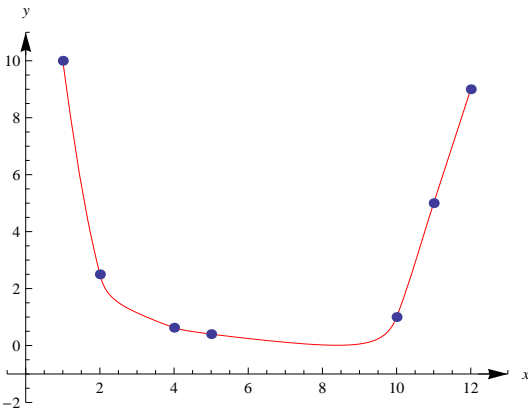


Fig. 14: Shape preserving using our proposed rational spline with $(\alpha_i = \beta_i = 0.5, \eta_i = 0.25)$ for data in Table 3

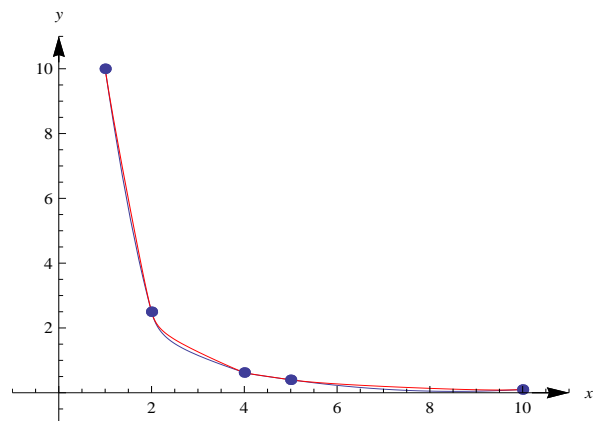


Fig. 17: Shape preserving with $\alpha_i = \beta_i = 0.05, \eta_i = 0.25$ (red) and Tian *et al.* (2005) (blue) for data in Table 2

works of Fritch and Carlson (1980) and Dougherty *et al.* (1989) is that our scheme does not require the modification of the first derivative, whereas Fritch and Carlson (1980) and Dougherty *et al.* (1989) require the first derivative modification in order to produce the convex curves for convex data sets. Even though the

rational cubic spline of Tian *et al.* (2005) works well for all tested data sets, it has no free parameter (s), meanwhile by using our proposed rational cubic spline, there are two free parameters α_i and β_i . By having extra free parameters, the user can changes the final shape of

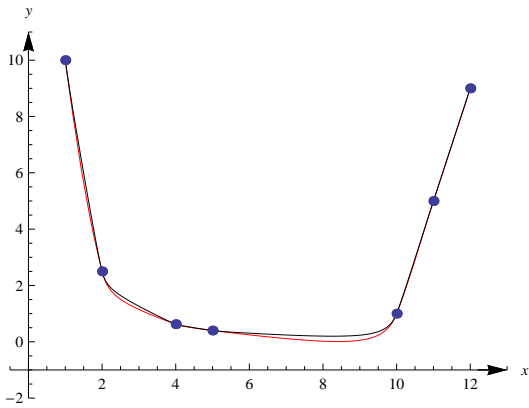


Fig. 18: Shape preserving with $(\alpha_i = \beta_i = 0.5, \eta_i = 0.25)$ (red) and Tian *et al.* (2005) (blue) for data in Table 2

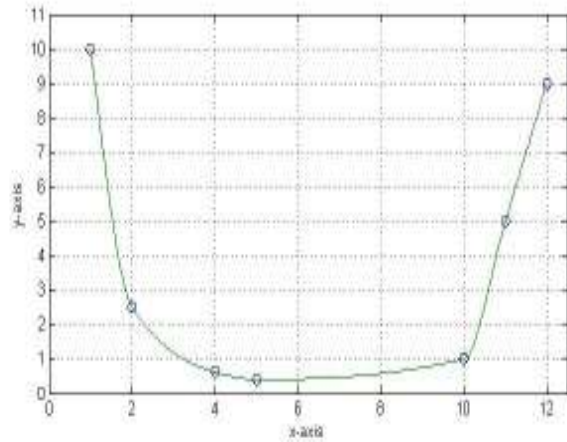


Fig. 21: Shape preserving interpolation using cubic spline for data in Table 3

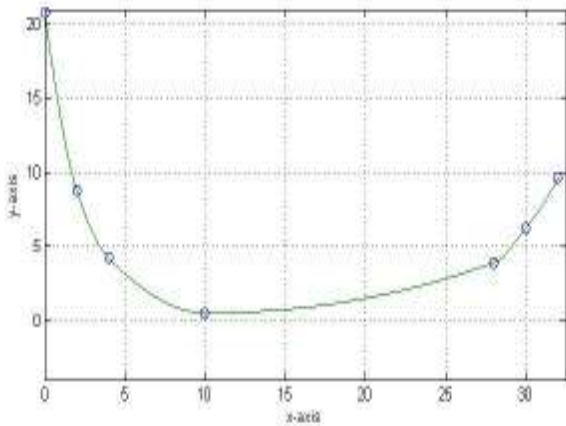


Fig. 19: Shape preserving interpolation using cubic spline for data in Table 1

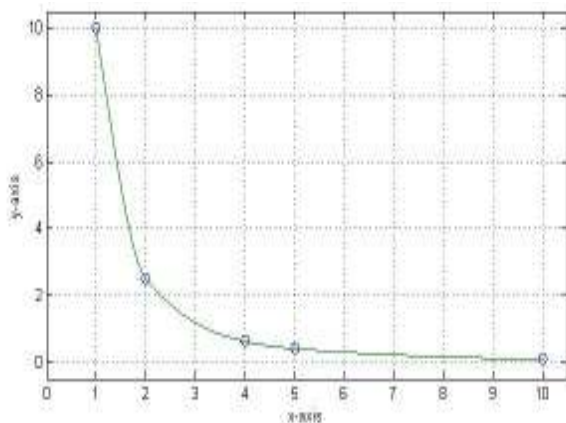


Fig. 20: Shape preserving interpolation using cubic spline for data in Table 2

the convex interpolating curves according to the what design requires. Indeed our rational scheme is easy to use and overall it gives a satisfactory results and comparable with the existing rational scheme such as Tian *et al.* (2005).

CONCLUSION

This study is devoted to the preserving the convex data sets by using rational cubic spline interpolant of the form cubic/quadratic with three parameters originally proposed by Karim and Kong (2013). The sufficient conditions for the convexity have been derived completely by using basic calculus approach. Those conditions are based to one parameter while the other two parameters can be changed in order to obtain the many convex curves. These curves not only preserve the convexity of the data provided, but it has C^1 continuity. Furthermore when $\gamma_i = 0$, the proposed scheme is the same as the work by Tian *et al.* (2005). Clearly our rational cubic interpolant with three parameters gives better freedom to the user in controlling the convexity of the curves by manipulating the values of free parameters α_i and β_i compare to the work of Tian *et al.* (2005) without any extra free parameter (s). Work on convexity-preserving by using C^2 rational cubic spline interpolant with three parameters is underway and it will be reported in our forthcoming study.

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