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Research Article

Definition of the Existence Region of the Solution of the Problem of an Arbitrary Gas-dynamic Discontinuity Breakdown at Interaction of Flat Supersonic Jets with Formation of Two Outgoing Compression Shocks

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Abstract: We have considered the modern theory of breakdown of an arbitrary gas-dynamic discontinuity for the space-time dimension equal to two. The regions of solutions existence for a one-dimensional non-stationary case and a two-dimensional stationary case have been compared. The Riemann problem of breakdown of an arbitrary discontinuity of parameters of two flat flows angle collision is considered. The problem is solved in accurate setting. The problem parameter areas where outgoing waves appear as two jumps are specified. Two depression waves solution are not covered. The special Mach numbers of interacting flows dividing the parameter plane into areas with different outgoing discontinuities are given.

Keywords: Computational gas dynamics, contact discontinuity, discontinuity breakdown scheme, Riemann wave, shock-wave

INTRODUCTION

Here we consider the problem of breakdown of an arbitrary gas-dynamic discontinuity in space-time with dimensionality equal to two. Let's remind the basics received and stated earlier.

Necessity of solution of the problem of breakdown of an arbitrary gas-dynamic discontinuity turns up in the numerical methods based on a scheme similar to the Godunov one. The requirement of the optimal combination of acceptable accuracy of approximation with high speed computation demands development of approximate methods, e.g., the Osher-Solomon scheme where for computation of weak shock-waves isentropic compression waves relations are used. In a number of technical applications (flow around the airfoil sharp edge, shock-wave reflection from an obstacle, shock-wave processes in jet streams, detonation burning), the problem of breakdown of discontinuities needs to be solved in accurate setting, with no simplification. This is actually especially for study such fine gas-dynamic phenomena as the Neumann paradox (Neuman, 1963). Let us remind that for the Mach numbers lower than the special number:

$$M_T = \sqrt{\frac{2-\varepsilon}{1-\varepsilon}}\,,\ \varepsilon = \frac{\gamma-1}{\gamma+1} \tag{1}$$

There is no solution for irregular reflection of a shock-wave from the wall, nevertheless, it is under experimental observation (White, 1952). Over almost 40 years there were experiments (Henderson and Siegenthaler, 1980; Colella and Henderson, 1990; Adachi *et al*., 1994), subtle at times, carried out which definitely demonstrated that the three-wave theory does not work (Olim and Devey, 1992) for reflection of weak shock-waves with the Mach number of the incoming flow less than M_T. For a long time it was impossible also to work out a numerical solution for such flows until Vasilev (1999) demonstrated that it was all about poor accuracy of numerical methods, effect of "circuit" computation viscosity and undesired oscillation of the solution and the flow meets the Guderley "four-wave" model (Guderley, 1960). For that e numerical method with separation of discontinuities (Vasilev and Olkhovsky, 2009). At interaction of stationary gas-dynamic discontinuities problems of breakdown of an arbitrary discontinuity appear also, which were solved for the first time for some particular cases by Kozhemyakin *et al*. (1999). For example, the angle $\beta_{1\text{-}2}$ collision of two supersonic gas flows with different thermodynamic variables (Fig. 1). Suppose, for definiteness $P_1 \geq P_2$, that an outgoing discontinuity R_1 depending on relation of values P_1v_1, P_2v_2 may be both a depression wave and a compression shock. Another discontinuity σ_2 is always a compression shock. It is understandably that nothing prevents interchanging of outgoing discontinuities R_1 and σ_2 and introducing designations R_2 and σ_1, considering that $P_1 \leq P_2$.

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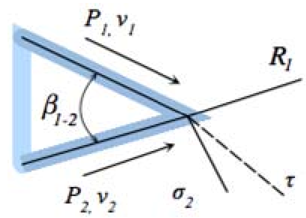

Fig. 1: Oblique interaction of two supersonic jets
P_1, P_2: Pressure in flows 1 and 2; v_1 and v_2: Velocity in flows 1 and 2; σ_2: Compression shock in flow 2; τ: Tangential discontinuity separating flows 1 and 2; R_1: outgoing discontinuity (compression shock or depression wave) in flow 1

It is important to know how to specify definite regions of existence of Shock-Wave Patterns (SWP) of different types and how to create effective algorithms of this important problem solutions. In part, these problems are solved in the Candidate's dissertation of Kozhemyakin (2000) and by Uskov *et al*. (2000) in the book, but there have been considered not all modes and SWP possible types and the computation has been given fragmentarily.

The following are qualitative pictures for the region of existence of the solution with two outgoing jumps.

MATERIALS AND METHODS

Possible types of solution: As you know, simplectic geometry specifies the reflection of the gas-dynamic variables space, as well as the specifics of transformation of shock-waves and wave fronts in the even-dimensional space. As for one-dimensional non-stationary flows and two-dimensional stationary ones the problem dimensionality is equal to two, the classifications must be equal. Previously it was shown that a one-dimensional case goes beyond collision of two flows, there can be other possible variants including with two outgoing depression waves of Riemann (Volkov, 2005). Therefore, in a two-dimensional case there must be flows around breakdown of an arbitrary discontinuity with two outgoing waves of Prandtl-Mayer.

On the first glance, it is impossible. And in case of converging supersonic jets it is really so. But there can be flows in which spreading of supersonic jets happens, for example, at gas injection in the flow through the porous surface of a return wedge (Fig. 2a), when two depression waves ω_1 and ω_2 are separated with a tangential discontinuity τ, or on the axis of supersonic under expanded flow in the point of reflection of the first discontinuity characteristic v_1 from the symmetry axis (Fig. 2b), where are two Prandtl-Mayer ω_1 and ω_2 just following it which are separated with the stream area similar to the stream from source R and there is no contact (tangential) discontinuity. In case of discontinuity breakdown originating in the triple point of the shock-wave configurations, the picture can be even more complicated. Generally, depending on an amount incoming discontinuities, there can be outgoing discontinuities more than one up to three.

Figure 2b, the lower insert, shows a stream around the nozzle edge scaled-up. A point marks the place of origin of a barrel shock (endpoint of a shockwave), which forms due to intersection of the performances of the v_1 family forming a compression wave originating on the jet edge and limited with the barrel shock. The barrel shock can be compared with one-dimensional non-stationary shock-wave travelling along its surface. In this case, the point of origin of barrel shock will match the shock-wave endpoint. The matching of one-dimensional non-stationary and two-dimensional stationary wave fronts is given in the insert on the right below. The arrows show travel of one-dimensional wave fronts. The round black labels mark end points. Further we consider a special case of the problem of discontinuity breakdown-interaction of colliding supersonic jets at a set angle (Fig. 1) which results in two shocks. The problem for a set angle of interaction of flows is finding the region of the Mach numbers M_1 and M_2 specifying the region of existing solutions.

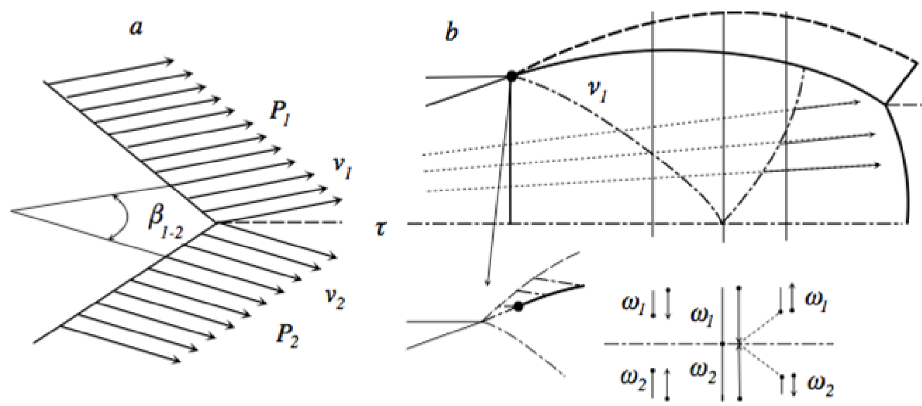

Fig. 2: Examples of streams with outgoing depression waves
P_1, P_2: Pressure in flows 1 and 2; v_1 and v_2: Velocities in flows 1 and 2; v_1: Front of the depression; ω: One-dimensional non-stationary analogs of wave front

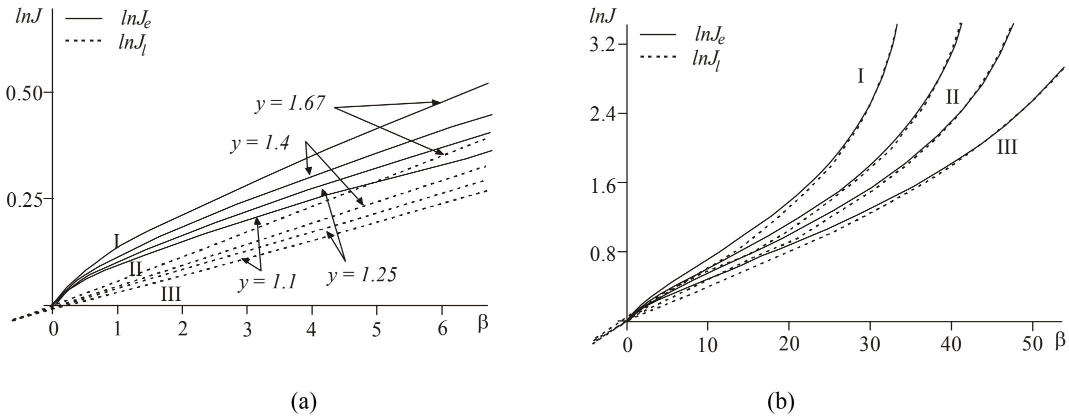

Fig. 3: Dependences meeting the limit angles of deviation $J_l(\beta)$ and envelope $J_e(\beta)$

RESULTS AND DISCUSSION

Analysis of breakdown of discontinuity with formation of two outgoing jumps: Suppose, for definiteness, that discontinuity R_1 is a compression shock. Then we need to specify the region of solutions existence for the SWP consisting of two jumps and tangential discontinuity.

Let's plot a polar matching compression shock σ_2 and the Mach number $M = M_2$, from the origin of coordinates $\{\Lambda = 0, \beta = 0\}$. And a polar meeting the wave R_1 (in this case, a compression shock σ_1), we will plot from the point with coordinates $\{\Lambda_{1\text{-}2}, \beta_{1\text{-}2}\}$. All region is divided into three parts with curves J_l (limit turn angle for the given M) and J_e (envelope) of shock polars:

$$J_l = \frac{M^2-2}{2} + \sqrt{\left(\frac{M^2-2}{2} + (1+2\varepsilon)(M^2-1)+2\right)},$$

$$\beta_l(J_l) = arctg\left(\sqrt{\left(\frac{1-E_l}{1-\varepsilon}\right)}\frac{(1-E_l)}{2\sqrt{E_l}}\right),$$

$$J_e = M^2-1, \beta_e(J) = arctg\left(\frac{1-E}{2\sqrt{E}}\right), \tag{2}$$

where, E = E (J) - the Rankin-Hugoniot adiabat. Area I meets the solution with two outgoing compression shocks. If point $\{\Lambda_{1\text{-}2}, \beta_{1\text{-}2}\}$ gets in area I in Fig. 3, outgoing discontinuities will be compression shocks. In this case, the point of polar cross ("*O*" in Fig. 4) will meet the fulfillment of conditions of dynamic compatibility and the problem mathematical solution (Fig. 4, on the left). The set of such points built at given $\{\Lambda_{1\text{-}2}, \beta_{1\text{-}2}\}$ for different M_1 and M_2 forms the region of solution existence. To find it, you need to build the envelope e_1 of polars 1 and the envelope e_2 of polar 2 (Fig. 4).

Any cross point "*O*" of polars inside the region limited with two envelopes e_1 and e_2 (shaded in Fig. 4), is belonged to the region of existence of solutions. In this case, for the set angle of interaction of flows $\beta_{1\text{-}2}$ and the set relation of pressures $\Lambda_{1\text{-}2}$ in these flows there are two limit Mach numbers M_{1e} and M_{2e}, limiting from above the area of the Mach numbers in the flows 1 and 2, where is no solution.

Apparently that they meet contact of the polar 1 of the polar envelope e_2 and, on the contrary, contact of the polar 2 released from point $\{\Lambda_{1\text{-}2}, \beta_{1\text{-}2}\}$ of the envelope of the polar e_1. Figure 4 shows two such polars $\Lambda = \Lambda(M_{1e}, \beta)$ and $\Lambda = \Lambda(M_{2e}, \beta)$. Therefore, for the Mach numbers $M_1 < M_{1e}$ there is no solution at any value M_2. And on the contrary, for the Mach numbers $M_2 < M_{2e}$ there is no solution at any value M_1. If the angle $\beta_{1\text{-}2}$ is more than the maximal for the given gas turning angles on jumps 1, 2:

$$\beta_{\lim}(\gamma) = arctg\frac{1-\varepsilon}{2\sqrt{\varepsilon}} \tag{3}$$

The envelopes e_1 and e_2 cannot cross under no Mach numbers and there is no solution. In the points of contact of two curves a condition of equality of ordinates and equality of partial derivative difference to zero is carried out. For the contact point of the polar 1 and the envelope e_2, it results in the equation system:

$$J_1 = J_{1-2}J_e, J_e = J_2,$$

$$\beta_e(J_1) = \beta_{1-2} + \beta_2(M_{2e}, J_2),$$

$$\frac{\partial\beta_e(J_1)}{\partial\Lambda_1} - \frac{\partial\beta_2(M_2, J_2)}{\partial\Lambda_2} = 0, \tag{4}$$

$$\frac{\partial\beta_e(J_1)}{\partial\Lambda_1} = \frac{J_1(1-\varepsilon)}{(J_1+1)\sqrt{(J_1+\varepsilon)(1+\varepsilon J_1)}} \tag{5}$$

$$\frac{\partial\beta_2(M_2,J_2)}{\partial\Lambda_2} = \frac{\chi}{2\gamma J}\sqrt{\frac{\varepsilon}{A(J+\varepsilon)}} \times \frac{B + A(1+\varepsilon)}{\mu(J+\varepsilon) - J(1+\varepsilon J)(1-\varepsilon)}, \tag{6}$$

$$A = \mu(1+\varepsilon) - (1+\varepsilon J),$$

$$B = A(J+\varepsilon) - \varepsilon(1+\varepsilon J)(J-1); \mu = 1 + \varepsilon(M^2-1).$$

For the polar 2 and the envelope e_1 you can similarly find the point of contact, but indices 1 in expressions for all derivatives should be changed for 2. Suppose that the Mach number $M_2>M_{2e}$ and a solution is possible. We find the Mach numbers M_1 range when the problem possesses a solution. At small Mach numbers, the polars 1 and 2 do not cross (Fig. 5), and there is no solution. With increase of the Mach number, there comes a moment when the polars 1 and 2 contact in the point ($M = M_{1t}$). With further increase of M, the curves cross in two points, the lower one corresponds to a physically realizable solution. Further, with increase of M_1, two cross points merge together in one point ($M = M_{2t}$) and at $M>M_{2t}$ polars don`t cross any more. The range (M_{1t}, M_{2t}) is a definitional domain. The range (M_{1t}, M_{2t}) for the number M_2 is defined similarly. It is obvious that if an interaction angle β_{1-2} is more than the limit angle $\beta_{\lim}$ for the given γ which is calculated according to the formula 2, then the polar 2 will never cross the ordinate and the Mach number M_{2t} always exists. And conversely, if β_{1-2} is less than the limit angle $\beta_{\lim}$ for the given γ, the Mach number M_l exists at which the polar 2 touches the ordinate. In this case, $M_{2t} \to \infty$, and at $M_2>M_l$ a solution exists for any M_l. The number M_l is defined under the formula:

$$M_l = \frac{1}{J_2 + 2\varepsilon + 1}\sqrt{(J_2 + 2\varepsilon + 1)((J_2 - 1)^2 + 2(J_2 + \varepsilon))} \tag{7}$$

The special numbers M_{1t}, M_{2t} are defined from the equation system meeting the condition of two polars contact. The system possesses two solutions meeting M_{1t}, M_{2t}:

$$\begin{gathered} J_1 = J_{1-2} J_2, \\ \beta_1(M_1, J_1) - \beta_2(M_2, J_2), \\ \frac{\partial \beta_1(M_1, J_1)}{\partial \Lambda_1} - \frac{\partial \beta_2(M_2, J_2)}{\partial \Lambda_2}. \end{gathered} \tag{8}$$

Partial derivatives in (8) are computed with the help of formulae (6) with substitution of appropriate values $J_1, J_2, \beta_1, \beta_2$. The analysis result is given in Fig. 6.

The greatest challenge of finding regions of solution existence is solution of two nonlinear systems of Eq. (4)-(6) and (8). If it is known in advance that interaction is regular, the problem becomes simpler, as the nature of variable change is monotonic and the equation system I solved by any standard numerical method, e.g., the tangent method.

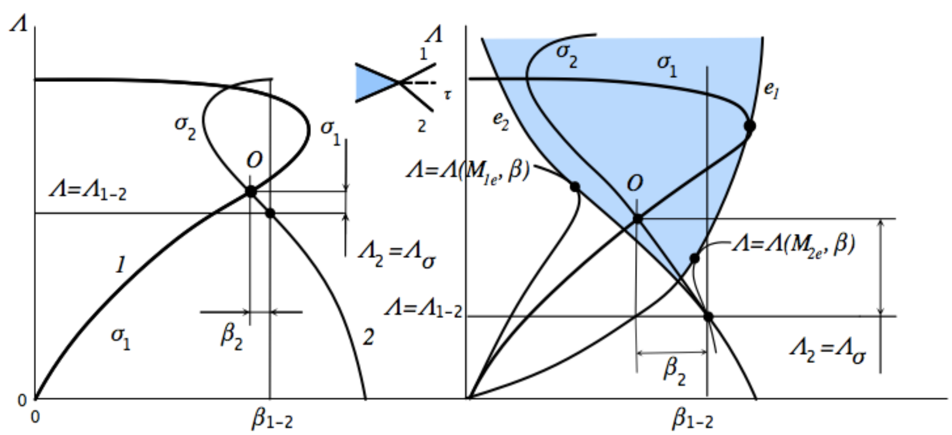

Fig. 4: Region of existence of the solution with two outgoing compression shocks at the given angle of interaction of flows β_{1-2} and relation of pressures Λ_{1-2}

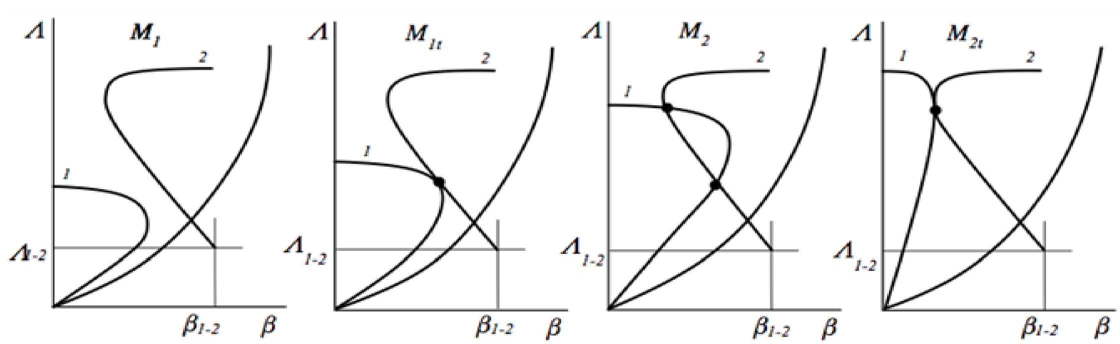

Fig. 5: Picture of solution on the polar plane at different mach numbers $M_1<M_{1t}<M_2<M_{2t}$

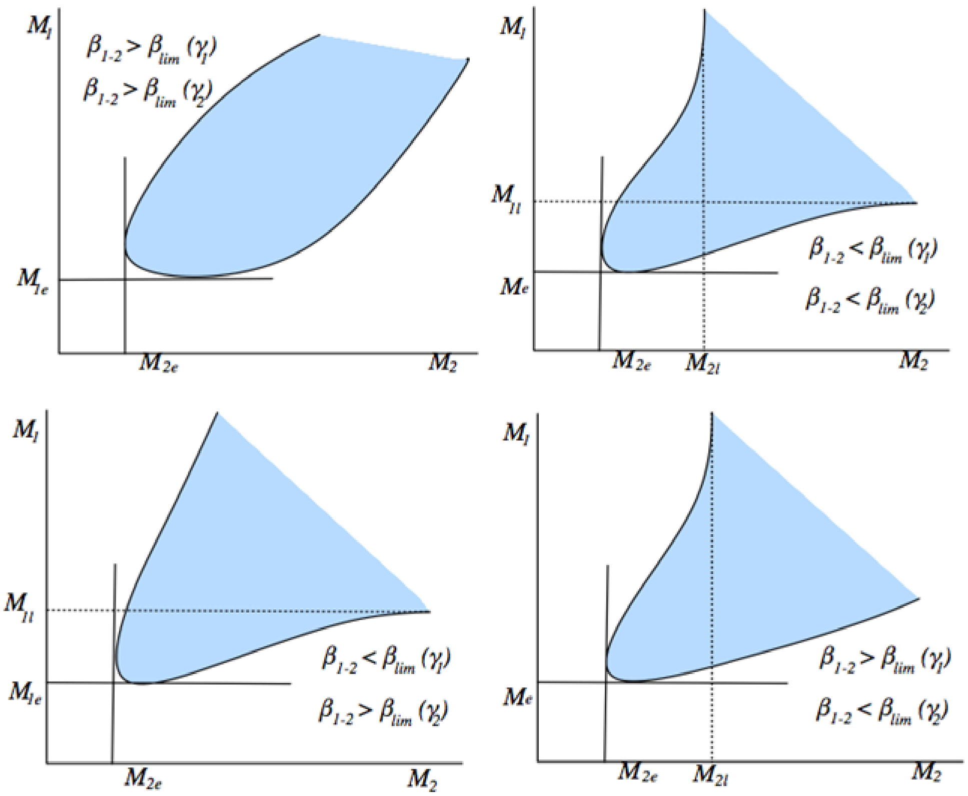

Fig. 6: Regions of solution existence for the problem of breakdown of an arbitrary discontinuity with formation of two outgoing compression shocks

CONCLUSION

The development of new algorithms for computation of regions of existence of different solutions is topical. In a two-dimensional case, the conditions dynamic compatibility are not enough for solution selection. The developed by the Russian mathematicians geometrical theory of shock-waves and wave front transformation allows not only uniquely to select physically realizable solutions from the set of solutions meeting the dynamic compatibility conditions, but also to take into account the hysteresis depending on a direction of change of the problem parameter. Analysis of one-dimensional non-stationary problems and their revealed full analogy to a flat stationary case have shown that breakdown of an arbitrary discontinuity can originate not only with formation of two jumps, but also with development of depression waves. As the three-wave theory of non-regular interaction of a shockwave with an angled wall corresponds to a case with two outgoing jumps, it is obviously important during the numerical study of such fine phenomena as the Neumann paradox and the Guderley four-wave reflection to know how accurately and with no simplifications to select the case with two outgoing jumps. In this study the analysis of regions of existence of such solutions for a two-dimensional case is given. In the form of comfortable diagrams, the basic dependences permitting completely to define a type of outgoing discontinuities and a nature of the problem of breakdown of an arbitrary discontinuity are presented. The qualitative appearance of the region of existence of the solution with two outgoing jumps for four variants of combination of the flow interaction angle and adiabatic index in two flows is presented.

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REFERENCES

Adachi, T., T. Suzuki and S. Kobayashi, 1994. Mach reflection of a weak shock waves. T. Jpn. Soc. Mech. Eng., 60(575): 2281-2286.

Colella, P. and L.F. Henderson, 1990. The von Neumann paradox for the diffraction of weak shock waves. J. Fluid Mech., 213: 71-94.

Guderley, K.G., 1960. Theory of Transonic Flows. Publishing House of Foreign Literature, Moscow, pp: 421.

Henderson, L.F. and A. Siegenthaler, 1980. Experiments on the diffraction of weak blast waves: The von Neumann paradox. P. R. Soc. London, A369: 537-555.

Kozhemyakin, A.O., 2000. Solution of problem of breakdown of an arbitrary stationary discontinuity. M.Sc. Thesis, in the Physico-Mathematical Science. Manuscript Copyright, BSTU Voenmeh, SPb, Russia, pp: 122.

Kozhemyakin, A.O., A.V. Omelchenko and V.N. Uskov, 1999. Angled interaction of supersonic jets. Izv. USSR Acad. Sci. Mech. Fluid., 4: 116-124.

Neuman, J., 1963. Collected Works. Pergamon Press, Oxford, 6: 239-299.

Olim, M. and J.M. Devey, 1992. A revised three-shock solution for the Mach reflection of weak shocks. Shock Waves, 2: 167-176.

Uskov, V.N., V.N. Glaznev *et al*., 2000. Jet and non-stationary flows in gas dynamics. Publ. House SO Russian Academy of Science, Novosibirsk, pp: 200.

Vasilev, E.I., 1999. Four-wave scheme of weak Mach shock waves interaction under the von Neumann paradox conditions. Fluid Dyn., 34(3): 421.

Vasilev, E. and M. Olkhovsky, 2009. The complex structure of supersonic patches in the steady mach reflection of the weak shock waves. Proceeding of 26th International Symposium on Shock Waves, pp: 322.

Volkov, K.N., 2005. Difference schemes of computation of flows with enhanced resolution and their application for solution of gas-dynamics problems. Comput. Methods Programm., 6: 146-167.

White, D.R., 1952. An experimental survey of shock waves. Proceeding of 2nd Midwest Conference on Fluid Mechanics. Ohio State University Press, Columbus, 3: 253-262.