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Research Article

Spherical Shock-wave-2D Surface Interaction

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Abstract: The purpose of research is the study of the transformation of the shock-wave configuration, caused by the reflection of a spherical shock wave from a flat surface. The blast of HE charge heightened over earth surface leads to formation of shock-wave triple configuration. In spite of static pressure equality of gas streams after the different wave sequences, the velocities, densities and other flow parameters are not equal. In view of the fact that flow velocities are sufficiently different, wind loads on objects subjected to blast wave action differ also. So blast shock wave hazard degree (in particular, for human organism at body translation) depends on both object and HE charge blast height. The mathematical model to calculate and analyze the propagating shock-wave triple configurations occurring at the heightened blast is provided in this study. The model is useful for calculation and comparison of the velocities and dynamic pressures of the streams behind the different sequences of shock waves in the triple configuration, i.e., it allows us to estimate the basic parameters characterizing the tertiary blast wave hazards.

Keywords: Dynamic pressure, heightened blast, triple configuration

INTRODUCTION

Object of study is the process of reflection from the surface of the Earth spherical shock wave that occurs when the air blast. Here we consider the task of the space shock-2D surface interaction. The task of the shock-wave protection in-situ occurs, particularly, for an aircraft flying at the supersonic speed. At the same time, the shock front leaves a trace on the surface (Fig. 1).

If the substrate is not flat, the trace from the shock front (Fig. 1 in yellow), apparently, will experience modifications (metamorphoses). At that, there can be special points on the shock trace. The contact geometry studies similar modifications (Arnold, 1996). Its own parameter space geometry lays at the bottom of every section of the science. The Minkowski geometry describes the space of the Special Relativity Theory. The Riemannian geometry is the basis for the General Theory of Relativity. The symplectic geometry is the phase space of classical mechanics. It always has even dimension.

The geometries differ in how the space metric is introduced. The metric is expression for the length of the arbitrary curve element the vector length shall not depend on choice of the coordinate system. The surface concept generalization in the modern geometry is the term ‘diversity’. The diversity is the arbitrary set of points introduced as the union of the finite number of areas of the Euclidean space with the local coordinates preset in every area. The smooth (Lagrangian) mapping is usually understood as a projection of the surface (diversity) of gas-dynamic parameters on the plane. The mapping can have its features and critical points. The set of critical points is called the mapping caustic. In the space of gas-dynamic variables the shock front meets the caustic (Arnold and Givental, 1985). So, the shock-wave is a set of features of the smooth mapping of projecting diversity of gas-dynamic variables.

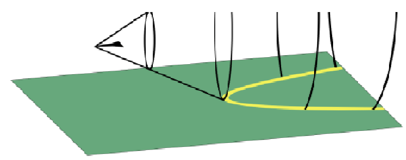

Fig. 1: Supersonic airplane blast action on the place

The contact geometry studies projective mapping of diversity of gas-dynamic parameters onto the space with the dimension one less. Such mapping is called Legendre mapping. The contact geometry is the twin of the symplectic geometry, but with the uneven dimension of space. The shock front trace on the substrate (Fig. 1) is called the Legendre cobordism. When travelling the shock front trace (cobordism) on the uneven surface can be subject to intricate modifications, including with formation of self-intersection points, closed loops and «bows» (Fig. 2).

The soviet mathematicians of the V. Arnold scientific school devoted their researches to this

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Fig. 2: Samples of nontrivial modifications of the shock-wave trace *in-situ*

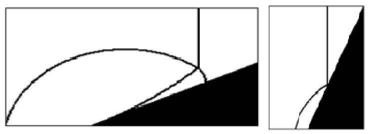
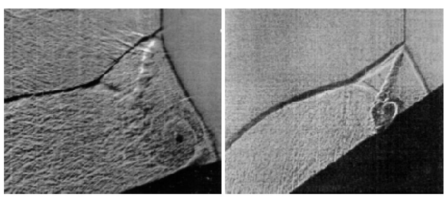


Fig. 3: Irregular Mach reflection (on the left) and regular reflection of the shock-wave from the sloping wall in the experiments of Mach (1878)



(a) (b)

Fig. 4: (a) Complex, (b) double Mach reflection

problem. In Arnold's production work (Arnold, 1976) the approach to the shock-waves and shock fronts is formulated according to the features of smooth mapping. The classification of the features and critical (special) points of differentiated mapping is given (Arnold *et al*., 1982).

Today, the theory of the features of smooth mapping modifications by force of the famous mathematicians Whitney, Arnold, Tom, Poincaré has been developed very good. For practically important cases of codimension (dimension of the tangent space) 2 and 3 the complete classification of the mapping features and their modifications is made up (Arnold, 1978).

First, the progressive shock-waves and generated shock-wave structures were discovered by (Mach, 1878). In his work he described two kinds of the shock-wave mapping from the sloping surface (Fig. 3):

- The regular reflection, which consists of two shock-waves: incident wave coming onto the solid surface and reflected wave outgoing from the impact point.
- The irregular reflection, which consists of three shock-waves: incident, reflected and basic ones having the common point; such a kind of reflection is named as simple Mach reflection and the correspond configuration, unless it contains other normal discontinuities, Triple Configuration (TC) of shock-waves.

Experimental research of interaction of the progressive shock-wave with the immobile wedge executed later by Smith (1945) and White (1951) discovered an opportunity of existence of a few kinds of irregular reflection which can appear at different values of the wave velocity and its contact angle with the surface (Fig. 4).

The analogous effects of formation of movable triple configurations at the shock-wave sloped reflection are well-known (Bazhenova and Gvozdeva, 1977; Arutyunyan and Karchevsky, 1973) and stationary configurations at the Mach reflection of compression shocks in the supersonic jets (Omel'chenko *et al*., 2003; Hadjadj *et al*., 2004). The model of triple configurations of shock-waves is studied in detail by V. Uskov scientific school (Uskov *et al*., 1995; Uskov and Mostovykh, 2008). It is shown that the triple configurations can be extreme under some gas-dynamic parameters (Tao *et al*., 2005; Uskov and Chernyshov, 2006), including under the rate of velocity head following them, which is actually for study of the ways of protection against the shock-wave effect (Gelfand and Silnikov, 2006).

MATERIALS AND METHODS

Schematic circuit of the shock-wave-2D surface under the blast interaction: Let's consider the simplest sample of the shock-wave-surface interaction when the surface is plane (Fig. 5). Analogous triple configurations of movable shock-waves appear at the surface explosion in the open on top reservoir and are formed with the shock-wave enveloping the device edges (Gelfand and Silnikov, 2003).

The shock-wave s_1 formed at the elevated blast in point *O* at height *H* (Fig. 5) reaches the ground in point *A* and then spreads over the surface with regular (e.g., in point *B*) reflection of the wave s_2. Under the wave front s_1 further moving off the blast epicenter (in point *C*) its regular reflection transforms into irregular (Mach) reflection: a new (major, Mach) wave appear s_3. At every point of time the shock-waves s_1, s_2 and s_3 have the common (triple) point *T*, where they form the triple configuration. The further movement of the blast wave front s_1 results in elevation of the triple point (its trajectory *t* in shown in Fig. 5), in the Mach wave upsizing s_3 and gradual junction of the incident wave s_1 and reflected wave s_2.

The cocurrent flows following the triple configuration moving along the contact discontinuity τ are collinear and they have equal static pressure. However, their velocities (u_2 and u_3, Fig. 5), dynamic pressure are not equal to each other which results in difference (occasionally essential) of wind (dynamic pressure) loads on the objects located at the same distance *R* up to the blast center, but at different height *h*.

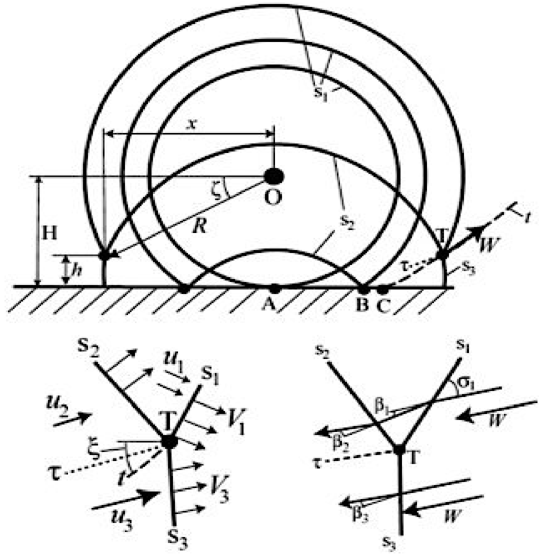


Fig. 5: Interaction of the shock-wave generated by the blast some height over the plane surface
O: Blast epicenter; s_1: Original shock-wave; s_2: Shock-wave reflected from the plane surface; s_3: Mach stem by height *h* in the triple configuration of shock-waves caused by the origin shock; Wave: Reflected shock-wave interaction; T: Triple point; t: Triple point travel; τ: Tangential discontinuity following point T; *W*: Velocity of the triple point

Just that very dynamic pressure of moving air mass entrained by the blast shock-wave is the reason of destructive effect of the shock-wave, instead of increase of static pressure in its front, as many think.

The model of triple configuration of progressive shockwaves is built based on the known correlations for the similar structure of waves at rest (Compression shocks) in the supersonic jet (Fig. 5). Such a stationary configuration consists of three stationary shock-waves (s_1, s_2 and s_3) in the incoming supersonic jet with Mach number $M>1$ and also tangential discontinuity τfollowing their common point *T*. For generalizing the results of analysis of the triple configurations of stationary shock-waves on the similar structures of progressing shock-waves, the principle of inversion of motion is used (Uskov, 2000) relatively to the triple point (Fig. 5). At motion inversion the triple point becomes immovable and the gas flow leaks on it with velocity *W* in the direction opposite to the trajectory of its movement shown in Fig. 5.

Conditions of dynamic compatibility in the triple point: The parameters of shocks $s_{1...3}$ are connected to each other and to the following flow property with the conditions of equality of pressure and co directional streams on the discontinuity sides τ:

$$J_1 J_2 = J_3 \tag{1}$$

$$\beta_1 + \beta_2 = \beta_3 \tag{2}$$

where, $J_i = p_i/p_j$ is relation of pressure of streams following the shock p_i and before it ипереднимp_j; $i = 1...3$ and β_i is an angle of the flow turning on the shock connected to its intensity (J) and Mach number (M_j) before it:

$$|\beta_i| = \text{arctg}\left[\sqrt{\frac{(1+\varepsilon)M_j^2 - \varepsilon - J_i}{J_i + \varepsilon}} \frac{(1-\varepsilon)(J_i - 1)}{(1+\varepsilon)M_j^2 - (1-\varepsilon)(J_i - 1)}\right] \tag{3}$$

The Mach number M_i of the flow following the shock is also specified by its intensity:

$$M_i = \sqrt{\frac{(J_i + \varepsilon)M_j^2 - (1-\varepsilon)(J_i^2 - 1)}{J_i(1+\varepsilon J_i)}} \tag{4}$$

Here, $\varepsilon = (\gamma - 1) / (\gamma + 1)$ and γ is the gas adiabatic index (in this study we consider the air shock-waves and $\gamma = 1, 4$).

Dependences (1-4), the shock adiabat and the equations of the medium condition specify also other parameters of the streams following the triple configuration shocks. Particularly, velocities u_2 and u_3 of the gas streams on the sides of the tangential discontinuity can be found from the correlations:

$$u_2 = a_0 M_2 \sqrt{E_1 E_2 J_3}\, , u_3 = a_0 M_3 \sqrt{E_3 J_3} \tag{5}$$

where, $E_i = (1 + \varepsilon J_i) / (J_i + \varepsilon)$ and a_0 is the sonic speed in the unperturbed medium.

The ravel speed, shock intensity and dynamic pressure following it: Intensity J_1 and J_3 of shock-waves s_1 and s_3 spreading on the medium at rest are specified with velocities v_1 and v_3 of the normal propagation of these waves fronts:

$$J_1 = (1+\varepsilon) M_{D1}^2 - \varepsilon,$$
$$J_3 = (1+\varepsilon) M_{D3}^2 - \varepsilon, \quad (6)$$
$$M_{D1} = V_1/a_0,$$
$$M_{D3} = V_3/a_0.$$

At the same time, the condition of belonging the triple point to both waves at every point of time is written as:

$$V_1/\sin\sigma_1 = V_3/\sin\sigma_3 = W \quad (7)$$

where, W is velocity of the triple point travel, $\sigma_1 = \pi/2 - \xi - \varsigma$ (Fig. 1a, b), $\sin\varsigma = (H-h)/R$, $\mathrm{tg}\,\xi = y'(x)$ and y (x) is the equation of the point travelling trajectory T. At the known dependence V_1 (R) and the given equation of the point travelling T of intensity of waves s_1 and s_3 (therefore, value $J_2 = J_3/J_1$) are specified at every point of time.

The intensity of waves s_1 and s_2 in the equivalent triple configuration of compression shocks are specified with correlations:

$$J_1 = (1+\varepsilon)(W\sin\sigma_1/a_0)^2 - \varepsilon,$$
$$J_3 = (1+\varepsilon)(W\sin\sigma_3/a_0)^2 - \varepsilon \quad (8)$$

As one can see from (6, 7), they remain invariant as compared with the considered case of the triple configuration of movable waves. The system (1-5) allows, due to correlation (5), to find both the velocities of streams on the discontinuity sides τ and on their projection on the coordinates in the equivalent configuration of shocks at rest:

$$u_{2x} = u_2\cos(\beta_3 - \varsigma),$$
$$u_{2y} = u_2\sin(\beta_3 - \varsigma), \quad (9)$$
$$u_{3x} = u_3\cos(\beta_3 - \varsigma),$$
$$u_{3y} = u_3\sin(\beta_3 - \varsigma).$$

At back transfer to the coordinates related to the flow at rest before the progressing waves, vectorial addition of velocities of the triple point and streams following the stationary configuration is carried out:

$$\vec{U}_2 = \vec{u}_2 + \vec{W},\ \vec{U}_3 = \vec{u}_3 + \vec{W} \quad (10)$$

which allows to find the velocity vectors $\vec{U}_2$ and $\vec{U}_3$ of the streams following the movable triple configuration, to compare their absolute values.

RESULTS AND DISCUSSION

The system (6-10) specifying the velocity of the streams following the triple configuration of movable blast shock-waves closes with the help of the correlation for the intensity of shock-wave in the triple point and the height of the Mach stem. These values can be determined numerically and with the help of the known auto model solutions. Also, suitable empirical dependences are known which are obtained as a result of processing of numerous results of computation and experiments (Henrych, 1979). One of them is the dependence of the amplitude $\Delta p_1 = (J_1 - 1)\cdot p_0$ of the incident wave s_1 on the distance R to the air blast epicenter, which specifies, according to (6), velocity V_1 of this wave travel:

$$\Delta p_1 = 1.38 R_*^{-1} + 0.543 R_*^{-2} - 0.035 R_*^{-3},$$
when $0.05 < R_* \le 0.3$ (11)

$$\Delta p_1 = 0.607 R_*^{-1} + 0.032 R_*^{-2} - 0.209 R_*^{-3},$$
when $0.3 < R_* \le 1.0$ (12)

$$\Delta p_1 = 0.065 R_*^{-1} + 0.397 R_*^{-2} - 0.322 R_*^{-3},$$
when $1.0 \le R_* < 10$ (13)

where, $R_* = R/G^{1/3}$, m/kg$^{1/3}$ is the distance in the Sadovsky-Hopkinson variables; G, kg is the trinitrotoluene equivalent of the blast energy; Δp_1 is the wave amplitude s_1, MPa. By analogy with variable R in this study we use the reduced values of the blast height $H_* = H/G^{1/3}$, the triple point $h_* = h/G^{1/3}$, the distance $x_* = x/G^{1/3} = \sqrt{R^2 - (H-h)^2}/G^{1/3}$ between the projections of the blast epicenter and triple point of the earth surface. Also dimensionless quantities $\bar{R} = R_*/H_* = R/H$, $\bar{h} = h_*/H_* = h/H$, $\bar{x} = x_*/H_* = x/H$ are used.

The second empirical relation which closes the system (6-10) is the equation $\bar{h} = \bar{h}(\bar{x})$ of the trajectory of point T (Balagansky and Merzhievsky, 2004):

$$\frac{1-\bar{h}}{\bar{x}} = \mathrm{ctg}\left[\alpha^* + 1,2\ln\frac{\bar{R}}{1,3}\right] \quad (14)$$

where, $\alpha^* \approx 2/9\pi$

In accordance with the correlation (13) at $\bar{h} = 0$ deviation of the triple point from the reflecting surface starts at $\bar{x} = 3,443$ ($\bar{R} = 3,585$). The intensity of the shock-wave transferring from the regular to Mach reflection according to (14) is close to two known criteria of the reflection type change (Fig. 6). The diagram of changing of the triple point dimensionless

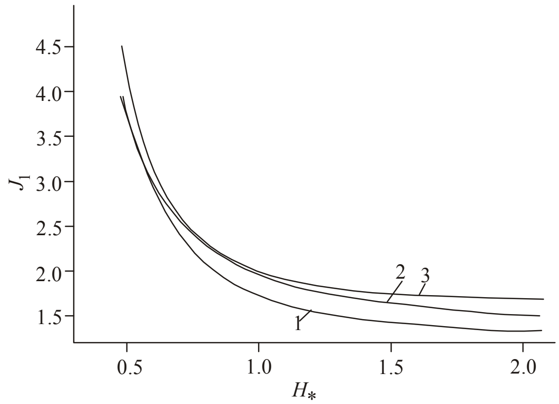


Fig. 6: Dependence of the incident shock-wave intensity, with transition to its irregular reflection, on the reduced height of the blast (N_*, m/kg$^{1/3}$)
1: Empirical relation (13); 2: Spectral conditioning criterion; 3: Criterion of "maximal angle of the flow turning"

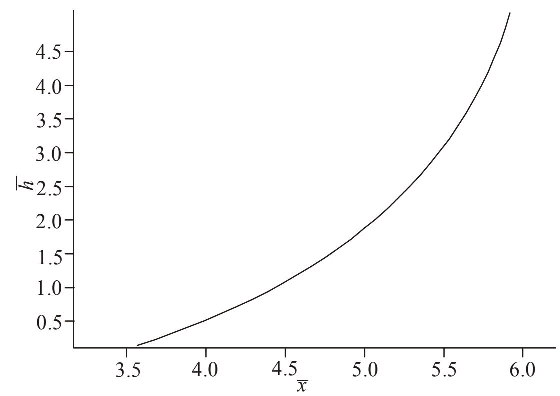


Fig. 7: Trajectory of the triple point specified with the empirical relation

height $\bar{h} = \bar{h}(\bar{x})$ depending on horizontal moving off the blast source is given in Fig. 7.

The system (5-13) allows calculating and comparing both parameters of the progressing shock-waves in triple configurations appearing at the elevated blast and the velocity of following gas streams. The higher the Mach stem, the more area with high dynamic pressure.

CONCLUSION

The mathematical model built based on the accurate and semi empirical correlations describes the shock-wave structure of the elevated blast and allows evaluating the wind loads following different shock-waves of this structure. Usage of semiempirical dependences for the wave intensity and the Mach stem height essentially simplifies calculating with preservation of acceptable quality. The results of such calculation can be used as the first approximation for evaluation of effect of the dynamic pressure following the shock-wave, on the objects located on in-situ. Considering the geometry of particular objects with the help of methods of the theory of features of the caustic and wave fronts, it is possible to reveal modifications of the wave front happening at some little distance from the object.

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