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Research Article

Investigating Students' Conceptual Knowledge and Procedural Skills in the Learning of College Level Trigonometry

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Abstract: This research study investigated 51 Pre-University (or College level) students' difficulties in solving trigonometric function problems. It sets to determine the correlation of conceptual knowledge and procedural skills of students' trigonometry test achievements using pre, post and delayed-post tests on six selected trigonometric questions. Results from the three tests revealed that there were improvements in the questions that tested on procedural skills; however, they did not perform as well as was predicted in the conceptual questions due to the lack in the conceptual knowledge of trigonometry. In contrast, there was evidence on the retention of procedural skills from the delayed-post tests. This indicated that learning had taken place for the students. However, it was noticed that there was also a slight decrease on conceptual knowledge. The greatest obstacle was that students memorize the sequence or steps. The common misconceptions depicted were misinterpreted language, distorted definition and technical mechanical error.

Keywords: Conceptual knowledge and procedural skills, pre-university students, trigonometry

INTRODUCTION

Trigonometry is one of the topics that will be examined in the Cambridge AS Level Examinations and questions on trigonometry are among the most frequently tested topics every year in both June and November papers. Trigonometry has been known to be one of the most difficult topics teachers find hard to explain and for the students to comprehend (Shahrill, 2009). Even though the same concepts were taught in Additional Mathematics when the students were in Year 10, students generally find it difficult to cope when they are taught the same concepts again in the Pre University Lower Sixth (Year 12 or College level) and they encounter various problems when solving trigonometry problems. To avoid students from further confusion and lose interest in the topic, majority of the teachers will choose to skip the explanations on triangles and the derivations of trigonometry formulae. Often, they will immediately teach how to answer examination questions without actually explaining the conceptual aspects of trigonometry.

On the other hand, students who already find difficulties in understanding triangles and the abstract concepts of the topics will therefore focus on the procedural understanding to help them obtain better marks for trigonometry tests (Appendix). The understanding of the procedural aspects will definitely allow the students to obtain better results; however, this only provides the students with short-term learning (Orhun, 2001). Most students will memorize the trigonometry formulae in order to solve the problems with minimal understanding, as they did not see the understanding of the conceptual aspects as important as they are more focused in getting the high grades in mathematics and trigonometry tests. Because of this, some tertiary education level students such as university students encountered difficulties in learning trigonometry, especially with reference to concepts, definitions, theorems and proofs of trigonometry. Many teachers neglect the fact that if the students actually understand both the conceptual knowledge and procedural skill of trigonometry, they will be able to develop thinking, reasoning, communication and modelling skills through a mathematical approach to problem solving and the use of the mathematical language. Through the understanding and applications of trigonometry, students will be able to connect ideas within and between mathematics and other disciplines.

This study investigated the correlation of conceptual knowledge and procedural skill of trigonometry by comparing the students' marks on the pre, post and delayed post tests. This will be followed by the study of the relationship of conceptual knowledge and procedural skills with difficulties of students in solving trigonometry problems.

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Conceptual knowledge is defined as the implicit or explicit knowledge of the principles that govern a domain and of the relationships between units of knowledge in a domain. Procedural skill, in contrast, is defined as the aptitude to execute a sequence of steps to solve problems (Johnson *et al*., 2001). Many researchers had placed eminent focus on whether conceptual or procedural understanding developed first (Haapasalo, 2003; Johnson *et al*., 2001). Nevertheless, it is believed that conceptual knowledge and procedural skill are strongly associated to one another thus this study will set out to determine the significant associations, if any, with trigonometry. Besides that, it is also important to investigate the misconceptions and the mathematical errors students always make in tests.

Many studies have been done to identify mathematics difficulties of students when solving mathematical problems. One of them is Gur (2009), who conducted his study on trigonometric learning and identified the types of misconceptions students made. He classified the five types of trigonometry misconceptions students usually make such as misused data, misinterpreted data, logically invalid inference, distorted definition and technical mechanical errors. Another study done by Seah (2005) who developed a conceptual framework classified the students' errors in integration problems into three categories namely conceptual, procedural and technical error. It will be interesting to investigate whether the selected students from the AS Level in one of the colleges in Brunei Darussalam will have comparative outcomes as those mentioned by Gur (2009) and Seah (2005). In addition to that, the misconceptions and common mathematical errors of local students will be identified simultaneously in order to analyze the students' understanding of the concepts in further depth.

Hence, this research is designed to investigate and seek answers to the following research questions:

- Is there any significant relationship in conceptual knowledge and procedural skill with the difficulties of problem solving in trigonometry?
- What are the mathematical misconceptions and common errors committed by the lower sixth students when solving trigonometric questions? From the research questions, three hypotheses are formulated. They are:
  - There is no significant relationship in students' procedural skills and conceptual knowledge.
  - There is no significant difference in students with difficulties in problem solving before and after the pre and post tests.
  - There is no significance difference in students with difficulties in problem solving before and after post and delayed post tests. Eventually, the acceptance and rejection for the hypotheses will be able to answer the research questions for this study.

SCOPE AND LIMITATIONS OF THE STUDY

The study is limited to one academic year (2013) to Pre University Lower Sixth AS Level students (Year 12) from a college in the Brunei Muara District. Two classes from this college were selected therefore this sample will not be a representative sample of the overall Pre University AS Level students' population in Brunei Darussalam. The scope of the study is only focused on Pure Mathematics on the topic of Trigonometry. The topic trigonometry was taught at selected time according to the scheme of work hence the study can only be done with respect to when the topic was taught by the tutor.

LITERATURE REVIEW

Piaget (1972) mentioned the importance of conceptual and practical knowledge and the similar features were reported in Gelman and Meck (1986) concerning conceptual and procedural competence. Subsequently, many studies concentrated on which type of knowledge developed first. Halford (1993) was one of the many researchers who agreed to concepts-first theories. In concepts-first theories, students either develop initially or born with them the conceptual knowledge which they would generate and solve problems using this knowledge. On contrary, procedures-first theories mentioned that students learn as they solve problem and hence develop the concepts from the repeating experiences (Siegler and Stern, 1998). According to Johnson *et al*. (2001) conceptual knowledge and procedural skills are interrelated to one another. They claimed that conceptual knowledge and procedural skills could be developed and improved through iterative process when they gave the tests on decimals and fractions. As far as is known, there had been no study on the relations of conceptual knowledge and procedural skill on difficulties of problem solving in trigonometry. Therefore, it will be worthwhile for this study to be carried out in order to understand how much impact it will exhibit on AS Level students in Brunei Darussalam who either have better trigonometry conceptual knowledge or procedural skill or probably possess both of them.

There were also a number of researches, which studied the mathematics difficulties faced by students when solving mathematical questions. Among them, Seah (2005) developed a conceptual framework to identify the three different types of errors made by students when performing integration test. He classified the first type as conceptual error. Conceptual errors were due to the failure to understand the concepts taught and were not able to interconnect the relationships involved in the problem. The second type was procedural error. In procedural error, students understood the concepts behind the problem but they were unable to carry out manipulations or algorithms on the problems. The third type of error was called the technical error whereby students lack in mathematical content knowledge from other topics or even due to carelessness. His results indicated that procedural errors were the most, followed by technical errors and conceptual errors. It was not easy and there was no fine line to distinguish completely between misconceptions and errors. Seah's research was compared to Gur (2009) "Trigonometry Learning" whereby Gur did a diagnostic test using seven trigonometric questions on 140 tenth grade high school students in Turkey and identified five trigonometry misconceptions such as misused data, misinterpreted language, logically invalid inference, distorted definitions and technical mechanical errors. Gur specifically identified the misconceptions mainly on the topic trigonometry and he identified the misconceptions of trigonometry in more details compared to Seah. For example, in misused data, students implied that tanx multiply with cotx would produce one. Another example would be that misinterpreted language was misconceptions on a concept that produced a mathematical object and symbol such as students who were confused to which side of the triangle was the adjacent side which eventually will give rise to technical error when students were already confused with the concepts.

In this research, three out of seven questions set by Gur (2009) in his study will be used as the syllabuses were not the same in Brunei Darussalam and Turkey. The tests collected were then analyzed by comparing the types of errors and misconceptions to that of Seah (2005) and Gur's study. Seah's study would be used mostly for reference purposes as he did not subdivide the errors in his study into greater details. On the other hand, Gur's study classified misconceptions in trigonometry more explicitly, which could be easily related to this research with the same topic. However, the drawback with Gur's study was that the examples given by him were not sufficient enough to fully understand his definitions in the misconceptions and no elaborations were given as well.

Orhun (2001) concluded that even though students at the basic level understood trigonometry, students treated the topic as short term lessons, which they will easily forget in the future. In the conference paper she mentioned that it was not easy to convey the knowledge to the students in a teacher centered classroom. Thus, it was suggested that some teaching methods should be eliminated and misunderstanding to be emphasized when teaching the topic. The conference paper provided a basis for comparing how students worked and behaved when doing trigonometry, however it did not provide detailed discussion on the teachers' teaching methods. Sadly, the abovementioned was again emphasized by Gur (2009) whereby he mentioned that mathematics teachers focused on algorithm approaches to solve problems and not conceptual learning. The fact was that the high school curriculum had too much to cover and the overwhelming syllabus was becoming a real barrier to actually concentrate on conceptual learning.

In another study by Blackett (1990), he mentioned that both boys and girls could understand basic trigonometry better if the teachers used computer graphics consisted of visual and numerical representations on trigonometry lessons. This again highlighted findings from Orhun (2001) and Gur (2009) regarding the importance of teaching methods and making the lessons interesting in trigonometry although computer graphics was not included in the intervention for this research. Hence, with reference to the studies above, class observations were done to understand to what extent teachers' teaching methods influenced the conceptual understanding of the trigonometric function. It was also anticipated that, post and delayed post test were given to students to further diagnose the students' understanding on the topic, misconceptions and mathematical mistakes of the students.

RESEARCH METHODOLOGY

Design: A correlational design, with pre, post and delayed-post test measurements with two classes was employed. The dependent variable was 'difficulty in problem solving of Trigonometry'. The independent variables were 'conceptual knowledge' and 'procedural skill'. The intervening variable was 'students' misconception and mathematical errors'. The same tutor taught the two classes that were selected in the college.

Participants: The participants of the study consisted of AS Level students (mean age 17.0) studying in one of the government college in Bandar Seri Begawan, Brunei Darussalam. The school was selected conveniently so that the pilot and full study were carried out successfully. Permission letter to carry out the study was acknowledged by the administrations of the college and the mathematics department. They were aware of the study being carried out in the college. Both classes took part in the pre, post and delayed post test. A total of 51 students took part in the tests. All the students in these two classes took Mathematics Syllabus D and Additional Mathematics for the GCE O Level Examinations. Hence, all of them had subject groundings on the topic trigonometry.

Instrumentation: Pre, post and delayed post tests were given to students to test the development of their understanding on trigonometry. There were a total of six questions in the trigonometry test. The first three questions were taken from Cambridge AS Level Examination question papers of 2009 and 2010. These

Table 1: The different category of students if they improved in the conceptual knowledge and procedural skills using the pre and post-tests

Improvements?	Conceptual knowledge	Procedural skills
Yes	There is a higher level thinking skills in the students.	Students are capable to answer examination questions on trigonometry.
No	Students did not learn the conceptual aspect of trigonometry.	Students do not understand how to solve trigonometry questions.

questions were selected because they were testing on the procedural skill of the students. The allocations of marks followed exactly the marking scheme of Cambridge International Examinations (CIE). Questions four to six were questions taken from Gur (2009) diagnostic test. These questions were intentionally selected as the mathematics syllabuses were not the same in Brunei Darussalam and Turkey where the diagnostic test was held. Therefore, only three appropriate questions based on conceptual knowledge of trigonometry were chosen. The question paper is attached in the appendix.

The six questions tested the students on the objectives in the syllabus pertaining to trigonometry (University of Cambridge Local Examination Syndicate, 2001):

- Graphs of the sine, cosine and tangent functions (for angles of any size and using either degrees or radians).
- Use the exact values of the sine, cosine and tangent of special angles and related angles.
- Use the notation of sine, cosine and tangent inverses to denote the principal values of the inverse trigonometric relations.
- Use the identities $tan\theta = \frac{sin\theta}{\cos\theta}$ and $sin^2x + cos^2x = 1$.
- Find all the solutions of simple trigonometric equations lying in a specified interval (general forms of solution are not included).

When the pre, post and delayed post tests had been completed, the results were analyzed in detail and categorized according to the types of errors. SPSS 20.0 for Windows was used to evaluate the data collected from the pre, post and delayed post tests. Paired Sample T Test was used to compare and to find the differences in conceptual knowledge and procedural skill. In addition to that, Pearson Correlation Coefficient was calculated as well from the SPSS to find the relationship of conceptual knowledge and procedural skill. This was followed with the analysis of conceptual knowledge and procedural skill with the difficulty of problem solving in trigonometry using Paired Sample t Test.

Ethics: In order to make sure that this research followed the rules of ethics, a number of steps were taken. Firstly, permission to carry out the research in school had been sought from the Department of School, Ministry of Education and the approval was received to conduct the research.

Procedure: Since the classroom teacher carried out the tests, the teacher was informed about several issues. During the test, students were allowed to use calculator, similar to the instructions given in the Cambridge AS Level Examination. In addition to that, students were given thirty minutes to complete the paper. They were requested to give explanations on the questions they were not able to solve so that the researcher could understand the difficulties the students encountered. At the same time, this also provided the opportunities to find out the misconceptions and the errors performed by the students.

Pilot study: A pilot study was carried out to a total of 31 students in one class in the first quarter of the year, before the actual implementation of the main study. The instructions were followed as mentioned in the procedure above. By doing this pilot study, a preliminary testing was successfully carried out whereby a more precised hypothesis was made concrete and alternative measures were set up to obtain the clearest results for the main study. With reference to Table 1 on the different predicted outcomes when the pre and post tests results were compared and analyzed for any improvements. For example, by comparing the pre and post test, if the student improved in the first three questions (questions on procedural skill) of the test but not the last three questions(conceptual knowledge), then the student was considered to be capable to answer examination questions on trigonometry, nonetheless, the student had not learned the conceptual aspect of the topic yet.

The test answers done by the 31 students for the pilot study were entered and analyzed using the SPSS 20.0 to investigate the reliability test. The reliability test, Cronbach Alpha, was performed and the value was 0.699. As a result, this showed that the trigonometry test was reliable to be carried out for the main study (Appendix).

General predictions can be obtained from the pilot study that was carried out. It can be seen that AS Level students from this college had good understanding of the procedural skill in trigonometry and they were able to solve the past year questions, putting the sequence of workings in a correct manner. On the other hand, it can also be seen that the conceptual knowledge of the students in the pilot test was at the lower side. This was worrying as it was observed that the students did not understand the basic of trigonometry at all. A handful of students commented that they were not taught on the conceptual aspects hence they did not know the answers. Students lack the initiative to explore and reach out to understand the topic in depth. Additionally, a number of common mistakes were spotted and misconceptions had been noted in the pilot test, which provided great benefit and insights before conducting and reporting on the main study.

RESULTS AND DISCUSSION

For the ease of analyses, the students' test papers were labeled A for pre test, B for post test and C for delayed post test. The students' test papers were also numbered 1, 2, 3 and so forth. For instance, student number one will be labeled A1 for responding to the pre test and student number forty nine would have the label C49 for the delayed post test response. By doing so, the analysis would be smoother as any mistakes could be tracked easily and locating a particular paper would be easier as well. To answer the first research question, Pearson Correlation Coefficient and Paired Sample T Test in the SPSS would be utilized. The second research question would be answered with the usage of percentage and comparison between the three tests.

For the first hypothesis of this research, there was no significant relationship in students' procedural skills and conceptual knowledge in trigonometry function problems. The Pearson Correlation Coefficient of the pre, post and delayed post tests were obtained in order to compare the two qualities. Refer to Table 2 for the results of Pearson Correlation Coefficient on the three tests. The results from the three tests indicated that indeed there was no significant relationship in students' procedural skills and conceptual knowledge, having $p>0.05$ for all the tests mentioned. This showed that the concepts or workings that were learned on procedural skill and conceptual knowledge were not used or linked to one another. This evidence was more obvious in the delayed post test as most students managed to solve problems on procedural skills but not conceptual knowledge. The same test was given to the same students three times. It was observed that there were improvements in the procedural skills but very minimal improvements on conceptual knowledge. Students seemed to have learned procedural skills but not conceptual knowledge. In addition to that, the students did not show any initiative to investigate the methods of solving these questions or make any effort to find out the correct answers to the conceptual questions and yet they still obtained wrong or no answer during the delayed post test. Despite that, this research also investigated if the conceptual knowledge or the procedural skills or both were retained before and after the tests, which gave rise to the second and third hypotheses of the research.

For the second hypothesis, there was no significant difference in students with difficulties in problem solving before and after the pre and post tests. A Paired Sample t Test was conducted for the pre and post tests. The results showed that $t = 6.219$ with $p = 0.000$ whereby $p<0.05$. Because the probability was less than 0.05, therefore the hypothesis was not accepted. Hence, the second hypothesis was rejected which denoted that there was a significance difference in students with difficulties in problem solving before and after the pre and post tests. In other words, this indicated that the students improved in the post test when compared to the pre test. Consequently, we would want to find out as well if the same group of students improved from the post test to delayed post test.

Subsequently, the third hypothesis of this research study, there was no significant difference in students with difficulties in problem solving before and after the post and delayed post tests. Similar to the second hypothesis, a Paired Sample t Test was utilized to understand the outcomes of the results. It was found that $t = 0.437$ and $p = 0.664$ whereby $p>0.05$. Since the probability was more than 0.05, hence the hypothesis was accepted. Therefore, the third null hypothesis was accepted indicating that there was no significant difference in students with difficulties in problem solving before and after the post and delayed post tests. This was a good indication as the trigonometry knowledge learned by the students was retained after a month when the delayed post test was administered.

From hypotheses two and three it was found that the students improved from pre to post test and the knowledge was retained when delayed post test was given to the students. However, the above hypotheses only mentioned the tests as a whole. There was no clear indication showing if the students improved and retained the knowledge on conceptual knowledge or

Table 2: The correlation coefficient and hypotheses of students' procedural skills and conceptual knowledge for all the tests

Correlation coefficient, r						
		Procedural skills				
		Pre test	Post test	Delayed post test	p	Null hypothesis
Conceptual knowledge	Pre test	0.165	-	-	0.257	Accept
	Post test	-	0.217	-	0.127	Accept
	Delayed post test	-	-	0.045	0.753	Accept

Table 3: The paired sample t test for the procedural skill and conceptual knowledge of the three tests

Paired differences	t	p	Mean
Total procedural skill and total conceptual knowledge (delayed post test)	21.382	0.000	8.333
Total procedural skill and total conceptual knowledge (post test)	18.739	0.000	7.922
Total procedural skill and total conceptual knowledge (pre test)	8.258	0.000	4.061

Table 4: Definitions of the three added categories to the trigonometric misconceptions

No.	Categories	Definitions
1	No working	No working is given to the particular question
2	Incomplete	Incomplete workings and no final answer is given
3	Full mark	Students obtain all the marks and no misconceptions or errors are noted

procedural skills. Hence, another hypothesis was set up to investigate whether the students improved either in conceptual knowledge or procedural skill or both.

For the fourth hypothesis, there was no significant difference in students' procedural skills and conceptual knowledge before and after the tests. The implications of the tests in the hypothesis were the improvements from the pre to post test and the retention of knowledge from the post to delayed post test. Refer to Table 3 for more information on the Paired Sample t Test for procedural skills and conceptual knowledge for the tests.

The fourth hypothesis was rejected with $p<0.05$ in all three cases. This indicated that there was a significant difference in students' procedural skills and conceptual knowledge before and after the tests. The rejection of this hypothesis suggested that between conceptual knowledge and procedural skill, the students did better in one of them but not both. To find out whether the students did better in conceptual knowledge or procedural skill, the mean values from the Paired Sample t Tests were analyzed. The mean value in the pre test as shown in Table 3 was 4.061, which signified that students did better in procedural skills compared to conceptual knowledge. A higher mean was obtained for the post test with a value of 7.922. The higher the mean value meant that the students showed greater improvements in procedural skills than conceptual knowledge. In addition to that, the mean value in the delayed post test increased to 8.333 again strengthened the analysis that procedural skills were improved and learned more than conceptual knowledge.

Item analyses: Another investigation on the misconceptions and errors were performed as well by analyzing the data on SPSS. To study the misconceptions and errors in greater details, every single question was analyzed to understand the mistakes of the students followed by the categorizing them into the respective five trigonometric misconceptions as mentioned by Gur (2009). To make sure that all the data were completely entered into SPSS, another three categories were created. The three categories were 'No Working', 'Incomplete' and 'Full Marks'. A detailed definition of the three categories can be found in Table 4.

When the students did not know how to solve a problem, they would just leave it blank or if there were no mistake in the question, then that particular question would not be categorized into the five misconceptions. Thus three extra categories were set up in this case. 'No Working' would be labeled to questions whereby no working was produced at all. 'Incomplete' would be given to workings that were incomplete and no final answer was given. Last but not the least, 'Full Marks' meant that the students obtained all the marks thus no misconceptions or errors were noted. For the item analyses, only selected errors and categories that had an impact on the questions and answers were discussed and elaborated:

Question 1(a) Show that the equation $2\,sin\,x\,tan\,x + 3 = 0$ can be expressed as $2cos^2x - 3\,cos\,x - 2 = 0$ (1)

Referring to Eq. (1), Question 1(a) was testing on the procedural skill of the students on rearranging the quadratic equation using the formula. This question required the students to understand the formula of trigonometry and apply them appropriately. There was a great improvement from the pre test to the delayed post test as the percentage of students who obtained full marks increased from 36.7 to 82.4% (Table 5). In addition, the percentage of students who did not attempt the question also reduced by approximately 43%. The common misconception that was noticed from the three tests as compared to Gur (2009) research was distorted definition. Students tend to come out with their own definition without any valid reasoning thus deviated from the correct formula and resulted in wrong answers. Not only that, it could be seen that the percentage of misconception on distorted definition increased in post test which indicated that students actually got confused after the topic trigonometry were taught to them. Fortunately, the same misunderstanding was reduced again in the delayed post test:

Question 1(b) Solve the equation $2cos^2x - 3\,cos\,x - 2 = 0$ for $0^o \leq x \leq 360^o$ (2)

Table 5: Types of errors, in percentages, done by the students for all the three tests in question 1(a)

Types of error for question 1(a)	Pre test	Post test	Delayed post test
Distorted definition	2.0	11.8	2.0
Technical mechanical error	-	2.0	-
Logically invalid inference	-	-	5.9
No working	49.0	9.8	5.9
Full marks	36.7	74.5	82.4

Table 6: Types of errors, in percentages, done by the students for all the three tests in question 1(b)

Types of error for question 1(b)	Pre test	Post test	Delayed post test
Distorted definition	6.1	23.5	29.4
Technical mechanical error	4.1	5.9	2.0
Logically invalid inference	14.3	-	-
No working	18.4	2.0	7.8
Full marks	32.7	68.6	60.8

Table 7: Types of errors, in percentages, done by the students for all the three tests in question 2

Types of error for question 2	Pre test	Post test	Delayed post test
Misinterpreted language	2.0	-	-
Logically invalid inference	16.3	5.9	2.0
Distorted definition	8.2	5.9	3.9
Technical mechanical error	-	-	2.0
No working	20.4	-	2.0
Full marks	44.9	86.3	86.3

Question 1(b) as shown in Eq. (2) was also testing on the procedural skills of the students. It was a typical question on solving trigonometric equation. This again required the students to have a comprehensive understanding of the formulae and abilities to apply them. It can be seen from Table 6 that there was improvement from pre to post test, however, the percentage of students obtaining full marks decreased in the delayed post test. Once again, this strengthened the findings by Orhun (2001) whereby she concluded that even though students understood trigonometry at the basic level, they treated the topic as short term lessons, which they would easily forget in the future. The most common misconception observed was distorted definition. Students still performed mistakes on distorted definition, which was repeated from Question 1(a) since both questions tested on the trigonometry formula. In addition to that, the percentage of students who made this mistake increased from pre test to delayed post test. As mentioned previously, the students did trigonometry when they were in upper secondary and they were learning the topic trigonometry for the second time. This again showed that the students were lacking in the conceptual knowledge of trigonometry regardless the number of times they studied and went through the topic in the classroom:

Question (2) Prove the identity $\frac{\sin x}{1-\sin x} - \frac{\sin x}{1+\sin x} \equiv 2tan^2 x$ (3)

For Question 2, the question was asking the students to prove a given identity as given in Eq. (3). This ideally chosen test question gave a good definition on procedural skills as the aptitude to do the manipulations of the formula and executions of a sequence of steps for this particular question was tedious and complicated. If the students had performed ample practices on proving identities, no doubt, they would be able to obtain excellent marks in this question. Even when the pre test was given to the students, this question proved to be the question, which most students were able to answer. The evidence can be seen from Table 7 whereby the post and delayed post tests results showed that there was a tremendous improvement of approximately 42% from the pre test. Additionally, students seemed to do very well in this question with minimal misconception. Therefore, this was one of the evidences that students performed very well in procedural skills:

Question 3(a) Sketch the curve $y = 2\sin x$for $0 \leq x \leq 2\pi$ (4)

Although students did better in procedural skills compared to conceptual knowledge questions, there was one sub topic on trigonometry that the students were weak in-trigonometric graphs. Question 3(a) as shown in Eq. (4) was a simple and direct question asking students to sketch $y = 2\sin x$. Referring to Table 8, misinterpreted language was claimed to be the main misconception in the pre test as students did not remember how to draw the sine graphs. Fortunately, after the topic was taught, there was no more misinterpreted language occurred. However, students got themselves confused in misconceptions such as distorted definition and technical mechanical error. The common errors found in this question were careless mistakes such as students forgot to change the angle into radians, failure to write down the angles and the sine values. The percentage of students who obtained full marks was from 22.4% in the pre test, improved to 66.7% in the post test but decreased again in delayed post test to 56.9%. Evidently, these showed that short term lessons were learned for trigonometry:

Table 8: Types of errors, in percentages, done by the students for all the three tests in question 3(a)

Types of error for question 3(a)	Pre test	Post test	Delayed post test
Misinterpreted language	46.9	-	-
Distorted definition	-	17.6	19.6
Technical mechanical error	20.4	13.7	13.7
No working	10.2	2.0	7.8
Full marks	22.4	66.7	56.9

Table 9: Types of errors, in percentages, done by the students for all the three tests in question 3(b)

Types of error for question 3(b)	Pre test	Post test	Delayed post test
Misinterpreted language	46.9	-	-
Distorted definition	-	17.6	19.6
Technical mechanical error	20.4	13.7	13.7
No working	10.2	2.0	7.8
Full marks	22.4	66.7	56.9

Table 10: Types of errors, in percentages, done by the students for all the three tests in question 4

Types of error for question 4	Pre test	Post test	Delayed post test
Misinterpreted language	6.1	45.1	33.3
Logically invalid inference	2.0	-	-
No working	91.8	54.9	66.7
Full marks	-	-	-

Question 3(b) By adding a suitable straight line to your sketch, determine the number of real roots of the equation $2\pi \sin x = \pi - x$. State the equation of the straight line (5)

Question 3(b), as shown in Eq. (5), was a continuation question from question 3(a) on the applications of trigonometric graphs. This question asked the students to what extent their knowledge on trigonometric graphs to find the number of roots for the equation given. All the students needed to do was to rearrange the equations so that they could find the straight line equation, followed by drawing the straight line on the same sketched graph from question 3(a) and attained the number of points of intersection. Nevertheless, it can be seen from Table 9 that a large number of students failed to do well in this question in the pre test with only 22.4% of students getting full marks. The results were much better in the post test but decreased again in the delayed post test. It was surprising to see that the number of students who did not respond to this question increased from 2.0 to 7.8%, showing that no effort was given to even attempt the question. The misconception on misinterpreted language was the biggest barrier with 46.9% at the beginning when the students did their pre test. This misconception was removed after the topic was taught and instead, the increase of the misconception on distorted definition still remained a main concern on students' misconceptions on trigonometry. A very common careless mistake made by a large number of students was that they forgot to write down the number of roots in the answer sheet as required by the question, which gave rise to the high percentage of technical mechanical error:

Question (4) $sin^2x + cos^2x = 1$.
Please explain why? (6)

Referring to Eq. (6), Question 4 was the first question testing on the ability of the students on conceptual knowledge of trigonometry. The question was asking the students to explain why $sin^2x + cos^2x = 1$. Undoubtedly, all the students knew or had seen this formula before. However, none of them could actually give a correct explanation on the question, which was the reason nobody obtained full marks for this question in all the three tests. A high percentage of 91.8% (Table 10) of students in fact did not even try to work out the reasoning during the pre test. For all the students who attempted the tests three times, it was seen that they either did not know how to solve this problem or maybe also, they had misconception on misinterpreted language. Of all the mistakes that the students made, one common error majority of the students did was writing $sin^2x + cos^2x = 1$ and using the same formula to replace cos^2x to become $sin^2x + (1 - sin^2x) = 1$. According to Gur (2009), he stated that the students simply just memorised the formula and then calculated it using the unit circle. He also mentioned that this was attributed to the fact that the mathematics textbooks and teachers almost always commenced this topic with the same technique as shown by the students. The students would retain the knowledge learned by them in secondary school right up to pre university level that $sin^2x + cos^2x = 1$ formula but they could not give a precise explanation of how to obtain one. The justification given by Gur had the similar aspects with the pre university level students who did the tests for this research study:

Question (5) $\tan 90^o$ is undefined.
Please explain why? (7)

Another question that tested on conceptual aspects of trigonometry learning of students were again proven that the students' conceptual knowledge was weak. All

Table 11: Types of errors, in percentages, done by the students for all the three tests in question 5

Types of error for question 5	Pre test	Post test	Delayed post test
Misinterpreted language	40.8	60.8	56.9
No working	59.2	37.3	43.1
Full marks	-	-	-

Table 12: Types of errors, in percentages, done by the students for all the three tests in question 6

Types of error for question 6	Pre test	Post test	Delayed post test
Misinterpreted language	26.5	29.4	39.2
No working	71.4	45.1	43.1
Full marks	2.0	25.5	17.6

the students were either not producing any workings or giving workings with misconceptions on misinterpreted language. Question 5, as can be seen in Eq. (7), was a simple question asking the students to explain why tan 90° was undefined. A handful of students drew the tangent graphs without any explanation, considering maybe the students had difficulties explaining the answers in English. Some gave the answer 'error' shown in the calculator when they punched in the numbers indicating that tan 90° was undefined. Comparing Gur (2009) research and this research study, he had three students out of 140 (approximately 2%) who showed misconception in this question and one student (less than 1%) did not respond to this question. The remaining of the high school students who sat for his diagnostic test in Turkey managed to understand and gave the correct answers.

Nevertheless, Gur (2009) did not mention the types of misconceptions the three students did. For this study, each item was analyzed and categorized into their respective misconceptions groups. Surprisingly, the pre university students who did this research study in Brunei Darussalam had an average percentage of 52.8% from the three tests performed misconception on misinterpreted language and an average of 46.5% who did not show any working at all, whereby the results can be seen from Table 11:

Question (6) $\cos(-\theta) = \cos\theta$
Please explain why? (8)

Equation (8) showed the last question of the test, Question 6. This question was asking the students to explain why $\cos(-\theta) = \cos\theta$, the final question on conceptual knowledge and also of the test paper. During lesson observation, it was noted that the tutor mentioned about the four quadrants and negative angles. The increased in the percentage of the students obtaining full marks ranging from 2.0% in the pre test to 25.5% in the post test (Table 12) indicating that there were learning taking place in the classroom. The number of students who attempted the question also increased after the lessons on trigonometry was taught. However, the misconception on misinterpreted language also increased from pre test to delayed post test showing that students were confused and not performing well in conceptual knowledge of trigonometry. Once again, Gur (2009) pointed out in his research that the students either had forgotten the memorized formula or the formula was remembered incorrectly.

The students were asked to write down the reasons if they were unable to solve a problem. A few mentioned that since they memorized the formulae, therefore they were not certain on how or why these formulae were obtained. A number of them gave the reason they were not able to solve the problem was because they forgot the formulae or identities but most of the students gave the reason that they did not know or understand the topic. All these reasons summed up to disclose that only short term learning had taken place in students which is very similar to the research done by Orhun (2001).

Other than the pre, post and delayed post tests that were carried out for this research study, classroom observations were also made during the trigonometry lessons. The teacher had five lessons on trigonometry with the students and three randomly selected lessons were observed. From the lesson observations, it was seen that the teacher covered reasonable number of theories and problems with none of the lessons pace observed too fast or too slow. Lots of practices were cultivated during the lessons with minimal explanations on the conceptual aspects of trigonometry. Teacher went through the answers in a chalk and talk manner and exercises were given to students as homework. There was no communication barrier between the teacher and the students whereby the students happily provided responses appropriately according to questions asked by the teacher. The teacher also mentioned about common mistakes and errors on trigonometry so that the students were aware and did not repeat the misconceptions.

CONCLUSION, IMPLICATIONS AND RECOMMENDATIONS

Comparing the pre, post and delayed post tests, it can be concluded that there was no relationship or connections between conceptual knowledge and procedural knowledge. From the tests it was also found that students improved tremendously in problems on procedural skills and the knowledge on procedural skills were retained as well. On the other hand, a minimal improvement of conceptual knowledge was recorded. This again showed that the students would be able to score in the examinations as long as they have the procedural skills. Poor performance in questions on conceptual knowledge will not cause the loss of majority marks. Thus, this shifted the students' perceptions on the subject that frequent practice will achieve good marks and conceptual learning is less crucial when it comes to getting excellent grades in examinations. Eventually, this dangerous shift will perhaps make the students lose any curiosity and lack the initiatives to find out and investigate the conceptual aspects of trigonometry.

Students who were able to solve problems on procedural skills will still be able to obtain good marks in the examinations although they may have weak conceptual understanding of trigonometry. The students, for sure, may be able to obtain excellent marks for the examinations but the knowledge retained may be short term. The findings found in this study may be taken as evidence because when students obtained good marks for the post test after they had learned the lesson recently and produced unsatisfactory marks in the pre test as most of them claimed to forget the theory, not forgetting that they only learned the topic the year before when they were in Year 11. This again demonstrated the diverse learning styles of students (Hamid *et al*., 2013; Matzin *et al*., 2013; Shahrill *et al*., 2013), which for some, also placed more emphases on getting good marks for examinations rather than understanding the conceptual aspects of the topic (Botty and Shahrill, 2014).

The greatest obstacle is that students memories the sequence or steps to solve a trigonometry problem which resulted in bad performance in the long run. The common misconceptions depicted from the tests were misinterpreted language, distorted definition and technical mechanical error when compared to Gur (2009) research. Also, the students made very little careless mistakes and comparing to Seah (2005) research, the students in Brunei Darussalam had more conceptual error, which was the biggest problem for students who were not able to solve the questions on trigonometry.

Although only two classes were involved in this research, this can be seen sufficient enough to allow insight on our schools today. A further research investigation can be done if more time is given to this research and a number of studies can be conducted in the future such as researches on the poor performance of students in trigonometry graphs and its applications, the ability to use mathematical language in relation to trigonometry in the classroom, comparative teaching strategy in trigonometry and also to investigate further to what extent conceptual understanding on trigonometry is taught and learned in the classroom.

APPENDIX

Trigonometry Test

Instruction: Answer All Questions. State the reason/s if you cannot answer a question or the part question:

- AS Level Paper 13, June 2010 Question 4
  - Show that the equation $2\sin x \tan x + 3 = 0$ can be expressed as $2cos^2x - 3\cos x - 2 = 0$ [2 marks]
  - Solve the equation $2cos^2x - 3\cos x - 2 = 0$ for $0^o \leq x \leq 360o$ [3 marks]
- AS Level Paper 13, June 2009 Question 1 Prove the identity $\frac{\sin x}{1-\sin x} - \frac{\sin x}{1+\sin x} \equiv 2tan^2x$ [3 marks]
- AS Level Paper 13, November 2010 Question 4
  - Sketch the curve $y = 2\sin x$ for $0 \leq x \leq 2\pi$ [1 mark]
  - By adding a suitable straight line to your sketch, determine the number of real roots of the equation $2\pi \sin x = \pi - x$. State the equation of the straight line [3 marks]
- $sin^2x + cos^2x = 1$. Please explain why? [1 mark]
- $\tan 90^o$ is undefined. Please explain why? [1 mark]
- $\cos(-\theta) = \cos\theta$. Please explain why? [1 mark]

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