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Research Article

Gas-dynamic Variable Relation on Opposite Sides of the Gas-dynamic Discontinuity

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Abstract: The goal of this study is to study the conditions of dynamic compatibility on gas-dynamic discontinuities written in the form of a generalized adiabat. We have considered the basic concepts of the gas-dynamic discontinuity theory, the ratios permitting to calculate pressure shocks. Recommendations for rational problem definition and methods of solution of the typical computational problems are given. The dependences for calculation of parameters behind the shock according to the known parameters of a stream and the shock intensity recorded for the first time with the help of a generalized adiabatic line are considered. Substituting in these relations equations of adiabatic line of Laplace-Poisson, Rankine-Hugoniot and Chapman-Jouget, you can calculate the parameters behind, accordingly: simple waves, shockwaves and detonation waves. There are given in friendly graphic form the dependence on the Mach number of incoming flow and gas adiabatic index of the most relevant parameters of shocks: maximum intensity, stream deviation angle on the shock, critical angle of the stream deviation, shock angle according to the critical angle of a the stream deviation. The work can be recommended to the experts, engineers and scientists working in the field of aerospace engineering, metallurgy and metal hardening, for usage of control technologies for hypersonic currents containing gas-dynamic discontinuity.

Keywords: Maximum shock-wave amplitude, shock, shock intensity, shockwave

INTRODUCTION

The objects of study are relations connecting the gas-dynamic variables on both sides of the gas-dynamic discontinuity (shock, shock wave or detonation), known as Dynamic Compatibility Conditions (DCC). In many practical cases it is necessary to know how to expect gas-dynamic discontinuities in hypersonic streams, in particular, shocks. The shock calculation is determination of its intensity, shock angle, stream turn angle on the shock, presence determination of relations between the basic gas-dynamic variables and their derivatives before the shock and behind it.

At a given Mach number the dependence of the shock wave intensity on flow turning angle on it was called shock polar. For the characteristic shape of this function's plot it is often called a heart-shaped curve. Since the shock polar is plotted for a given Mach number, it is called iso-mach. Gas-dynamic discontinuities may occur in smooth flow regions due to various physical phenomena. It may be shock waves, centered isentropic wave and detonation waves. For each of these physical processes their own dependence between density and pressure exists, which is called an adiabat. It is useful to be able to write the ratio of gas-dynamic discontinuity in a form that would be independent of the physical processes leading to the field discontinuity in gas-dynamic variables. For this purpose, this study introduces the concept of generalized adiabat. Differential equations for the density and velocity potential describing the one-dimensional unsteady motion of inviscid perfect isothermal gas were first introduced in 1788 in the book of Lagrange (1788). Poisson (1808) introduced the concept of the sonic speed when considering the propagation of a plane compression wave.

Stokes (1848) first introduced the concept of discontinuity in the field of a continuous environment flow and received two conditions for the density p ρ and velocity of the gas u on the sides of the discontinuity resulting from the laws of mass and momentum conservation. Earnshaw (1858, 1860) considered the one-dimensional unsteady gas flows, both isothermal and adiabatic. He obtained the solution in form of a plane wave, in which areas of sharp parameters changes occurs over time. He, same as Stokes, called them discontinuities. An important role in the analysis of gas-dynamic discontinuities, supersonic gas motions plays the speed of disturbances propagation-the sonic speed.

Conditions, formulated by Stokes are insufficient to determine the two unknown parameters of the flow behind the discontinuity and the propagation speed the discontinuity itself. The first attempt to close the

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equations system, written by Stokes was published in 1860 in the work of Riemann (1860). In this study, the author suggested that during passing through normal discontinuity the entropy is constant and supplemented Stokes' system with the third equation. In the meantime, Riemann (1860) couldn't explain the changes in energy when passing through the discontinuity that occurred with this assumption.

Independently of Riemann (1860) and Rankine (1869, 1870) obtained the third equation, supplementing the Stokes' system in another form in 1869-1870. He determined a link between the parameters on the sides of shock wave, having considered constantly changing state of the environment within it, in which the equilibrium heat exchange occurs. The total amount of heat obtained by the environment must be zero. Using the relations of equilibrium thermodynamics and the Stokes formula, Rankine (1869, 1970) obtained an expressions for the normal discontinuity propagation velocity in a stationary environment a (not to be confused with sonic velocity a) and for the flow rate in terms of known pressures in front of the discontinuity and behind it, as well as known relative volume before the discontinuity for a perfect gas.

The most important Rankine's result is the assertion that normal discontinuity always propagate with supersonic speed relatively to stationary environment with, while, relatively to the environment behind the discontinuity it is always subsonic. A method of obtaining DCC on a shock wave used by Rankine (1869, 1870) leads to the implementation of all conservation laws, but it takes into account the gas' thermal conductivity and neglects its viscosity, which is very valid, because viscosity and thermal conductivity are interrelated. Hugoniot obtained the condition on normal discontinuity more strictly than Rankine, as a consequence of the law of energy conservation, avoiding consideration of the gas' state "inside" of the shock wave (Hugoniot, 1889). This condition coincides with the previously obtained Rankine condition, but to obtain it Hugoniot didn't require additional assumptions. Detailed analysis of gas-dynamic waves (isentropic expansion and compression waves) and oblique shocks arising in plane steady inviscid flows of non-conducting perfect gas was published in T. Mayer (1908). In the same paper the parameters of oblique shocks, formed around a plane acute angle were defined. This is an important task for the practice, since flow around an inclined barrier is one of the most common causes for a shock wave in the gas stream. Starting with this study of Mayer the shock wave intensity (the ratio between the static pressures on its sides) is considered as the main parameter characterizing it. In their modern form DDC at the shocks were formulated (Uskov, 1980). They were later developed for the case of one-dimensional traveling waves (Uskov, 2000) and for the oblique shock waves (Uskov *et al*., 2002). The research of heart-shaped curves, performed by Uskov *et al*. (1995) allowed to determine their important properties: the presence of the envelope, limiting deflection angles at the discontinuity, points corresponding to the discontinuities, behind which the Mach numbers equals to 1. It may be noted that the presence of the envelope is important in problems of supersonic aerodynamics (Uskov and Chernyshov, 2014) as it corresponds to pressure extremums on the sides of the body, flying with a predetermined angle attack, but at variable velocity.

The problem is more than a hundred years old, but its solution still causes difficulties. The situation is getting more complicated, if it is required to find some optimal solution, i.e., to select from a series of possible realizations of the shock the unique one which meets the given optimal criterion.

Relations describing the heart-shaped curves have long been known, but their use still often causes trouble due to computational traits and necessary selection from a variety of formal roots. Let us consider the laws giving relations on the discontinuities as well as the properties of shock polars.

MATERIALS AND METHODS

Gas-dynamic discontinuity image as features of reflection of projection of gas-dynamic parameters variety: Relations of variables $\hat{f}$ and f on opposite sides of gas-dynamic discontinuities are produced from the Conditions of Dynamic Compatibility (CDC) on them. In the lab coordinate system, CDC on the stationary discontinuities are balances of specific streams.
Substance:

$$\left[\rho \upsilon_n\right] = \hat{\rho}\hat{\upsilon}_n - \rho\upsilon_n = 0 \tag{1}$$

Normal:

$$\left[p + \rho\upsilon_n^2\right] = 0 \tag{2}$$

And the pulse tangent component:

$$\left[\rho\upsilon_n\upsilon_\tau\right] = 0 \tag{3}$$

The energy component:

$$\left[\rho\upsilon_n h_0\right] = 0 \tag{4}$$

where, υ_n and υ_τ projections of the velocity vector on the discontinuity plane. It follows from (1-4) that there are 2 kinds of discontinuities: tangent (τ, where $\upsilon_n = 0$) and normal (shock) through which the gas flows. It follows from (2) that on the both sides τ the static pressure is the same and (3) shows that the tangent components can differ, i.e., τ they can be slip lines. The density, temperature, full heat content and entropy of streams divided with tangent discontinuities can differ.

The given system allows to simply receive the Laplace-Poisson adiabatic lines (isentropy):

$$JE^{\gamma} = 1 \tag{5}$$

And Rankine-Hugoniot (shock adiabatic line):

$$E = \frac{1+\varepsilon J}{J+\varepsilon} \tag{6}$$

where, $\varepsilon = (\gamma - 1)/(\gamma + 1)$, γ – the adiabatic line index, $E = \rho / \hat{\rho}$, $J = \hat{p} / p$ -the intensity of the shockwave process (J>1) or rarefaction (J<1).

The isentropic curve (1.5) is true for simple pressure waves (J>1) and rarefaction waves (J<1) stationary waves (Prandtl-Mayer) or running waves (Reimann). The shock adiabatic line appeared caused by modeling of shocks and shockwaves because of discontinuity surfaces.

In the direct shocks relations $\hat{f}$ and f are specified from the system (1-4), where velocity D is the velocity of the shock travel on the source stream having the velocity U:

$$\bar{\upsilon}_n = |U - D| \geq a \tag{7}$$

Which transforms the given system CDC into the form of CDC-*D*:

$$[pu] = [\rho] D \tag{8}$$

$$\left[p + \rho u^2\right] = [\rho] D = [\rho u] D \tag{9}$$

$$[h_0] = [u] D \tag{10}$$

At *D* = 0 the system (7-10) describes CDC on the direct shock, $\upsilon_\tau = 0$ in (3).

From (7) and (1.10) it follows that at $D = U$ ($\upsilon_n = 0$) the value $\hat{U} = U$, i.e., $[u] = 0$ and there is a surface of the variables discontinuity which gas cannot flow through. Such discontinuity is contact one ($\bar{K}$). It travels at the rate of gases $\hat{U} = U = D$ and divides the streams with different thermodynamic variable (except for statistic pressure $\hat{P} = P$, like on the tangent discontinuities). In consequence of the Clapeyron equations ($p = \rho R T = \hat{\rho}\hat{R}\hat{T}$) for perfect gas ($R = \hat{R}$) on $\bar{K}$ the following equtions in progress:

$$\hat{\rho}\hat{T} = \rho T, \frac{\hat{M}}{M} = \frac{a}{\hat{a}} - \sqrt{\frac{T}{\hat{T}}}$$

So, the contact discontinuity is a special trajectory dividing the gases with different thermodynamic parameters (except for pressure).

From CDC-D (10) it also follows that, as opposed to shocks on the shockwaves, there is a discontinuity of total heat content. The very direct shocks are particular cases of the standing shocks (*D* = 0) in the supersonic streams of gas.

The oblique shock intensity ($\bar{\sigma}$):

$$J_\sigma = (1+\varepsilon)\left(\frac{\upsilon}{a}\right)^2 \sin^2\sigma - \varepsilon = (1+\varepsilon) M^2 \sin^2\sigma - \varepsilon \tag{11}$$

Depends on the normal component of the velocity $\upsilon_n = \upsilon sin\sigma$. Here, σ is a shock angle of the velocity vector to the shock plane which can change within the limits of $\alpha \leq \sigma \leq 90°$, where $\sin\alpha = 1/M$ is the Mach angle whereby the shock degenerates into the Mach line ($J_\sigma = 1$).

The maximum intensity meets the direct shock:

$$J_m = (1+\varepsilon) M^2 - \varepsilon \tag{12}$$

Or direct shockwave:

$$J = (1+\varepsilon)\left(\frac{U-D}{a}\right)^2 - \varepsilon \tag{13}$$

The values of J specify other gas-dynamic variables after discontinuities: density (with the help of shock adiabat) (6), temperature ($\hat{T}/T = EJ$), acoustic speed ($\hat{a}/a = \sqrt{EJ}$).

Braking parameters after these discontinuities can be specified with the help of the generalizing formula:

$$J_0 \equiv \left(\frac{\hat{p}_0}{p_0}\right) = \left(\frac{H_0^{\gamma}}{JE^{\gamma}}\right)^{\frac{1}{\gamma-1}} \tag{14}$$

where, the value of $H_0 = \hat{h}_0 / h_0$ on the shock does not support the discontinuity on shocks and $H_0 \neq 1$ on shockwaves. On shocks at $H_0 = 1$, as it follows from (14), the loss factor of the total pressure:

$$J_0 = \left(JE^{\gamma}\right)^{-\frac{1}{\gamma-1}} \tag{15}$$

Depends on the intensity with use of the shock adiabatic line Eq. (6). And is entropy (5) shows that in the Prandtl-Mayer stationary waves ($\bar{\omega}$) the total pressure does not change.

Using the function:

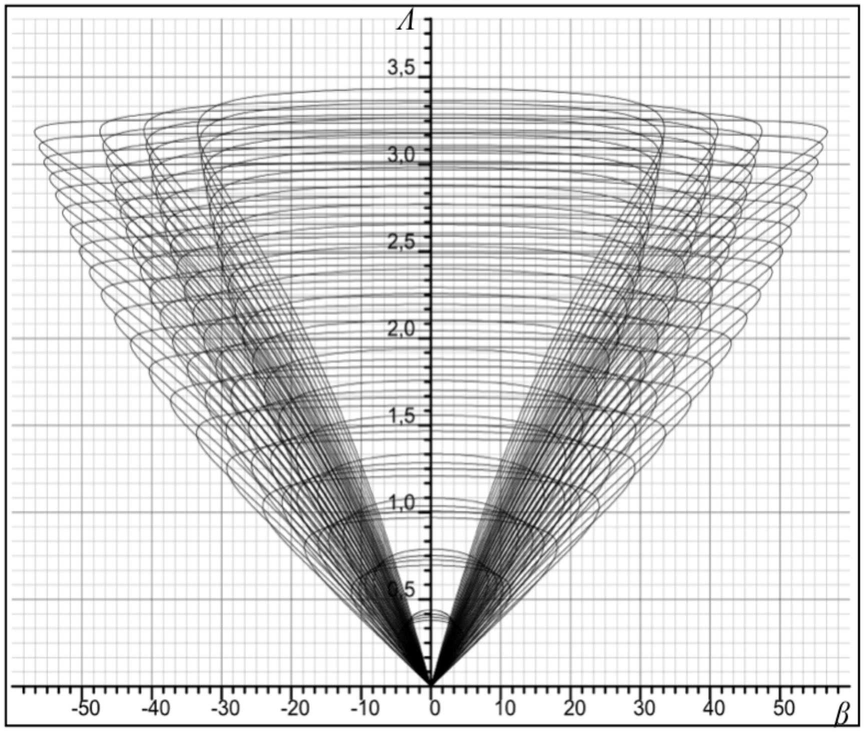

Fig. 1: Cardioform curves for the Mach numbers = 1.5-5 and indexes of the adiabatic line γ = 1.1 (external group of curves), 1.25, 1.4, 1.67 (internal group of curves)

$$\Pi\left(M\right) = p / p_0 = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\gamma/(\gamma-1)}$$

It is easy to acquire the generalized formula linking the Mach numbers on the discontinuity (wave):

$$\frac{\mu}{\hat{\mu}} = EJ \tag{16}$$

where, $\mu = (1 + \varepsilon(M^2 - 1)$ and $\hat{\mu} = 1 + \varepsilon\left(\hat{M}^2 - 1\right)$. Formula (16) allows to calculate the Mach numbers after the depression waves $\overline{\omega}$ and $\overline{\sigma}$ shocks.

The stream turn angle on shocks is also specified by intensity J_σ and J_m:

$$tg\beta = ctg\sigma \frac{(1-\varepsilon)(J-1)}{J_m + \varepsilon - (1-\varepsilon)(J-1)} \tag{17}$$

Here, $ctg^2\sigma = (J_m - 1)/(J + \varepsilon) = (E - E_m)/(E_m - \varepsilon)$ in coordinates $\{\Lambda \equiv \ln J, \beta\}$ formula (17) describes the family of cordiform curves (Fig. 1). Another its name is a shock polar. The curve form depends on the Mach number of the incoming stream M_1 (index 1 is often omitted, only M, P is written, etc.) and the gas adiabatic index γ, equal to the relation of heat content at constant pressure c_p to heat content at constant volume c_v.

Lower branches of cordiform curves are not physical, as they meet the depression shocks which do not exist in nature.

The study in Uskov and Mostovykh (2010) of these curves allowed to specify the significant properties of these families: presence of an envelope, limit angles of the stream angularity on the discontinuity, discontinuity appropriate points, after which the Mach number is equal to 1.

The relations describing cordiform curves are known for a long time, but they are still difficult in use because of the computing specifics and necessity of solution selection according to the real shocks. Let`s consider the most friendly definition of the angular shock calculation.

Angular shock calculation method: From the physical point of view, the shock is specified by the stream turn angle, when the supersonic stream flows onto an obstacle (Fig. 2 above), or by the relation of pressure P_2/P_1 at interaction of two supersonic streams with different pressure, for example, when flowing of over-expanded ($P_1<P_2$) supersonic stream (Fig. 2 below) from the nozzle.

Angle shocks can appear as the result of interference of other gas-dynamic discontinuities by the zero order and the first order: shocks, simple and centered waves, discontinuity characteristics. However,

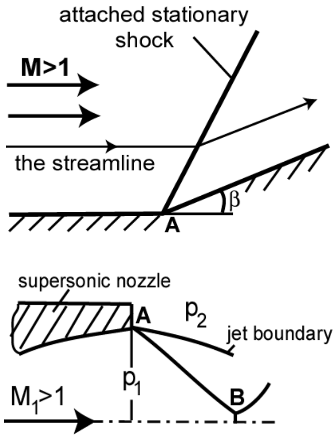

Fig. 2: Formation of an angle shock when the stream is flowing onto the wall (above) and at interaction of two streams with different pressure (below)

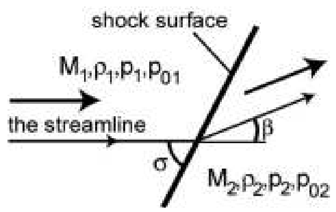

Fig. 3: Diagram of the stream before the angle shock and after it

all these cases, from the point of view of the calculation method, come to the two abovementioned.

The shock angle σ, its intensity J and the stream angle on the shock β (Fig. 3) at the flow specified parameters before the shock (M_1, P_1, P_{01}, ρ) are mutually definitely connected to each other. Assignment of any of three parameters allows to calculate the two others.

If we know the stream angle β, as in Fig. 2 (above), e.g., the wedge angle is assigned, which the supersonic stream goes onto, it is possible to calculate the intensity and angle of the angel shock under formation.

Dependence of the shock intensity J on the stream turn angle β, obtained, for its characteristic appearance, the name of cordiform curves.

It is comfortable to assign the shock polars in the parametric form, where the shock angle is used as a parameter σ. Actually, if to change it within limits 0-90°, one can easily calculate the shock intensity:

$$J = (1+\varepsilon)M^2 \sin^2\sigma - \varepsilon$$

where, $\varepsilon = \frac{\gamma-1}{\gamma+1}$ and the stream angle on the shock:

$$tg\beta = \sqrt{\frac{J_m - J}{J+\varepsilon}}\frac{(1-\varepsilon)(J-1)}{(J_m+\varepsilon)-(1-\varepsilon)(J-1)}$$

Recording of relations on the shock or shockwave with the help of generalized adiabatic line: If we know the intensity of shock and parameters before it, we can calculate all gas-dynamic variables after the shock. The intensity relation is specified by the Rankine-Hugoniot adiabatic line (6).

It is comfortable with the help of (6) to write the relations for calculation of all key parameters of the shock and the following stream. In such a way, the stream angle can be written in the form:

$$tg\beta = \frac{1-E}{2\sqrt{E}}\frac{2}{\sqrt{\frac{J_m-J}{1+\varepsilon J}}+\sqrt{\frac{1+\varepsilon J}{J_m-J}}} \tag{18}$$

The Mach number after the shock is specified from the condition of total heat content (enthalpy) when travelling over the shock:

$$M_2^{\,2} = \frac{(J+\varepsilon)M^2-(1-\varepsilon)(J^2-1)}{J(1+\varepsilon J)} = \frac{M^2-(1-E)(J+1)}{EJ} \tag{19}$$

Temperature rate:

$$\theta = \frac{T_2}{T} = EJ \square 1+\varepsilon J \tag{20}$$

The sonic speed:

$$\frac{a_2}{a} = \sqrt{EJ} \tag{21}$$

Recovery factor of total pressure:

$$I_0 = \frac{P_{02}}{P_0} = (E^\gamma J)^{\frac{-1}{\gamma-1}} \tag{22}$$

Written in such from relations (19-22) are true for any types of waves: Simple, shock and detonation ones. If we substitute the Laplace-Poisson adiabat equations (White, 1998) for E in the relation, we will obtain relations for simple and centered isentropic waves. If we insert adiabat Chapman-Jouget (Dremin, 1999), we will obtain the equations for detonation waves. All variables following the shock in Eq. (19)-(22) change monotonically dependently on the shock intensity J.

RESULTS AND DISCUSSION

Polar analysis: Even a few significant works have been dedicated to the polar analysis, it make sense to show here the relations for individual extreme characteristics important in practice (Uskov and Omelchenko, 1995, 1997, 1998). Looking at the cordiform curves we can make a key conclusion. For every M and γ there is a limit angle β of the stream possible deviation by an angle shock. Consequently, the stream picture shown in

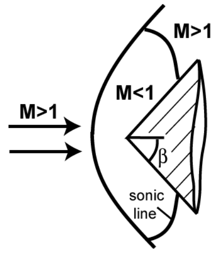

Fig. 4: Picture of the streamline if the wedge angle is more than the stream limit angle β_l

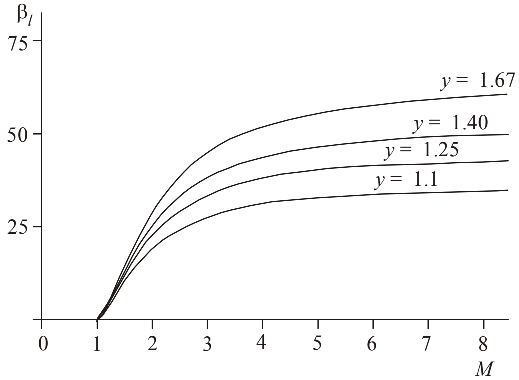

Fig. 5: Dependence of the stream limit angle β_l on the Mach number and adiabatic index $\gamma = 1.1$ (upper curve), 1.25, 1.4, 1.67 (lower curve)

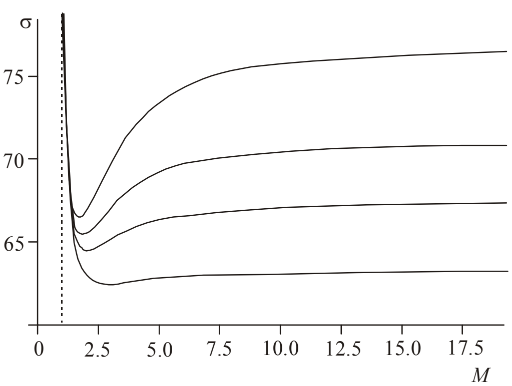

Fig. 6: Dependence of the shock angle σ_l for the stream limit angle β_l on the Mach number and the adiabatic index $\gamma = 1.1$ (upper curve), 1.25, 1.4, 1.67 (lower curve)

Fig. 2 (below) is possible only for small wedge angles β. If is exceeds a limit value for this M, which is traditionally designated as β, a detached curved shock is formed (Fig. 4). The intensity of the shock which is able to turn the stream for the maximal angle is expressed by the relation:

$$J_l = \frac{M^2-2}{2} + \sqrt{\left(\frac{M^2-2}{2}\right)^2 + (1+2\varepsilon)(M^2-1)+2} \quad (23)$$

Inserting (23) into (18), we obtain the value of the limit angle of the stream turn:

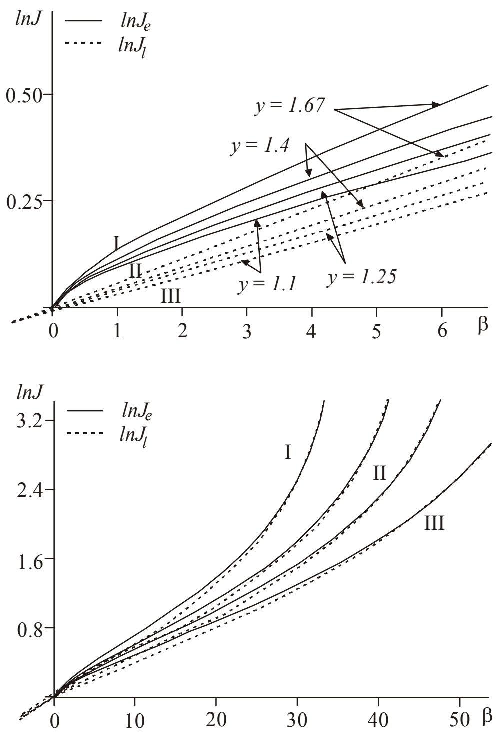

Fig. 7: Comparison of dependences $J_e(\beta_e)$ и $J_l(\beta_l)$

$$tg\beta_l = \frac{1-E_l}{2\sqrt{E_l}} \frac{2}{\sqrt{\frac{J_m - J_l}{1+\varepsilon J_l}} + \sqrt{\frac{1+\varepsilon J_l}{J_m - J_l}}} \quad (24)$$

The point on the cordioform curve meets J_l and divides the polar into two parts. The curve part lower this point meets the attached shocks. The curve part upper this point meets detached ones. The limit angle β_l increases with increase of M (Fig. 5) and for $M \to \infty$ is equal to 48.58° for $\gamma = 1.4$. The shock angle σ_l, at which the stream limit angle β_l is reached, depends on the Mach number non-monotonically (Fig. 6).

The second singular point on the shock polar is associated with the concept of heart-shaped envelope curve bounding the region of a single shock wave's existence on a plane $\Lambda - \beta$. The envelope can be found from the condition $\frac{\partial\beta}{\partial M} = 0$:

$$J_e = M^2 - 1,\ \beta_e(J) = arctg\left(\frac{1-E}{2\sqrt{E}}\right) \quad (25)$$

where, $E = E(J_e)$- an expression of Rankine-Hugoniotadiabat.

From (25) we see that the envelope only exists for $M \geq \sqrt{2}$. Plots of flow's rotation angle dependence at the shock polar's point of tangency with the envelope of polars family are shown in Fig. 7.

If the pressure relation P_2/P_1, is assigned as in Fig. 2 (below), the intensity of shock $J = P_2/P_1$ is known. If $P_2/P_1 < J_m$ (M), than according to formula (1) one can calculate the shock angle σ and according to formula (2) -the stream angle β. If $P_2/P_1 > J_m$ (M), there is no solution for the angle shock, inside the nozzle a starting shock appears with the intensity and position making the pressure on the nozzle edge be equal to the environment pressure.

Often, a practical problem arises how to brake the stream until the velocity lower than the sonic speed, therefore, it is useful to know how with the assigned M in the incoming stream to calculate the intensity of the shock after which $M = 1$:

$$J_S = \frac{M^2-1}{2} + \sqrt{\left(\frac{M^2-1}{2}\right)^2 + \varepsilon\left(M^2-1\right)+1} \qquad (26)$$

The topical task is an inverse problem: calculation of the Mach number of the incoming stream according to the shock assigned intensity, when the stream following the shock gets sonic stream:

$$M_S = \sqrt{1+\frac{J^2-1}{J+\varepsilon}} \qquad (27)$$

CONCLUSION

We have considered the calculation ways covering 90% of practical problems connected to computation of shocks. In spite of blanket distribution of computational approaches in the gas dynamics in a series of applications, the topical problem is direct computation of shocks, especially if the optimal solution is needed. In numerous subject literature, the shock computation ways, as a rule, are given in the form which makes their application difficult for optimization and control of supersonic streamlines. The things are becoming more complicated because of the equations connected to the shock computation often have a few solutions, calculation specifics or, often, cannot be solved relatively to the desired variable. For selection of the solutions which meet physically realized shockwave configurations, obtaining the values around the special points, it is necessary to attract extra considerations. On the other hand, there is the minimal set of the important characteristics of the shocks for which it is possible to solve the computation problem in a friendly form. Knowledge of the special and limit parameters of shocks allows to easily divide the solutions into categories. This study covers such an approach allowing to easily solve 90% of key practical problems on computation of single angle shocks.

Here are given the universal formulae to calculate the parameters after the shock recorded with the help of the generalized adiabat and applied also for simple waves and detonation waves (with use of proper expressions for the adiabatic line). These formulae allow calculating the shock parameters are you know even the only gas-dynamic variable following the shock. If you know parameters of the stream before the shock and the shock intensity, these equations allow computing all the parameters following the shock. The computation results for dependence of the significant shock characteristics on the Mach number and the stream adiabatic line index are given in a friendly form.

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