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**Research Article**

# Evaluation of Routing Flexibility of a Flexible Decision Making Units Using Hybrid Intelligent Algorithm: Modeling and Applications

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**Abstract:** For the same of two fuzzy sets with different supports, this study investigates the entropy relationship between both of them. For the fuzzy numbers with triangular membership function, the entropy change between the resultant fuzzy numbers through some arithmetic operations and the original fuzzy numbers is studied. Since, the entropy can be used as a crisp approximation of a fuzzy number, therefore the resultant value is used to rank the fuzzy numbers. The main advantage of the proposed approach is that the proposed ranking method provides the correct ordering of fuzzy numbers and also the proposed approach is very simple and easy to apply in the real life problems.

**Keywords:** Defuzzification, efficient, fuzzy numbers, ranking

## INTRODUCTION

Fuzzy set theory is a powerful tool to deal with real life situations. Real numbers can be linearly ordered by $\geq$ or $\leq$, however this type of inequality does not exist in fuzzy numbers. Since fuzzy numbers are represented by possibility distribution, they can overlap with each other and it is difficult to determine clearly whether one fuzzy number is larger or smaller than any other. An efficient approach for ordering the fuzzy numbers is by the use of a ranking function $R : F(R) \rightarrow R$, where $F(R)$ is a set of fuzzy numbers defined on real line, which maps each fuzzy number into the real line, where a natural order exists. Thus, specific ranking of fuzzy numbers is an important procedure for decision-making in a fuzzy environment and generally has become one of the main problems in fuzzy set theory. The method for ranking was first proposed by Yager and Filev (1993). Saneifard and Rasoul (2011) proposed four indices which may be employed for the purpose of ordering fuzzy quantities in $[0, 1]$. In Saneifard (2010), an approach is presented for the ranking of fuzzy numbers. Wang and Kerre (2001) proposed a subjective approach for ranking fuzzy numbers. Cheng (1999) developed a ranking method based on integral value index. Chu and Tsao (2002) presented a method for ranking fuzzy numbers by using the distance method. Chen (1985) considered the overall possibility distributions of fuzzy numbers in their evaluations and proposed a ranking method. Cheng (1998) proposed a ranking method based on preference function which measures the fuzzy numbers point by point and at each point the most preferred number is identified. Chu and Tsao (2002) proposed a method for ranking fuzzy numbers with the area between the centroid point and original point. Wang *et al*. (2006) presented a centroid-index method for ranking fuzzy numbers. Wang and Kerre (2001) also used the centroid concept in developing their ranking index. Chen (1985) presented a method for ranking generalized trapezoidal fuzzy numbers. Chen (1985) presented a method for fuzzy risk analysis based on ranking generalized fuzzy numbers with different heights and different spreads. The main aim of this study is to propose a new approach for the ranking of generalized trapezoidal fuzzy numbers. The proposed ranking approach is based on membership function so it is named as entropy approach. The main advantage of the proposed approach is that the proposed approach provides the correct ordering of generalized and normal trapezoidal fuzzy numbers and also the proposed approach is very simple and easy to apply in the real life problems.

## PRELIMINARIES

In general, the membership function of a fuzzy set is determined by users subjectively. The shape of a membership function always presents the knowledge grade of the elements in the fuzzy set. In other words, every membership function also presents the fuzziness of the corresponding fuzzy set in the idea of users. Therefore, it is necessary having some measurements to measure the fuzziness of fuzzy sets. Until now, there have been several typical methods being used to measure the fuzziness of fuzzy sets. Chen (1985) utilized the conception of the \entropy" to indicate the fuzziness of a fuzzy set. Saneifard and Rasoul (2011) proposed that the fuzziness of a fuzzy set can be

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measured through the distance between the fuzzy set and its nearest non-fuzzy set. Yager and Filev (1993) suggested the measure of fuzziness can be expressed by the distances between the fuzzy set and its complement. We cannot deny that the entropy is indeed a proper measurement of a fuzzy set and has received a lot of attention recently. There is lots of literature talking about the entropies of fuzzy sets. Pedrycz (1994) is the motivation of this study's work. Pedrycz (1994) showed the entropy change when the interval size of the universal set is changed. He considered the fuzzy set with triangular membership function. In this study, the result of Pedrycz (1994) is extended to any type of fuzzy sets. Next, we consider a set of Triangular Fuzzy Numbers (TFN) through arithmetic operations. These operations include addition, subtraction and multiplication. The resultant fuzzy number after arithmetic operations has its entropy. What is the relationship between the entropies of the resultant fuzzy number and the original fuzzy numbers is the main task to be discussed in this study.

**The entropy change between fuzzy sets:** Let $\xi$ be a fuzzy variable with membership function $\mu$. For any $x \in R$, $\mu(x)$ represents the possibility that $\xi$ takes value $x$. Hence, it is also called the possibility distribution. For any set $B$, the possibility measure of $\xi \in B$ was defined by Dubois and Prade (1978) as:

$$Pos\{\xi \in B\} = sup_{x \in B}\mu(x)$$

which is used to express the possibility that $\xi$ takes values in $B$. In addition, Dubois and Prade (1978) defined a necessity measure as the dual part of possibility measure. That is:

$$Nec\{\xi \in B\} = 1 - sup_{x \in B^c}\mu(x)$$

It is proved that both possibility measure and necessity measure satisfy the properties of normality, non-negativity and monotonicity. However, neither of them are self-dual. Since the self-duality is intuitive and important in real problems, Pedrycz (1994) defined a credibility measure as the average of possibility measure and necessity measure:

$$Cr\{\xi \in B\} = \frac{1}{2}\big(sup_{x \in B}\mu(x)\big) \\ +1 - sup_{x \in B^c}\mu(x)) \quad (1)$$

It is easy to prove that credibility measure is self-dual. That is:

$$Cr\{\xi \in B\} + Cr\{\xi \in B^c\} = 1$$

for any set $B$.

**Definition 1 (Pedrycz, 1994):** Let $\xi$ be a fuzzy variable. Then its expected value is defined as:

$$E[\xi] = \int_0^{\infty} Cr\{\xi \geq r\}\, dr - \int_{-\infty}^{0} Cr\{\xi \leq r\}\, dr \quad (2)$$

Provided that at least one of the above two integrals is finite. Expected value is one of the most important concepts for fuzzy variable, which gives the center of its distribution.

**Definition 2 (Pedrycz, 1994):** Let $\xi$ be a fuzzy variable with finite expected value $e$. Then its variance is defined as:

$$V[\xi] = E[(\xi - e)^2] \quad (3)$$

If $\xi$ is a fuzzy variable with expected value $e$, then its variance is used to measure the spread of its distribution about $e$.

Please note that variance concerns not only the part $\xi$ is less than its expected value, but also the part $\xi$ is greater than its expected value. If we are only interested in the first part, then we should use the concept of semivariance.

**Definition 3 (Pedrycz, 1994):** Let $\xi$ be a fuzzy variable with finite expected value $e$. Then its semivariance is defined as:

$$S_v[\xi] = E[[(\xi - e)^-]^2] \quad (4)$$

where,

$$(\xi - e)^- = \min\{\xi - e, 0\}$$

Essentially, variance and semi-variance are all second moments and they take the same values for symmetric fuzzy variables.

**Definition 4 (Pedrycz, 1994):** Let $\xi$ be a fuzzy variable with finite expected value $e$. Then its skewness is defined as:

$$Sk[\xi] = E[(\xi - e)^3] \quad (5)$$

If $\xi$ is a fuzzy variable with symmetric membership function, then we have $Sk[\xi] = 0$.

**Definition 5 (Pedrycz, 1994):** Let $\xi$ be a continuous fuzzy variable. Then its entropy is defined as:

$$H[\xi] = \int_{-\infty}^{+\infty} S(Cr\{\xi = x\})\, dx \quad (6)$$

where,

$$S(t) = -t\, ln\, t - (1 - t) \ln(1 - t)$$

Fuzzy entropy is used to measure the uncertainty associated with each fuzzy variable. If $\xi$ has continuous membership function $\mu$, then it follows from (1) that

$Cr\{\xi = x\} = \frac{\mu(x)}{2}$ for any $x \in R$. In this case, it is easy to prove that its entropy is:

$$-\int_{-\infty}^{+\infty} \frac{\mu(x)}{2} . \ln \frac{\mu(x)}{2} + \left(1 - \frac{\mu(x)}{2}\right) . \ln \left(1 - \frac{\mu(x)}{2}\right) dx$$

**Definition 6 (Pedrycz, 1994):** Let $\xi$ and $\eta$ be two continuous fuzzy variables. Then the cross-entropy of $\xi$ from $\eta$ is defined as:

$$D[\xi, \eta] = \int_{-\infty}^{+\infty} T(Cr\{\xi = x\}, Cr\{\eta = x\})\, dx \quad (7)$$

where,

$$T(s, t) = \frac{s \ln s}{t} + (1 - s) \ln \frac{(1-s)}{(1-t)}$$

Fuzzy cross-entropy is used to measure the divergence of fuzzy variable from an a priori one. If $\xi$ and $\eta$ have continuous membership functions $\mu$ and $\nu$, respectively, then the cross-entropy of $\xi$ from $\eta$ is:

$$CRe[\xi, \eta] = \int_{-\infty}^{+\infty} \frac{\mu(x)}{2} . \ln \frac{\mu(x)}{\nu(x)} + \left(1 - \frac{\mu(x)}{2}\right) . \ln \left(\frac{2-\mu(x)}{2-\nu(x)}\right) dx \quad (8)$$

**Example 1:** A triangular fuzzy variable $\xi$ is fully determined by a triplet $(a, b, c)$ with $a < b < c$ and its membership function is given by:

$$\mu(x) = \begin{cases} \frac{x-a}{b-a} \; when \; a \le x \le b, \\ \frac{c-x}{c-b} \; when \; b < x \le c, \\ 0, otherwise \end{cases}$$

In what follows, we write $\xi = (a, b, c)$. If $\xi$ is a symmetric triangular fuzzy variable with $b - a = c - b$, then it can be proved that $E[\xi] = b$, $V[\xi] = (b - a)^2/6$ and $H[\xi] = (c - a)/2$.

**Example 2:** A trapezoidal fuzzy variable $\xi$ is fully determined by the quadruplet $(a, b, c, d)$ with $a < b \le c < d$ and its membership function is given by:

$$\mu(x) = \begin{cases} \frac{x-a}{b-a} \; when \; a < x \le b, \\ 1 \; when \; b \le x \le c, \\ \frac{d-x}{d-c} \; when \; c < x \le d, \\ 0, otherwise \end{cases}$$

For symmetric trapezoidal fuzzy variable $\xi$ with $b - a = c - b$, it is proved that $E[\xi] = b$, $V[\xi] = (b - a)^2/(24 + (c - a)^2/8)$ and $H[\xi] = (b - a) + (c - b)ln2$.

**Example 3:** A normally distributed fuzzy variable $\xi$ is defined by the membership function:

$$\mu(x) = 2\left(1 + exp\left(\frac{\pi|x-e|}{\sqrt{6}\sigma}\right)\right)^{-1}, x \in R$$

It is proved that $E[\xi] = e$, $V[\xi] = \sigma^2$ and $H[\xi] = \frac{\sqrt{6}\pi\sigma}{3}$.

## A NEW METHOD FOR RANKING FUZZY NUMBERS

In this section, we present a new approach for ranking fuzzy numbers based on the distance method. The method not only considers the central interval of a fuzzy number, but also the maximum crisp value of fuzzy numbers. For ranking fuzzy numbers, this study firstly defines a maximum crisp value $a_{max}$ to be the benchmark.

Assume that there are $n$ fuzzy numbers $A_1, A_2, \cdots, A_n$, where $A_j = (a_{1j}, a_{2j}, a_{3j}, a_{4j})$, $1 \le j \le n$. The maximum crisp value $a_{max}$ is the maximum value of the $a_{1j}, a_{2j}, a_{3j}, a_{4j}$ and $1 \le j \le n$.

The proposed method for ranking fuzzy numbers $A_1, A_2, \cdots, A_n$ is now presented as follow:

**Step 1:** Calculate the maximum crisp value $a_{max}$ of all fuzzy numbers $A_j$, where $1 \le j \le n$

**Step 2:** Use formula (8) to calculate the cross-entropy of $A_j$ from $a_{max}$ of each fuzzy numbers $A_j$, where $1 \le j \le n$ as follows:

$$CRe[A_j, a_{max}] = \int_{-\infty}^{+\infty} \frac{\mu_{A_j}(x)}{2} . \ln \frac{\mu_{A_j}(x)}{\mu_{a_{max}}(x)} + \left(1 - \frac{\mu_{A_j}^{(x)}}{2}\right) . \ln \left(\frac{2-\mu_{A_j}^{(x)}}{2-\mu_{a_{max}}^{(x)}}\right) dx \quad (9)$$

**Step 3:** Use the cross-entropy $CRe\left[A_j, a_{max}\right]$ to calculate the ranking value $CR\left[A_j, a_{max}\right]$ of the fuzzy numbers $A_j$, where $1 \le j \le n$, as follow:

$$CR[A_j, a_{max}] = \left|CRe[A_j, a_{max}] - a_{max}\right| \quad (10)$$

From 10, one may that $CR\left[A_j, a_{max}\right]$ could be considered as the Euclidean distance between the point $(CR\left[A_j, a_{max}\right], 0)$ and the point $(a_{max}, 0)$. We can see that the larger the value of $CR\left[A_j, a_{max}\right]$, the better the ranking of $A_j$, where $1 \le j \le n$.

Since this article wants to approximate a fuzzy number by a scalar value, the researchers have to use an operator $CR: F \to R$ (a space of all fuzzy numbers into a family of real line). $CR$ is a crisp approximation operator. Since ever above defuzzification can be used as a crisp approximation of a fuzzy number, therefore the resultant value is used to rank the fuzzy number. Thus, $CR$ is used to rank fuzzy numbers. The larger $CR$ the larger fuzzy number.

Let $A_1, A_2 \in F$ be two arbitrary fuzzy numbers. Define the ranking of $A_1$ and $A_2$ on $CR(.)$ as follow:

- $CR[A_1, a_{max}] > CR[A_2, a_{max}]$ if only if $A_1 \succ A_2$
- $CR[A_1, a_{max}] < CR[A_2, a_{max}]$ if only if $A_1 \prec A_2$
- $CR[A_1, a_{max}] = CR[A_2, a_{max}]$ if only if $A_1 \sim A_2$

However, this study formulates the orders $\succcurlyeq$ and $\preccurlyeq$ as $A_1 \succcurlyeq A_2$ if and only if $A_1 \succ A_2$ or $A_1 \sim A_2$, $A_1 \preccurlyeq A_2$ if and only if $A_1 \prec A_2$ or $A_1 \sim A_2$.

**Remark 1:** If $inf\ supp(A) \geq 0$, then $CR[A, a_{max}] \geq 0$.

**Remark 2:** If $sup\ supp(A) \leq 0$, then $CR[A, a_{max}] \geq 0$.

Here are some examples to illustrate the ranking of fuzzy numbers.

**An application:** In the following application, we use a geo-database of street excavations to estimate the possibility of a street to have been in activity after a given date. In the SIGRem project, information about Roman street excavations is stored in the ‘‘BDRues’’ geo-database with a fuzzy representation of their dates. Our goal is to obtain a visualization of the comparison between these fuzzy dates and a given fuzzy date (reference date). For example, archaeologists and historians want to know which streets were built after the first century. This application uses a GIS software to localize excavations and assigns a color to each excavation. According to a reference date $D_{ref}$, the query is ‘‘How anterior is $D_{ref}$ to the objects from the database?’’. Thus, for each excavation $e_i$ in the database, the anteriority between $D_{ref}$ and the excavation activity period $e_i d$: $Cr\ (D_{ref}, a_{max})$, $Cr\ (e_i d, a_{max})$ are computed and we assign it a color in accordance with this value. The example of the map allows us to visualize the query results (the anteriority index values) obtained by using the triangular fuzzy number (0, 200, 400) for $D_{ref}$. Those values could be interpreted as qualities of anteriority. In this map, an object is white if its date is not defined. Experts use this kind of visualization mode to select data in their diagnosis processes. During their prospection, experts evaluate which kind of objects they expect to find in a specific site. This application helps archaeologists to determine the evolution of the city during the Roman period. The anteriority index will guide archaeologists in their expertise processes.

## CONCLUSION

In this study, authors presented a new method for ranking fuzzy numbers. The proposed method considers the cross-entropy and maximum crisp value of fuzzy numbers to ranking fuzzy numbers. Cross-entropy method can overcome the drawback previous methods.

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