Research Journal of Environmental and Earth Sciences 6(7): 389-393, 2014
DOI:10.19026/rjees.6.5248
ISSN: 2041-0484; e-ISSN: 2041-0492

Submitted: May 04, 2014 Accepted: May 25, 2014 Published: July 20, 2014

Research Article

Solving Fuzzy Linear Systems by District Matrix's: An Application in GIS

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Abstract: In this study we want to investing the ways for solving the fuzzy linear systems that the coefficient is a fuzzy number, which is equals a classic number and our aim is to show the fact that whether the answer can be fuzzy systems or not and at the end complete the discussion by a numerical example.

Keywords: Block matrix, fuzzy linear systems, fuzzy numbers, inverse matrix

INTRODUCTION

In applied mathematics Linear functions systems and solving the large proportions is very important for getting the result. Normally in most applied program the result are shown with fuzzy numbers instead of classic hence the use of different methods of math and developing different solutions and getting a suitable is more important than the fuzzy systems or solving them.

A general solution for a common fuzzy system n×n that coefficient matrix is the fuzzy number and he right hand Column is a number into fuzzy vector and draw a system and replace the main fuzzy system with a classic linear system 2×2 n and solve it by matrix linear system approach. Ezzati (2008, 2011), Friedman *et al*. (1998, 2000), Ma *et al*. (1999) and Wu and Ma (1991) and some other suggested over the fuzzy systems which is the right hand column is fuzzy vector and they investigate the numerical fuzzy systems. In this study, the researchers want to investing the ways for solving the fuzzy linear systems that the coefficient is a fuzzy number, which is equals a classic number and our aim is to show the fact that whether the answer can be fuzzy systems or not and at the end complete the discussion by a numerical example.

Fuzzy systems:

Definition 1: by the use of a couple of functions such as: ($\underline{u}$ (r), $\bar{u}$ (r)), 0≤r≤1:

- $\underline{u}$ (r) is a bounded monotonic increasing left continuous function
- $\bar{u}$ (r) is a bounded monotonic decreasing left continuous function
- $\underline{u}$ (r) ≤$\bar{u}$ (r), 0≤r≤1

For example the fuzzy number (1+r, 4-2r) as illustrated in Fig. 1 a classic number a simply is calculated. With specialty of definition of fuzzy number

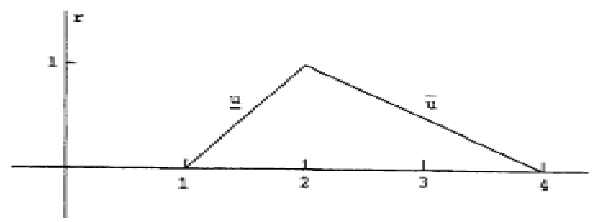

Fig. 1: A fuzzy number

aria {$\underline{u}$ (r), $\bar{u}$ (r)} is out come an E convex carroty which replacing in isomorphic, isometric shape into the Banakh space (Ezzati, 2011).

PROPOSED METHODOLOGY

Linear system n×n:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_{n1} &= y_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_{n1} &= y_2 \\ &\vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_{n1} &= y_n \end{aligned} \quad (1)$$

Which matrix coefficients $\tilde{A} = (a_{i\,j}) = (\underline{a}_{i\,j}, \bar{a}_{i\,j})$, 0≤i, j≤1 is a fuzzy matrix:

$n \times n, y_i$: 1≤i≤n

Is a part of classic vector?

With replacing fuzzy coefficients, we have:

$$\begin{aligned} &(\underline{a}_{11}, \bar{a}_{11})\ x_1 + (\underline{a}_{12}, \bar{a}_{12})\ x_2 + \ldots + (\underline{a}_{1n}, \bar{a}_{1n})\ x_n = y_1 \\ &(\underline{a}_{21}, \bar{a}_{21})\ x_1 + (\underline{a}_{22}, \bar{a}_{22})\ x_2 + \ldots + (\underline{a}_{2n}, \bar{a}_{2n})\ x_n = y_2 \\ &\vdots \\ &(\underline{a}_{n1}, \bar{a}_{n1})\ x_1 + (\underline{a}_{n2}, \bar{a}_{n2})\ x_2 + \ldots + (\underline{a}_{nn}, \bar{a}_{nn})\ x_n = y_n \end{aligned} \quad (2)$$

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So conversion to the two system like down which is a classic coefficient:

$$\begin{aligned}
&\underline{a}_{11}x_1 + \underline{a}_{12}x_2 + \ldots + \underline{a}_{1n}x_n = y_1\\
&\underline{a}_{21}x_1 + \underline{a}_{22}x_2 + \ldots + \underline{a}_{2n}x_n = y_2\\
&\vdots\\
&\underline{a}_{n1}x_1 + \underline{a}_{n2}x_2 + \ldots + \underline{a}_{nn}x_n = y_n
\end{aligned} \tag{3}$$

and,

$$\begin{aligned}
&\bar{a}_{11}x_1 + \bar{a}_{12}x_2 + \ldots + \bar{a}_{1n}x_n = y_1\\
&\bar{a}_{21}x_1 + \bar{a}_{22}x_2 + \ldots + \bar{a}_{2n}x_n = y_2\\
&\vdots\\
&\bar{a}_{n1}x_1 + \bar{a}_{n2}x_2 + \ldots + \bar{a}_{nn}x_n = y_n
\end{aligned} \tag{4}$$

If B was matrix coefficient of first system and C matrix coefficient of second system, matrix equation is such as:

$$\begin{bmatrix} \underline{a}_{11} & \cdots & \underline{a}_{1n} \\ \vdots & \ddots & \vdots \\ \underline{a}_{1n} & \cdots & \underline{a}_{nn} \\ 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \\ \bar{a}_{11} & \cdots & \bar{a}_{1n} \\ \vdots & \ddots & \vdots \\ \bar{a}_{1n} & \cdots & \bar{a}_{nn} \end{bmatrix} \times \begin{bmatrix} \underline{x_1} \\ . \\ \underline{x_n} \\ \overline{x_1} \\ . \\ \overline{x_n} \end{bmatrix} = \begin{bmatrix} y_1 \\ . \\ y_n \\ y_1 \\ . \\ y_n \end{bmatrix} \tag{5}$$

or,

$$\tilde{A}\,\mathrm{X} = \mathrm{Y} \tag{6}$$

$$\begin{bmatrix} B & 0 \\ 0 & C \end{bmatrix} \begin{bmatrix} \underline{X} \\ \overline{X} \end{bmatrix} = \mathrm{Y} \tag{7}$$

If district matrix $\begin{bmatrix} B & 0 \\ 0 & C \end{bmatrix}$ was invisible and suppose that it's inverse is $\begin{bmatrix} E & F \\ G & H \end{bmatrix}$ then:

$$\begin{bmatrix} E & F \\ G & H \end{bmatrix} \begin{bmatrix} B & 0 \\ 0 & C \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \tag{8}$$

$$\begin{bmatrix} E\,B + 0 & 0 + F\,C \\ G\,B + 0 & 0 + H\,C \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \tag{9}$$

E = B^{-1}
F = 0,
H = C^{-1}
G = 0

So $\begin{bmatrix} B^{-1} & 0 \\ 0 & C^{-1} \end{bmatrix}$ will be inverse of $\begin{bmatrix} B & 0 \\ 0 & C \end{bmatrix}$. Thus the answer of system is:

$$\begin{bmatrix} \underline{X} \\ \overline{X} \end{bmatrix} = \begin{bmatrix} B^{-1} & 0 \\ 0 & C^{-1} \end{bmatrix} \mathrm{Y} \tag{10}$$

Then the answer X = $(\underline{X}, \overline{X})$ is a fuzzy number.

NUMERICAL EXAMPLE

Example 1: In this example, the coefficients of trigonometric equations are fuzzy numbers. And in the right, the numbers are classics:

$$\begin{cases}(0.1, 0.4, 0.9)(\underline{x}_1, \overline{x}_1) + (1,1.4,1.9)(\underline{x}_2, \overline{x}_2) = 2\\(0.1,0.15,0.2)(\underline{x}_1, \overline{x}_1) + (0.11,0.14,0.2)(\underline{x}_2, \overline{x}_2) = 1\end{cases}$$

Conversion of fuzzy equations to interval form, we get:

$$\begin{cases}(0.3r, 0.1\,,\ -0.5r + 0.9)(\underline{x}_1,\ \overline{x}_1) + (0.4r + 1, -05r + 1.9)(\underline{x}_2,\ \overline{x}_2)\\ (0.05r + 0.1,\ -0.05r + 0.2)(\underline{x}_1,\ \overline{x}_1) +\\ (0.03r + 0.11,\ -0.06r + 0.2)(\underline{x}_2,\ \overline{x}_2)\end{cases}$$

If we write the matrix form:

$$\begin{bmatrix} 0.3r + 0.1 & 0.4r + 1 & 0 & 0 \\ 0.05r + 0.1 & 0.03r + 0.11 & 0 & 0 \\ 0 & 0 & -0.5r + 0.9 & -0.5r + 1.9 \\ 0 & 0 & -0.05r + 0.2 & -0.06r + 0.2 \end{bmatrix}\begin{bmatrix}\underline{x}_1\\ \underline{x}_2\\ \overline{x}_1\\ \overline{x}_2\end{bmatrix} = \begin{bmatrix}2\\1\\2\\1\end{bmatrix}$$

Assuming that:

$$\mathrm{C} = \begin{bmatrix}0.5r + 0.9 & -0.5r + 1.9\\ 0.5r + 0.2 & -0.06r + 0.2\end{bmatrix},\ \mathrm{B} = \begin{bmatrix}0.3r + 0.1 & -0.4r + 1\\ 0.05r + 0.1 & -0.03r + 0.11\end{bmatrix}$$

By calculating C^{-1}, B^{-1}:

$$B^{-1} = \frac{1}{11r^2 - 54r - 81}\begin{bmatrix}30r + 110 & -400r - 1000\\ -50r - 100 & 300r + 100\end{bmatrix}$$

$$C^{-1} = \frac{1}{5r^2 + 41r - 200}\begin{bmatrix}60r + 200 & 500r - 1900\\ 50r - 200 & -500r + 900\end{bmatrix}$$

Placement in the equation:

$$\begin{bmatrix}\underline{x}_1\\ \underline{x}_2\\ \overline{x}_1\\ \overline{x}_2\end{bmatrix} = \begin{bmatrix}B^{-1} & 0\\ 0 & C^{-1}\end{bmatrix} = \begin{bmatrix}2\\1\\2\\1\end{bmatrix},\ \begin{bmatrix}\underline{X}\\ \overline{X}\end{bmatrix} = \begin{bmatrix}B^{-1} & 0\\ 0 & C^{-1}\end{bmatrix} = \begin{bmatrix}Y\\ Y\end{bmatrix}$$

$$\begin{bmatrix}\underline{x}_1\\ \underline{x}_2\\ \overline{x}_1\\ \overline{x}_2\end{bmatrix} = \begin{bmatrix} \frac{30r+110}{11r^2-54r-81} & \frac{-400r-1000}{11r^2-54r-81} & 0 & 0\\ \frac{-50r-100}{11r^2-54r-81} & \frac{300r+100}{11r^2-54r-81} & 0 & 0\\ 0 & 0 & \frac{60r+200}{5r^2+41r-200} & \frac{500r-1900}{5r^2+41r-200}\\ 0 & 0 & \frac{50r-200}{5r^2+41r-200} & \frac{-500r+900}{5r^2+41r-200}\end{bmatrix} \times \begin{bmatrix}2\\1\\2\\1\end{bmatrix}$$

$$\underline{\tilde{x}}_1 = \frac{-340r - 890}{11r^2 - 54r - 81}$$

$$\underline{x}_2 = \frac{200r - 100}{11r^2 - 54r - 81}$$

$$\overline{x}_1 = \frac{620r - 1500}{5r^2 + 41r - 200}$$

$$\overline{x}_2 = \frac{-400r + 500}{5r^2 + 41r - 200}$$

And finally:

$$\widetilde{X_1} = \left(\frac{-340r - 890}{11r^2 - 54r - 81},\quad \frac{620r - 1500}{5r^2 + 41r - 200}\right)$$

$$\widetilde{X_2} = \left(\frac{200r - 100}{11r^2 - 54r - 81},\quad \frac{-400r + 500}{5r^2 + 41r - 200}\right)$$

The device answers, which is a fuzzy number.

Example 2: In this example, the coefficients are either trapezoidal fuzzy numbers. The right numbers are classics:

$$\begin{cases}(0.1,0.4,0.6,0.9)(\underline{x}_1,\overline{x}_1)+(1,1.4,1.6,1.9)(\underline{x}_2,\overline{x}_2)=1\\(5,5.1,5.2,5.4)(\underline{x}_1,\overline{x}_1)+(0.1,0.3,0.035,0.4)(\underline{x}_2,\overline{x}_2)=0.5\end{cases}$$

The conversion equations to form an interval fuzzy:

$$\begin{cases}(0.3\alpha+0.1,-0.3\alpha+0.9)(\underline{x}_1,\overline{x}_1)+(0.4\alpha+1,-0.3\alpha+1.9)(\underline{x}_2,\overline{x}_2)=1\\(0.1\alpha+5,-0.2\alpha+5.4)(\underline{x}_1,\overline{x}_1)+(0.2\alpha+0.1,-0.05\alpha+0.4)(\underline{x}_2,\overline{x}_2)=0.5\end{cases}$$

We write the above system as a matrix equation:

$$\begin{bmatrix}0.3\alpha+0.1 & 0.4\alpha+1 & 0 & 0\\0.1\alpha+5.0 & 0.2\alpha+0.1 & 0 & 0\\0 \quad 0 & & -0.3\alpha+0.9 & -0.3\alpha+1.9\\0 \quad 0 & & -0.2\alpha+5.4 & -0.05\alpha+0.4\end{bmatrix}\times\begin{bmatrix}\underline{x}_1\\\underline{x}_2\\\overline{x}_1\\\overline{x}_2\end{bmatrix}=\begin{bmatrix}1\\0.5\\1\\0.5\end{bmatrix}$$

If we consider coefficient matrix as block matrices $\begin{bmatrix}B & 0\\0 & C\end{bmatrix}$ so:

$$B=\begin{bmatrix}0.3\alpha+0.1 & 0.4\alpha+1\\0.1\alpha+5 & 0.2\alpha+0.1\end{bmatrix},\ C=\begin{bmatrix}-0.3\alpha+0.9-0.3\alpha+1.9\\-0.2\alpha+5.4\ -0.05\alpha+0.4\end{bmatrix}$$

And calculating B^{-1}, C^{-1} in the below:

$$B^{-1}=\begin{bmatrix}\frac{20\alpha+10}{2\alpha^2+2\alpha-499} & \frac{-40\alpha-100}{2\alpha^2+2\alpha-499}\\\frac{-10\alpha-500}{2\alpha^2+2\alpha-499} & \frac{30\alpha+10}{2\alpha^2+2\alpha-499}\end{bmatrix},\ C^{-1}=\begin{bmatrix}\frac{-50\alpha+400}{45\alpha^2+1835\alpha-9820} & \frac{300\alpha-1900}{45\alpha^2+1835\alpha-9820}\\\frac{200\alpha-5400}{45\alpha^2+1835\alpha-9820} & \frac{-300\alpha+900}{45\alpha^2+1835\alpha-9820}\end{bmatrix}$$

Answer Calculation of equation $X = A^{-1}Y$ or:

$$\begin{bmatrix}\underline{X}\\\overline{X}\end{bmatrix}=\begin{bmatrix}B^{-1} & 0\\0 & C^{-1}\end{bmatrix}=Y$$

$$\begin{bmatrix}\underline{x}_1\\\underline{x}_2\\\overline{x}_1\\\overline{x}_2\end{bmatrix}=\begin{bmatrix}\frac{20\alpha+10}{2\alpha^2+2\alpha-499} & \frac{-40\alpha-100}{2\alpha^2+2\alpha-499} & 0 & 0\\\frac{-10\alpha-500}{2\alpha^2+2\alpha-499} & \frac{30\alpha+10}{2\alpha^2+2\alpha-499} & 0 & 0\\0 \quad 0 & & \frac{-50\alpha+400}{45\alpha^2+1835\alpha-9820} & \frac{300\alpha-1900}{45\alpha^2+1835\alpha-9820}\\0 \quad 0 & & \frac{200\alpha-5400}{45\alpha^2+1835\alpha-9820} & \frac{-300\alpha+900}{45\alpha^2+1835\alpha-9820}\end{bmatrix}\times\begin{bmatrix}1\\0.5\\1\\0.5\end{bmatrix}$$

And finally:

$$\underline{x}_1=\frac{-40\alpha-100}{2\alpha^2+2\alpha-499}$$

$$\underline{x}_2=\frac{-20\alpha-495}{2\alpha^2+2\alpha-499}$$

$$\overline{x}_1=\frac{100\alpha-550}{45\alpha^2+1835\alpha-9820}$$

$$\overline{x}_2=\frac{-350\alpha-4950}{45\alpha^2+1835\alpha-9820}$$

AN APPLICATION

The SIGRem project is an archaeological GIS dedicated to the Roman period of Reims. The main goal of the project is to help experts in their decision processes. In this section, we use the anteriority index in the decision processes for simulation and data visualization. Dates in archaeological excavation data can be represented by the possibility theory using fuzzy numbers as “near 1330 A.D.” or “Middle Ages”. To give the archaeologists some predictive maps, we want to use the comparison decisions for spatial inference. The anteriority index helps us to resolve some spatial and architectural contradictions of excavation maps. On the “Galeries Remoises” site, there are two walls (WALL1 and WALL2) built during the first century, showing an architectural contradiction as one overlaps the other. Our goal is to design a new map to simulate the archaeological hypothesis on dating. The index proposes a weighed indication on the question “Was WALL1 built before WALL2?”. The two building dates are represented by fuzzy numbers F (date of WALL1) and G (date of WALL2). The membership function f (resp. g) of F (resp. G) is defined as a trapezoidal where:

- Support (f) = [(t1 - 0.2 (t2 - t1), (t2 - 0.2 (t2 - t1)]
- Kernel (f) = [(t1 - 0.2 (t2 - t1), (t2 0.2 (t2 t1)]
- t1 and t2 are respectively the beginning and the end of the period

For WALL1, t1 is equal to 0 and t2 to 100. For WALL2, t2 is equal to 50 and t2 to 100. So F = (-20, 20, 80, 120, 1) and G = (40, 60, 90, 110). Membership functions of the fuzzy numbers associated to WALL1 and WALL2 are illustrated in Fig. 1. The calculation of the values of the anteriority index between F and G gives Centr (F) = 0.95 and Centr (G) = 0.05. So, as defined in Sect. 3, F is rather anterior to G, thus WALL1 was rather anterior to WALL2. So this result suggests that WALL1 was built first and was destroyed before the construction of WALL2.

CONCLUSION

In the present study the authors want to investing the ways for solving the fuzzy linear systems that the coefficient is a fuzzy number, which is equals a classic number and our aim is to show the fact that whether the answer can be fuzzy systems or not and at the end complete the discussion by a numerical example.

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