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Research Article

Influence of Radiation and Mass Concentration on MHD Flow over a Moving Plate

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Abstract: This study investigates the influence of radiation and mass concentration on Magnetohydrodynamic (MHD) flow over a moving plate with time dependent suction in an optically thin environment. By introducing fairly realistic assumptions and for a small time dependent perturbation, the concentration temperature and velocity profiles are obtained. The effects of magnetic parameter M, radiation parameter F, porosity parameter X and G_r on the flow variables are discussed qualitatively.

Keywords: Mass concentration, MHD, optically thin, radiation, suction

INTRODUCTION

Flows over a moving plate have several industrial and technological applications in nuclear engineering, astrophysics, aerospace and chemical engineering. Bestman (1990) analyzed the free boundary layer flow with heat and mass transfer in a porous medium where the boundary wall moves in its own plane with large suctions and chemical reaction model of a laniary reaction with Arrhenius activation energy. Alagoa *et al*. (1999) considered the problem of MHD free convection flow with radiation in a porous medium due to time dependent suction in an optically transparent medium using time dependent perturbation. Previously, Pop and Soundalgekar (1975) had done an approximate analysis of the effects of suction on the unsteady rotating flow past an oscillatory plate when both the fluid and the plate are in solid body rotation. The work of Bestman and Adjepong (1988) incorporated radiative heat transfer, since in astrophysical environments the effect of radiation cannot be neglected. Soundalgekar (1974) investigated a two dimensional steady free convection flow of an incompressible, viscous, electrically conducting fluid past an infinite vertical porous plate with constant suction and plate temperature when the difference between the plate temperature and free steam is moderately large to cause free convection currents.

The study by Bestman (1989) considered the effects of velocity, temperature and chemical reaction on the stability of flow in porous media. That is, to ascertain when the flow is unstable with respect to infinitesimal disturbances. Here the equilibrium configuration corresponds to $\frac{\partial q}{\partial y} = 0$. Israel-Cookey and Tay (2002) applied Laplace transform techniques to the problem of MHD free convection and mass transfer flow on a porous medium in a rotating fluid due to heat transfer under arbitrary time-dependent heating of the plate in which the radiative heat flux is expressible in differential form. Also, Israel-Cookey *et al*. (2003) studied the effect of viscous dissipation and radiation on unsteady MHD free-convection flow past an infinite heated vertical plate in a porous medium with time-dependent suction. In another study, Israel-Cookey and Sigalo (2003) investigated the problem of unsteady MHD flow past a semi-infinite vertical plate in an optically thin environment with simultaneous effects of radiation, free convection parameters and time dependent suction.

However, in all these studies, the influences of radiation and mass concentration have been neglected. The present study combines the effects of radiation and mass concentration on MHD flow over a moving plate with time dependent suction in an optically thin environment.

Mathematical formulation: We consider the problem of an electrically conducting fluid over a moving plate with radiation, mass concentration and time dependent suction in the presence of a magnetic field in an optically thin environment. The physical model and the coordinate system are shown in Fig. 1. The x-axis is taken along the vertical infinite plate in the upward direction and y'-axis normal to the plate.

At time $t' = 0$, the plate is maintained at a temperature T'_w which is high enough to initiate radiative heat transfer. A constant magnetic field Bo is

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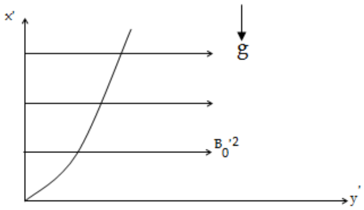

Fig. 1: The physical model and coordinate system of the problem

maintained in the y' direction and the plate moves uniformly along the positive x' direction with velocity U_o. Under Boussinesq approximation the flow is governed by the following equations:

$$\frac{\partial w'}{\partial y'} = 0 \tag{1}$$

$$\frac{\partial u'}{\partial t'} + w' \frac{\partial u}{\partial y'} = \frac{du'}{\partial t'} + V\frac{\partial^2 u}{\partial y^{12}} - \frac{V}{K}(u'-U') - \frac{\sigma c \mu^2 B_0^2}{\rho}(u' - U')$$

$$+ g\beta(T' - T_w) + g\beta c(C' - C_\infty) \tag{2}$$

$$\frac{\partial w'}{\partial t'} = -\frac{1}{P}\frac{\partial P'}{\partial y'} \tag{3}$$

The boundary conditions are:

$$\left.\begin{array}{l} u' \to U'(t') = w_o'(1+\varepsilon e^{i\omega' t'}), T' = T_w, C' = C_w \text{ on } y' = 0 \\ u' \to 0, T' \to 0,\ C' \to 0, \text{as } y' \to 0 \end{array}\right] \tag{4}$$

Since we are considering an optically thin environment such as in astrophysical and galactic environments where the plasma density is low and the temperature of the surrounding medium is not much different from that of the plasma, we assume that the radiative heat flux, $\frac{\partial q'}{\partial y'}$ in the energy equation takes the form:

$$\frac{\partial q'}{\partial y'} = 4\alpha^2 (T' - T_\infty) \tag{5}$$

where,

$$\alpha^2 = \int_o^\alpha \left(\partial \frac{\partial B}{\partial T'}\right) \tag{6}$$

Moreso, from Eq. (1) it is obvious that w^1 is a constant or a function of time only and so we assume:

$$w' = -w_o(1+\varepsilon A e^{i\omega' t'}) \tag{7}$$

Such that ε A<<1 and the negative sign indicates that the suction velocity is towards the plate.

Using Eq. (5) to (7), then Eq. (2) to (5) reduce to:

$$\left(\begin{array}{l} \frac{\partial u'}{\partial t'} - w_0'(1+\varepsilon A e^{i\omega' t'})\frac{\partial u}{\partial y'} = \frac{dU'}{\partial t'} + V\frac{\partial^2 u'}{\partial y'^2} - \frac{V}{K} \\ + \frac{(\sigma_c \mu^2 B_0^2)}{\rho}(u' - U') + g\beta(T' - T_\infty) + g\beta_c(C' - C'_\infty) \end{array}\right] \tag{8}$$

$$\frac{\partial T'}{\partial t'} - w_0'(1+\varepsilon A e^{i\omega' t'})\frac{\partial T'}{\partial y'} = \frac{\kappa}{\rho C_\rho}\frac{\partial^2 T'}{\partial y'^2} - \frac{4\alpha^2}{\rho_{C_P}}(T' - T_\infty) \tag{9}$$

$$\frac{\partial C'}{\partial t'} - w_0'(1+\varepsilon A e^{i\omega' t'})\frac{\partial C'}{\partial y'} = \frac{D}{\nu}\left[\frac{\partial^2}{\partial y'^2} - K_y'^2\right]C' \tag{10}$$

As a result of Eq. (5) to (10) become:

$$\frac{1\partial u}{4\partial t} - (1+\varepsilon A e^{i\omega t})\frac{\partial u}{\partial y} = \frac{1}{4}\frac{dU}{\partial t} + \frac{\partial^2 u}{\partial y^2} \tag{11}$$
$$(X^2 + M^2)(u - U) - M^2 u + G_r\theta + G_c C$$

$$\frac{1}{4}P_r\frac{\partial\theta}{\partial t} - P_r(1+\varepsilon A e^{i\omega t})\frac{\partial\theta}{\partial y} = \left(\frac{\partial^2}{\partial y^2} - F^2\right)\theta \tag{12}$$

$$\frac{1}{4}G_c\frac{\partial c}{\partial t} - G_c(1+\varepsilon A e^{i\omega t})\frac{\partial\theta}{\partial y} = \left(\frac{\partial^2}{\partial y^2} - K_y^2\right)C \tag{13}$$

where the following dimensionless variables were used:

$$t = \frac{w_0'^2 t'}{4\nu}, y = \frac{w_0'}{\nu}y', U = \frac{U'}{U_O}, u = \frac{u'}{U_O}, w = \frac{4\nu w'}{w_0'^2}$$

$$\theta = \frac{T' - T_\infty}{T_w - T_\infty}, c = \frac{C' - C_\infty}{C_w' - C_\infty}, G_r = \frac{\nu g\beta}{R_o w_0'^2}(T_w' - T_\infty),\ P_r = \frac{\mu c_p}{\kappa} \tag{14}$$

$$G_c = \frac{\nu g\beta c}{U_o w_0'^2}(C_w' - C_\infty), F^2 = \frac{4\alpha^2}{\rho C_p K w_0'^2}(T_w' - T_\infty)$$

$$X = \frac{V^2}{w_0'^2 K}, S_c = \frac{\nu}{D}, K_y^2 = K_y'^2$$

$$M = \sqrt{\frac{\mu O B_0^2}{\rho w_0^2}} \text{ Magnetic field}$$

Eq. (11) to (13) are now subject to the boundary conditions:

$$u \to U(t) = 1 + \varepsilon e^{i\omega t}, \theta = 1,\ C = 1;\ \text{on } y = 0 \tag{15}$$

$$u \to 0,\ \theta \to 0,\ C \to 0 \text{ as } y \to \infty$$

The mathematical statements of the problem are now complete and buttress the solution of Eq. (11) to (13) and (15) subject to boundary conditions (15).

Method of solution: The problem as seen in Eq. (11) to (13) subject to the boundary conditions are highly nonlinear partial differential equations and their solutions are intractable to analytical statement. Moreso, asymptotic approximation is in order since ε is small, we assume a perturbation expansion for the variables of the form:

$$u(y,t) = u_o(y) + \varepsilon e^{i\omega t}u_1(y) + O(\varepsilon)^2 \quad (16)$$

$$\theta(y,t) = \theta_o(y) + \varepsilon e^{i\omega t}\theta_1(y) + O(\varepsilon)^2 \quad (17)$$

$$C(y,t) = C_o(y) + \varepsilon e^{i\omega t}C_1(y) + O(\varepsilon)^2 \quad (18)$$

Substituting Eq. (16) to (18) in Eq. (11) to (13), neglecting the coefficients of O (ε)^2 and simplifying we have:

$$u_0'' + u_0' - (X^2 + M^2)u_o = -G_r\theta_o - G_cC_o - (X^2 + M^2) \quad (19)$$

$$\theta_0'' + P_r\theta_0' - F^2\theta_O = 0 \quad (20)$$

$$C_0'' + S_cC_0' - K_y^2 = 0 \quad (21)$$

Subject to the boundary conditions:

$$u_o = 1, \quad \theta_o = 1, \quad C_o = 1 \quad \text{on} \quad y = 0$$

$$u_o \to 0, \quad \theta_o \to 0, \quad C_o \to 0 \quad \text{as}\, y \to \infty \quad (22)$$

For 0(1) equations and:

$$u_1'' + u_1' - \left(X^2 + M^2 + \frac{i\omega}{4}\right)u_1 = -Au_0' - G_r\theta_1 - G_cC_1 - X^2 - M^2 - \frac{i\omega}{4} \quad (23)$$

$$\theta_1'' + P_r\theta_1' - \left(\frac{i\omega P_r}{4} + F^2\right)\theta_1 = -AP_r\theta_0' \quad (24)$$

$$C_1'' + S_cC_1' - (K_y^2 + \frac{i\omega S_c}{4})C_1 = -S_cAC_0' \quad (25)$$

Subject to:

$$\left.\begin{array}{l} u_1 = 1, \;\; \theta_1 = 0, \;\; C_1 = 0 \;\; \text{on} \;\; y = 0 \\ u = 0, \;\; \theta_1 = 0, \;\; C_1 = 0 \;\; \text{as} \;\; y \to \infty \end{array}\right] \quad (26)$$

The solutions of (19) to (21) for the steady flow variables satisfying the boundary conditions (22) are:

$$u_0(y) = (\beta_1 + \beta_2)\, e^{m_5 y} - \beta_1 e^{m_1 y} - \beta_2 e^{m_3 y} + 1 \quad (27)$$

$$\Theta_0(y) = e^{m_3 y} \quad (28)$$

$$C_0(y) = e^{m_1 y} \quad (29)$$

where,

$$\beta_1 = \frac{G_c}{m_1^2 + m_1 - (X^2 + M^2)} \quad (30)$$

$$\beta_2 = \frac{G_r}{m_3^2 + m_3 - (X^2 + M^2)} \quad (31)$$

$$m_1 = -\frac{1}{2}(S_c + \sqrt{S_c^{\,2} + 4K_y^{\,2}}) \quad (32)$$

$$m_3 = -\frac{1}{2}(P_{r_r} + \sqrt{P_r^2 + 4F^2}) \quad (33)$$

$$m_5 = -\frac{1}{2}(1_r + \sqrt{1 + 4(X^2 + M^2)}) \quad (34)$$

Superimposing the steady solutions into the first order approximations (23)-(26), we obtain:

$$U_1(y) = (D_1 + D_2 + D_3)\, e^{m_6 y} - D_1 e^{m_2 y} - D_2 e^{m_4 y} - D_3 e^{m_5 y} + 1 \quad (35)$$

$$\Theta_1(y) = (e^{m_4 y} - 1)\, L \quad (36)$$

$$C_1(y) = (e^{m_2 y} - 1)\, H \quad (37)$$

where,

$$D_1 = \frac{G_cHm_2}{m_2^2 + m_2 - \alpha_0} \quad (38)$$

$$D_2 = \frac{G_rLm_4}{m_4^2 + m_4 - \alpha_0} \quad (39)$$

$$D_3 = \frac{ARm_5}{m_5^2 + m_5 - \alpha_0} \quad (40)$$

$$L = \frac{AP_rm_3}{m_3^2 + P_rm_3 - \left(F^2 + \frac{i\omega P_r}{4}\right)} \quad (41)$$

$$H = \frac{S_cAm_1}{m_1^2 + S_cm_1 - \left(K_y^2 + \frac{i\omega s_c}{4}\right)} \quad (42)$$

$$\alpha_0 = X^2 + M^2 + iw/4 \quad (43)$$

$$R = (\beta_1 + \beta_2) \quad (44)$$

$$m_1 = -\frac{1}{2}\left(S_c + \sqrt{S_c^{\,2} + 4K_y^2}\right) \quad (45)$$

$$m_2 = -\frac{1}{2}(S_c + \sqrt{S_c^{\,2} + 4(K_y^2 + \frac{i\omega s_c}{4})} \quad (46)$$

$$m_4 = -\frac{1}{2}\left(P_r + \sqrt{P_r^{\,2} + 4\left(F^2 + \frac{i\omega P_r}{4}\right)}\right) \quad (47)$$

$$m_6 = -\frac{1}{2}\left(1 + \sqrt{1 + 4\left(X^2 + M^2 + \frac{i\omega}{4}\right)}\right) \quad (48)$$

$$= -\frac{1}{2}\left(1 + \sqrt{1 + \left(1 + 4\alpha_0\right)}\right)$$

This completes the mathematical analysis of the problem within the limits of our approximation.

RESULTS AND DISCUSSION

In the previous sections we have established and solved the problem of the influence of radiation and mass concentration on MHD flow over a moving plate. By imposing the optically thin differential approximation for the radiative heat flux in the energy equation and time dependent perturbation for the concentration, temperature and velocity. The complete expressions for the velocity temperature and concentration is given by the equations:

$$u(y,t) = u_0(y) + \varepsilon e^{i\omega t} u_1(y) \quad (49)$$

$$\theta(y,t) = \theta_0(y) + \varepsilon e^{i\omega t} \theta_1(y) \quad (50)$$

$$C(y,\ t) = C_0(y) + \varepsilon e^{i\omega t} C_1(y) \quad (51)$$

For A = 0.01, P_r = 0.71, ε = 0.01 and t = 0.2 and different values of reaction parameter, the concentration profiles in the Fig. 2 show that increase in suction velocity parameter leads to negligible increase in concentration. Figure 3 shows that the real temperature profile decays exponentially across the boundary layer. Increase in the radiation parameter leads to decrease in the real temperature whereas the reverse is the case with the frequency.

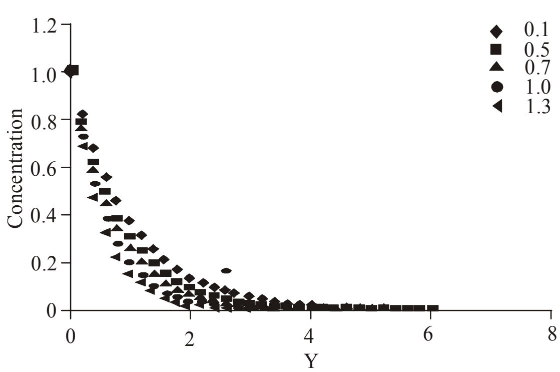

Fig. 2: Concentration profiles for G_c = 1, A = 0.01, t = 0.5 and different values of reaction parameter

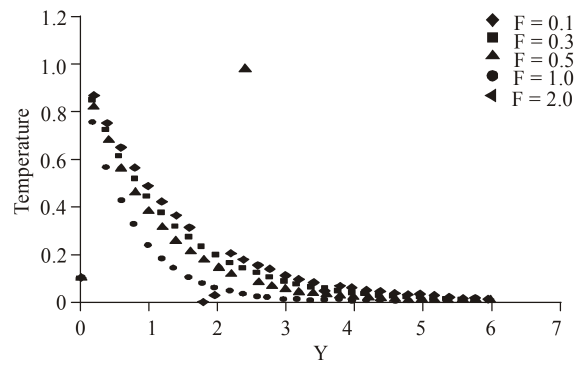

Fig. 3: Temperature profiles for G_c = 2, A = 0.01, t = 0.5 and different values of radiation parameter

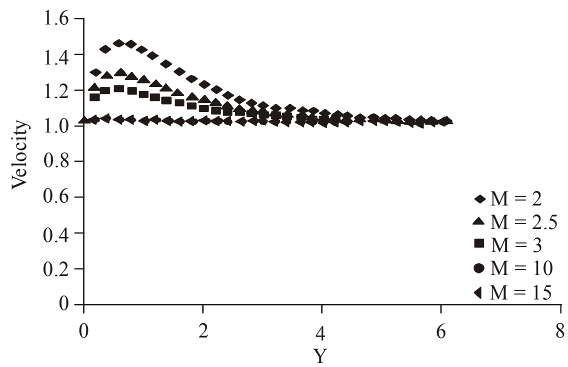

Fig. 4: Velocity profiles for G_r = 2, A = 0.3, G_c = 1, X = 0.5, F = 0.1, R = 0.1, t = 0.1 and different values of magnetic parameter

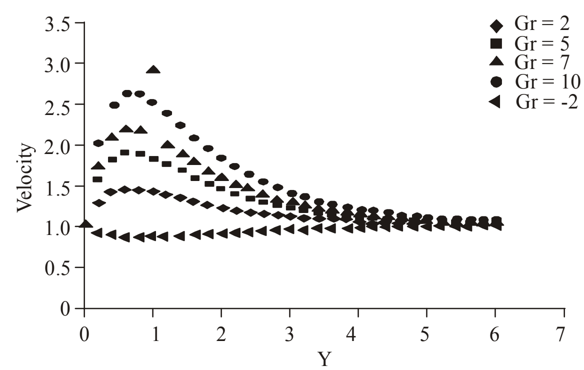

Fig. 5: Velocity profiles for A = 0.3, G_c = 1, M = 2, X = 0.5, F = 0.1, R = 0.1, t = 0.5 and different values of radiation parameter

Figure 4 gives the variation with increasing magnetic field parameter. It is observed that increasing magnetic field parameter gives rise to a rapid decrease in velocity which converges to a steady value towards free stream.

In Fig. 5 it is observed that cooling leads to increase in the velocity, but which converges to a steady value towards free stream. However, the real

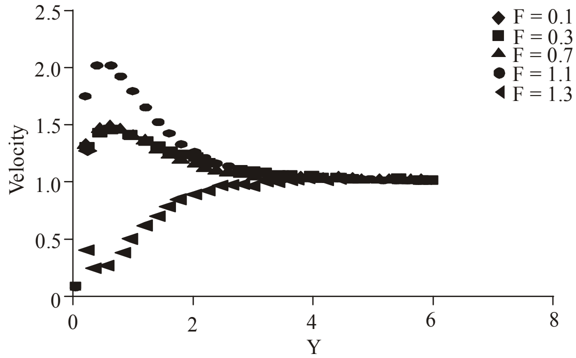

Fig. 6: Velocity profiles for $G_r = 2$, $A = 0.3$, $G_c = 1$, $M = 2$, $X = 0.5$, $R = 0.1$, $t = 0.5$ and different values of radiation parameter

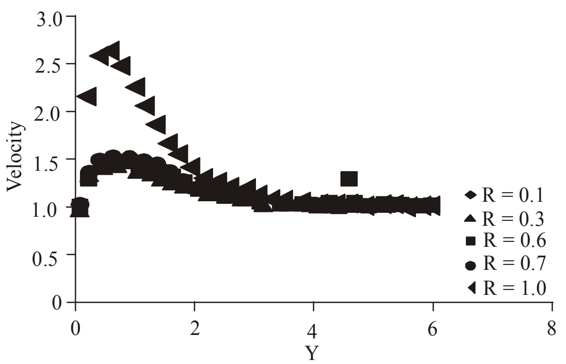

Fig. 7: Velocity profiles for $G_r = 2$, $A = 0.3$, $G_c = 1$, $M = 2$, $X = 0.5$, $F = 0.1$, $t = 0.5$ and different values of reaction parameter

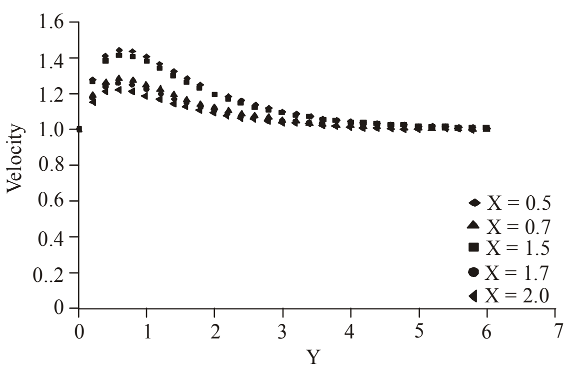

Fig. 8: Velocity profiles for $G_r = 2$, $A = 0.3$, $G_c = 1$, $M = 2$, $F = 0.1$, $R = 0.1$, $t = 0.5$ and different values of parameter

velocity profile falls rapidly from positive values with reversal and rises gradually converging to a constant value at free stream (Fig. 6). Furthermore, increasing the magnetic field parameter results in increase in the real velocity.

Figure 7 shows a correlation between reaction and fluid velocity, with the fluid velocity increasing with reaction and rapidly converging to a constant value at free stream.

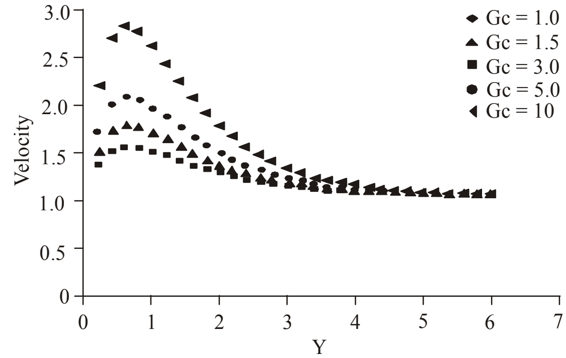

Fig. 9: Velocity profiles for $G_r = 2$, $A = 0.3$, $M = 2$, $X = 0.5$, $F = 0.1$, $R = 0.1$, $t = 0.5$ and different values of modified Grashof parameter

Also, the real velocity profile rises and falls rapidly across the boundary layer and converges to a constant value at free stream (Fig. 8). Increase in porosity parameter X, leads to increase in the real velocity. In Fig. 9 it is observed that greater cooling leads to increase in the velocity which also decays to the relevant free stream value.

There is much qualitative agreement between this present work and previous work in the literature (Israel-Cookey *et al*., 2003; Israel-Cookey and Tay, 2002; Kim, 2001; Niranjan *et al*., 1990).

CONCLUSION

Radiation, porosity and magnetic parameters together with frequency influence the damping velocity of the flow of a fluid over a moving plate with variable suction. In addition t the effects of radiation, Grash of and modified Grash of parameters on the behavior of the velocity profiles are infinitesimal to some extent, while high magnetic parameter M speeds the damping nature of the fluid, whereas radiation and frequency parameters play dominant role on the temperature profiles.

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