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Research Article

A Simple Air Deterministic Mathematical Model for Determining Oxygen and Carbon-dioxide Concentration in a Sealed Environment

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Abstract: This study presents a mathematical model that simulates oxygen and carbon-dioxide concentrations in a sealed environment without the actual measurement of the presence of these substances. Using Mathematical software for simulation, the model was tested using data (volume in cubic meters) of a sealed environment whiles varying the number of people in that environment. The results revealed that, the number of people in the sealed environment affected the concentrations of these substances in the air. Also the model revealed that there is a time (threshold) beyond which it will be unsafe to stay in such an environment.

Keywords: Carbon dioxide emission, deterministic model, oxygen depletion, sealed environment

INTRODUCTION

The quality of air, especially in a sealed environment where people inhabit is very important to health since human beings make use of the various gases contained in the air we breathe. Many sealed environments are fitted with air conditioners with the aim of reducing the room temperature in order to make the place habitable. The air conditioners however, do not affect the various concentrations of gases in the air but rather the people in the environment do affect the levels of oxygen and carbon dioxide concentrations in the sealed environment. As a result, the concentrations of oxygen and carbon dioxide in such an environment may change over time depending on the number of people in that environment.

Human life on earth is sustained by the frequent intake of oxygen and the giving out of carbon dioxide. For an open atmosphere, the percentage volume of oxygen and carbon-dioxide remains fairly unchanged because there are so many biological and chemical processes that replace the loss of these gases.

In a sealed environment (confined spaces, such as classrooms, meeting rooms, offices etc.) however, the situation is a bit different. This suggests that it is not healthy to stay in a sealed environment for so long because the air may not be safe for breathing.

Low oxygen concentrations are dangerous to living creatures. The minimal safe level of oxygen is 19.5%. Below this level breathing and heart rate will increase. Fatigue can set in. At levels lower than 10%, nausea and vomiting can occur as well as loss of consciousness. This is a very serious situation especially if it occurs within a confined space where someone may not have clear access to an exit or method for getting fresh air into the confined area (Rekus, 1999).

Studies have shown that being exposed to increased carbon dioxide in confined spaces, levels you find in a meeting room or classroom where there's lots of carbon dioxide being exhaled by multiple people can actually have a negative impact on your decision-making skills (Rekus, 1999).

From the above discussion, our objective is to determine what happens to the levels of oxygen and carbon dioxide concentrations in a sealed atmosphere over a period of time as well as the time beyond which a given number of people cannot stay in the sealed environment because the air will be unsafe for breathing.

EMPIRICAL LITERATURE REVIEW

Studies have been done to actually measure the concentration of oxygen and carbon dioxide concentrations in a sealed environment over a period of time. Most of the studies require a technical knowledge in chemistry and other related disciplines to measure concentration of these substances in the environment. Some of the works done in this area have been discussed in the following paragraphs.

Using an atmospheric measurement station that was established on the North Sea oil and gas production platform, where atmospheric concentrations of O_2 and CO_2 are continuously measured using fuel cell technology and compact infrared absorption instruments, respectively, Van der Laan *et al.* (2010) presented a paper using the station measurements and detailed methodologies. In comparison to land-based

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stations, the data showed low day-to-day variability, as they were practically free of nightly inversions as well as human influences, due to the station's remoteness. Therefore, the data set collected at this measurement station served directly as background data for the coastal northwest European region. Additionally, the first data presented showed the seasonal cycle as expected during August 2008 through June 2009. Furthermore, some short-term O_2 and CO_2 signals were presented. The observations at the platform included several large and fast changing negative atmospheric O_2 excursions without an accompanying change in the CO_2 signal, which most likely indicated marine O_2 uptake.

Rosenbaum *et al*. (2004) have developed a new clinical bymixer that bypasses a constant fraction of gas flow through a mixing arm. A separate bymixer was interposed in the expiratory and inspiratory limbs of the ventilation circuit to measure mixed gas fractions. By utilizing nitrogen conservation, the clinical bymixer allows the determination of airway carbon dioxide elimination (VCO2) and oxygen uptake (VO2), whenever basic expired flow and gas monitoring measurements are used for the patient. Neither an expiratory exhaust gas collection bag nor expensive, complex equipment are needed. This study tested the accuracy of airway by mixer-flow measurements of VCO2 and VO2 in a new bench apparatus. They compared airway by mixer-flow measurements of VCO2 and VO2 over a range of reference values generated by ethanol combustion in a new metabolic lung simulator, which was ventilated by a volume-cycled respirator. An airway humidity and temperature sensor permitted standard temperature and pressure, dry, correction of airway VCO2 and VO2. Bymixer-flow airway measurements of VCO2 and VO2 correlated closely (R2 = 0.999 and 0.998, respectively) with the stoichio metric values generated by ethanol combustion. The measurements are helpful in the assessment of metabolic gas exchange in the critical care unit. In contrast to using the gas collection bag or complex metabolic monitor, the bymixer should measure mixed gas concentrations in the inspired or expired limb of the common anesthesia circle ventilation circuit.

This formulation presents another perspective that does not require that actual measurements of the substances.

METHODOLOGY

Mathematical formulation: Before the problem is formulated mathematically, the following parameters were defined:

Let:

V = Volume of sealed environment

x = Concentrtion of oxygen at time t

y = Volume of carbondioxide at time t

r_{O2} = Rate of oxygen intake (240 mL or 0.00024 m^3

r_{cO2} = Rate of carbon dioxide outflow (200 mL or 0.0002 m^3)

n = Number of people $in\ the\ environment$

Also to derive a differential equation which describes this process, we applied the equation of continuity or conservation equation. The equation is explained as follows:

Let y be a function that represents the amount of any substance in a given container at time x, then the instantaneous rate of change of y with respects to x is given by:

$$\frac{dy}{dx} = (rate\ of\ substance\ addition) - (rate\ of\ substance\ removal) \quad (1)$$

This is the EQUATION OF CONTINUITY or a CONSERVATION EQUATION (Lomen and Lovelock, 1996).

If x represent the volume of carbon-dioxide in a room of volume V at time t, then concentration of carbon-dioxide is x/V.

In a sealed atmosphere, carbon-dioxide is added when the inhabitants exhale after taking in oxygen. As a result of this assumption, the continuity equation:

$$\frac{dx}{dt} = (rate\ at\ which\ co2\ is\ added) - (rate\ at\ which\ co2\ is\ leaving) \quad (2)$$

Becomes:

$$\frac{dx}{dt} = (rate\ at\ which\ co2\ is\ added) \quad (3)$$

The rate at which carbon dioxide is added into the environment per person is given by:

$$\frac{dx}{dt} = \frac{x\, r_{cO2}}{V} \quad (4)$$

Considering n number of people in the environment, the equation becomes:

$$\frac{dx}{dt} = \frac{x\, r_{cO2} n}{V} \quad (5)$$

Now $r_{cO2} = 0.0002$, so we have:

$$\frac{dx}{dt} = \frac{0.0002nx}{V} \quad (6)$$

Or,

$$\frac{dx}{dt} - \frac{0.0002nx}{V} = 0 \quad (7)$$

If y represents the volume of oxygen in a room of volume V at time t, then concentration of oxygen is y/V.

In a sealed atmosphere, oxygen leaves when the inhabitants inhale. As a result of this assumption, the continuity equation:

$$\frac{dy}{dt} = (rate\ at\ which\ o2\ is\ added) - (rate\ at\ which\ o2\ is\ leaving)$$

Becomes:

$$\frac{dy}{dt} = -(rate\ at\ which\ o2\ is\ leaving) \quad (8)$$

The rate at which oxygen is added per person is given by:

$$\frac{dy}{dt} = -\frac{yr_{O2}}{V} \quad (9)$$

Considering n number of people, we have:

$$\frac{dy}{dt} = -\frac{ynr_{O2}}{V} \quad (10)$$

Since $r_{O2} = 0.0024$, we have:

$$\frac{dy}{dt} = -\frac{0.00024ny}{V} \quad (11)$$

Or,

$$\frac{dy}{dt} + \frac{0.00024ny}{V} = 0 \quad (12)$$

Normal air contains 21% by volume of oxygen and 0.03% by volume of carbon dioxide. Supposing V is the volume of air, then oxygen content is 21%V and carbon dioxide is 0.03%V. (Rekus, 1999).

With the help of the above information, we arrive at initial conditions to help solve the differential equations.

The initial conditions are:

$y(0)_{02} = 0.21V$ and $x(0)c_{02} = 0.0003V$ where V (volume of sealed environment).

The formulation is thus to solve:

$$\frac{dx}{dt} - \frac{0.0002nx}{V} = 0$$

$$\frac{dy}{dt} + \frac{0.00024ny}{V} = 0$$

Such that:

$y(0)_{02} = 0.21V$ and $x(0)c_{02} = 0.0003V$

Assumptions of the model:

- There are no infected persons in the sealed atmosphere
- There are no inflows and outflows of air into the sealed atmosphere
- There is no production of any other gas in the sealed environment

METHOD OF SOLUTION

Solving the differential equations:

$$\frac{dx}{dt} - \frac{0.0002nx}{V} = 0 \quad (13)$$

$$\frac{dy}{dt} + \frac{0.00024ny}{V} = 0 \quad (14)$$

Such that:

$y(0)_{02} = 0.21V$ and $x(0)c_{02} = 0.0003V$

Equations (1) and (2) are first order differential equations with constant coefficients.

Solving Eq. (1):

$$\frac{dx}{dt} - \frac{0.0002nx}{V} = 0 \quad (15)$$

The integrating factor $I = e^{-\int \frac{0.0002n}{V} dt} = e^{-0.0002nt/V}$.

Multiply Eq. (1) by $e^{-0.0002nt/V}$

$$\frac{dx}{dt} e^{-0.0002nt/V} - \frac{0.0002nx}{V} e^{-0.0002nt/V} = 0$$

$$\frac{d}{dt}\left(xe^{-0.0002nt/V}\right) = 0$$

Integrating both sides of the equation above gives:

$xe^{-0.0002nt/V} = k$ where, k is constant

Applying the initial condition gives

$x(t) = 0.0003Ve^{0.0002nt/V}$

Solving Eq. (2):

$$\frac{dy}{dt} + \frac{0.00024ny}{V} = 0 \quad (16)$$

The integrating factor $I = e^{\int \frac{0.00024n}{V} dt} = e^{0.00024nt/V}$.

Multiply Eq. (2) by $e^{0.00024nt/V}$

$$\frac{dy}{dt} e^{0.00024nt/V} + \frac{0.00024ny}{V} e^{0.00024nt/V} = 0$$

$$\frac{d}{dt}\left(ye^{0.00024nt/V}\right) = 0$$

Integrating both sides of the equation above gives:

$ye^{0.00024nt/V} = c$

Table 1: Volume of rooms and capacity

Each persons occupies AN area of 0.45 m^2	Number of persons in the room (n)	Standard height of the rooms (3.5 m)	Volume of the room in m^3
0.45	10	3.5	15.750
0.45	20	3.5	31.500
0.45	30	3.5	47.250
0.45	40	3.5	63.000
0.45	50	3.5	78.750
0.45	60	3.5	94.500
0.45	70	3.5	110.25
0.45	80	3.5	126.00
0.45	90	3.5	141.75
0.45	100	3.5	157.50
0.45	120	3.5	189.00
0.45	140	3.5	220.50

(Ernst and Neufert, 2000)

where, c is constant.
Applying the initial condition gives:

$$y(t) = 0.21Ve^{-0.00024nt/V}$$

MODEL SIMULATION USING MATHEMATICA

Table 1 contains the data of the various room sizes that people usually inhabit together with their respective capacities (Table 2).

Figure 1 below is the result of the simulation using mathematical software. The y-axis represents the volume of oxygen and carbon dioxide and the x-axis represents the time. The point where the two curves meet is called the threshold point. The graph revealed that in a room with volume 47.25 m^3 and containing 10 people, it will take approximately 70000 min to reach the threshold. This means that beyond this point, it will be unsafe to stay in such as environment because the levels will have dropped so low.

Figure 2 below revealed that in a room with volume 47.25 m^3 and containing 15 people, it will take approximately 48000 min to reach the threshold. This means that beyond this point, it will be unsafe to stay in such as environment because the levels will have dropped so low.

Figure 3 below revealed that in a room with volume 47.25 m^3 and containing 20 people, it will take approximately 35000 min to reach the threshold. This means that beyond this point, it will be unsafe to stay in such as environment because the levels will have dropped so low.

Figure 4 below revealed that in a room with volume 47.25 m^3 and containing 25 people, it will take approximately 28000 min to reach the threshold. This means that beyond this point, it will be unsafe to stay in such as environment because the levels will have dropped so low.

Figure 5 below revealed that in a room with volume 47.25 m^3 and containing 30 people, it will take approximately 24000 min to reach the threshold. This means that beyond this point, it will be unsafe to stay in such as environment because the levels will have dropped so low.

Table 2: Description and parameter estimates

Parameter	Description	Estimated value
N	Number of people in a room	Variable
V	Volume of the room	Variable
r_{CO_2}	Rate of carbon dioxide outflow	0.0002 m^3
r_{O_2}	Per-capita latent rate for strain 2	0.00024 m^3

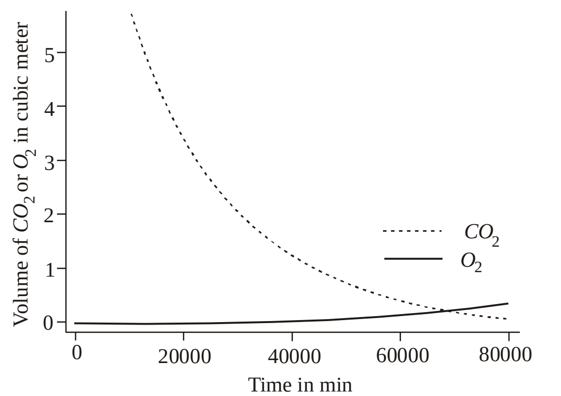

Fig. 1: N = 10, V = 47.25

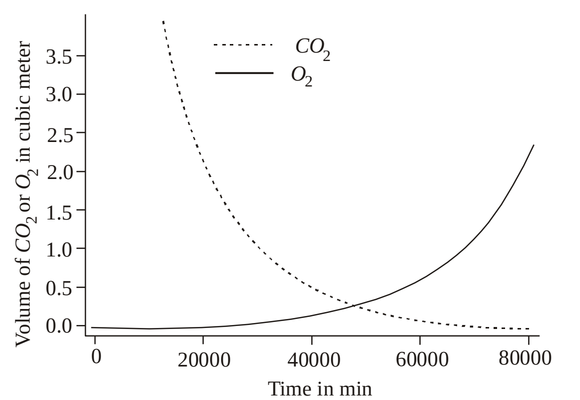

Fig. 2: N = 15, V = 47.25

Figure 6 below revealed that in a room with volume 47.25 m^3 and containing 35 people, it will take approximately 20000 min to reach the threshold. This means that beyond this point, it will be unsafe to stay in such as environment because the levels will have dropped so low.

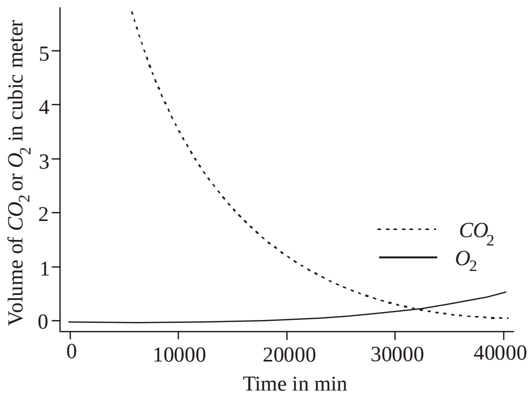

Fig. 3: N = 20, V = 47.25

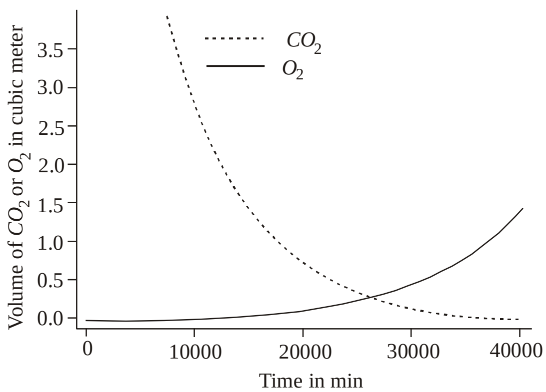

Fig. 4: N = 25, V = 47.25

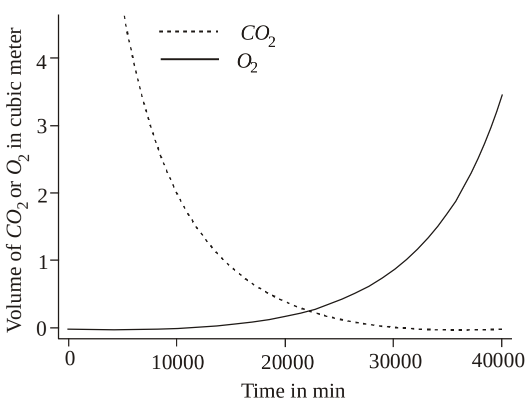

Fig. 5: N = 30, V = 47.25

Figure 7 below revealed that in a room with volume 47.25 m^3 and containing 40 people, it will take approximately 175000 min to reach the threshold. This means that beyond this point, it will be unsafe to stay in such as environment because the levels will have dropped so low.

Figure 8 below revealed that in a room with volume 47.25 m^3 and containing 45 people, it will take approximately 15000 min to reach the threshold. This means that beyond this point, it will be unsafe to stay in such as environment because the levels will have dropped so low.

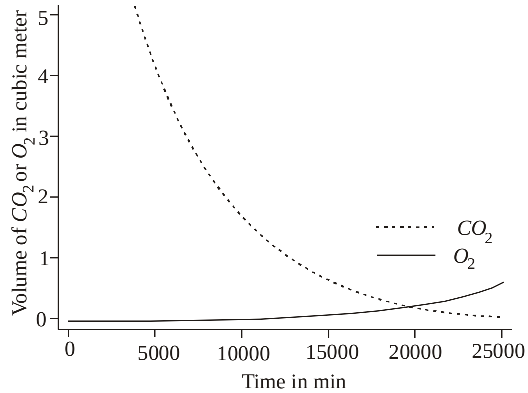

Fig. 6: N = 35, V = 47.25

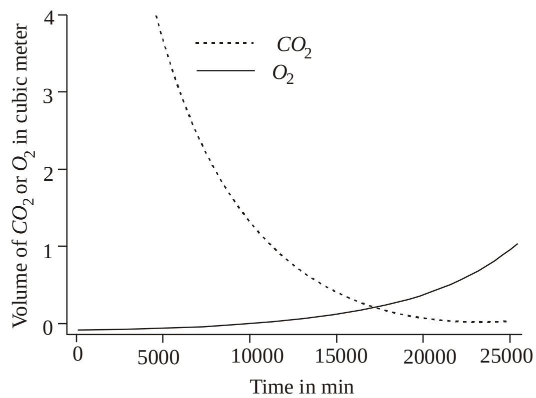

Fig. 7: N = 40, V = 47.25

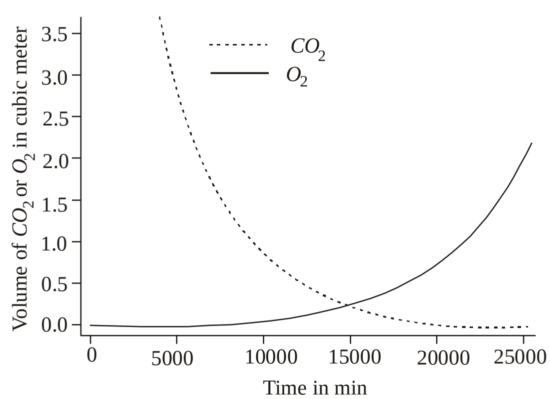

Fig. 8: N = 45, V = 47.25

Figure 9 below revealed that in a room with volume 47.25 m^3 and containing 50 people, it will take approximately 14000 min to reach the threshold. This means that beyond this point, it will be unsafe to stay in such as environment because the levels will have dropped so low.

Figure 10 below revealed that in a room with volume 47.25 m^3 and containing 55 people, it will take approximately 12500 min to reach the threshold. This means that beyond this point, it will be unsafe to stay in

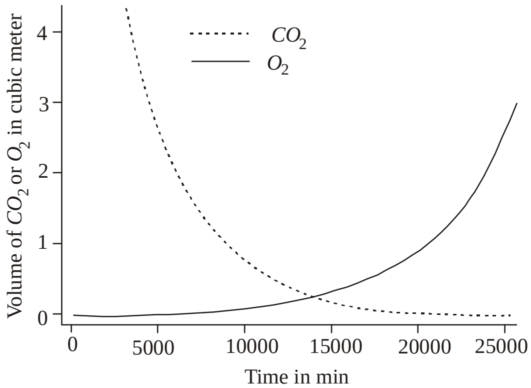

Fig. 9: N = 50, V = 47.25

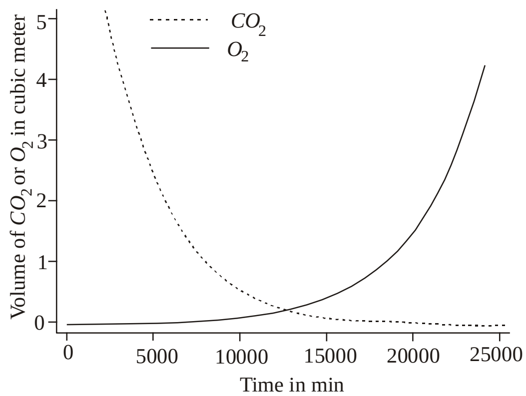

Fig. 10: N = 55, V = 47.25

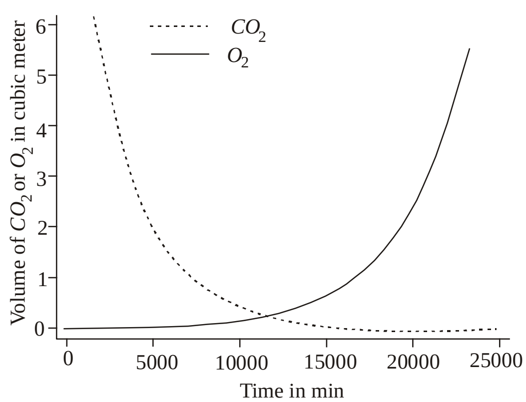

Fig. 11: N = 60, V = 47.25

such as environment because the levels will have dropped so low.

Figure 11 above revealed that in a room with volume 47.25 m^3 and containing 60 people, it will take approximately 11000 min to reach the threshold. This means that beyond this point, it will be unsafe to stay in such as environment because the levels will have dropped so low.

Figure 12 below revealed that in a room with volume 47.25 m^3 and containing 65 people, it will take approximately 10500 min to reach the threshold. This

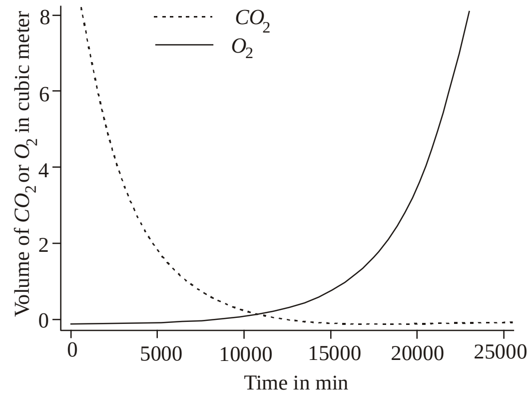

Fig. 12: N = 65, V = 47.25

means that beyond this point, it will be unsafe to stay in such as environment because the levels will have dropped so low.

RECOMMENDATIONS

The simulation was done using a room size of 47.25 m^3 and varied the number of people who stayed in the closed environment. The graph shows how the levels of carbon dioxide and oxygen changes over time. The graphs meet at a point in the course of time. We could see that the time the two graphs met decreased when we increased the number of people and kept the room constant. Beyond the point of intersection, oxygen decreased whiles carbon dioxide increased drastically.

The point the two graphs meet is the threshold point. Beyond this point, it will not safe to stay in such an environment because the concentration of carbon dioxide will be so high whiles that of oxygen will be low.

The simulation helped to determine the time beyond which it will be unsafe to stay in a sealed environment. This can easily be curtailed by reducing the number of people or spending less time in the environment.

CONCLUSION

The simple deterministic model revealed that one could roughly estimate the time beyond which a sealed environment such as an office, classrooms amongst others will be unsafe to inhabit since the oxygen and carbon dioxide would have dropped so low to unsafe levels.

The beauty of this formulation is that actual measurements are not required in this case in order to do the simulation.

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