

## Research Article

### Architectural Configuration and Probability Calculation of the Sequential Logit Model over a Household Budget Survey Data in Turkey

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**Abstract:** The aim of this study is to determine the architectural configuration and probability structure of the sequential logit model. In situations where there are more than two alternatives and where the choice between the alternatives are made sequentially, the situation turns into the estimation of sequential models with fewer alternatives and this reduces the number of calculations that need to be done. In multiple-choice models, individuals make a choice between more than two alternatives and the probability of this choice is calculated. In sequential models, dependent variable levels have a multi-staged sequence and response level on each stage contains the answer level on the previous stage. Success of each step depends on the success of the previous step. An important point regarding sequential models is the necessity that choice probability in each stage must be independent from the choice probability in other stages. In this study, the most important factors that affect an individuals indebtedness status were estimated using sequential logit model and interpretations were made by calculating odds ratios and marginal effect values related to this model. The architecture configuration and detailed interpretations of the sequential logit model were discussed over a real data set on indebtedness.

**Keywords:** Categorical dependent variable, household budget, logit model, sequential modelling

## INTRODUCTION

In multiple-choice models, individuals make a choice between more than two alternatives and the probability of this choice is calculated by creating a model. If a variable, which has more than one categories is taken as dependent variable, it is very important to determine the correct model according to data structure and type of the dependent variable. Akkuş and Özkoç (2012) discussed the most appropriate model or models that can be used depending on the dependent variable type and necessary assumptions. These are given in Table 1.

As can be seen from the Table 1, there is a need for an additional categorical dependent variable structure and a model, which will explain this structure in the best way possible. This structure involves the situations in which individuals' choices are nested and sequential and called sequential probability models.

The main purpose of this study is to introduce the structure of the sequential logit model, to make estimations for each sequential stage of the model and to explain how to calculate and interpret the odds ratio's and explanatory variables' marginal effect values on probability over indebtedness data. Additionally, important factors among the explanatory

variables of household size, marital status, education level, sex and income are determined while indebtedness status of an individual is the dependent variable.

## METHODOLOGY

If the case in hand involves a categorical dependent variable with more than two alternatives, it is important to determine which model in Table 1 will give the best results in terms of modelling the data. In multiple-choice models, individuals make a choice between more than two alternatives and the probability of this choice is calculated. For example, a model in which consumers are asked to express their level of satisfaction about a certain product and have the choices of very bad, bad, fair, good, very good and in which this choice is taken as the dependent variable can be considered to be a multiple choice model (McCullagh and Nelder, 1989).

Without any limitations regarding explanatory variables, in categorical dependent variable models with more than two levels:

- If the dependent variable is measured using nominal scale and choice based, Multinomial Logit

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Table 1: Some popular models used for modelling the psychotomous dependent variable (Akkuş and Özkoç, 2012)

Model	Type of the dependent variable	Model assumptions
Multinomial logit	Nominal	1. Only the characteristics of individuals are required. 2. Strict assumption of Independence of Irrelevant Alternative (IIA) has to be satisfied.
Multinomial probit	Nominal	1. Only the characteristics of individuals are required. 2. No other assumption is necessary including IIA.
Ordered logit	Ordered	1. Only the characteristics of individuals are required. 2. Parallel Slopes Assumption (PSA) is required.
Ordered probit	Ordered	1. Only the characteristics of individuals are required. 2. Parallel Slopes Assumption (PSA) is required.
Nested logit	Nested nominal design	1. Inclusive Value (IV) are required to be positive.
Conditional Logit	Nominal	1. Characteristics of the choice and individuals are both required.
New model added to the table		
Sequential Logit	Nested sequential design	1. Probability of preference in each sequential step is independent from the other probabilities.

\*Logit and Probit models only make difference from the link function they used

or Probit Models, if the dependent variable is measured using ordinal scale and situation based Ordered Logit or Probit Models are used.

- In choice based models, if the personal properties of individuals are taken into account as well, Conditional Logit or Probit Models are used.
- If sequential choices are made between alternatives, but there is not a significant sorting between levels in a nominal structure, Nested Nominal Logit or Probit Models are used.
- If sequential choices are made between alternatives and there is a significant sorting between levels and a nested structure, Sequential Logit or Probit Models are used. This provides us to satisfy the model assumptions and thus improves the quality and reliability of the estimated model.

**Sequential modelling for the categorical dependent variable:**

The sequential modelling that is created for a categorical dependent variable can also be called as a sequential response model or a hierarchic response model. In sequential models, dependent variable levels have a multi-staged sequence and dependent variable level on each stage contains the answer level on the previous stage (Liao, 1984). The multiple response model estimates can be reduced to sequential model estimates with fewer response levels (Amemiya, 1981). In sequential models, the transition from a level to another level is modelled and the success of each step depends on the success of the previous step (Tutz, 2014; Boes and Winkelmann, 2006).

In sequential modelling, the estimation of dependent variable models turn into the estimation of binary models and this reduces the number of calculations that need to be done. Therefore, general information regarding binary categorical dependent variable models is given below.

Here,  $\beta$  indicates the parameter vector,  $Y$  indicates the binary dependent variable and  $X$  indicates the explanatory variables vector. The expected value of  $Y_i$ , conditional on explanatory variables is defined as follows:

$$\begin{aligned} \eta_i &= \beta_1 + \beta_2 X_{i1} + \dots + \beta_k X_{ik} \\ \beta &= (\beta_1 \beta_2 \dots \beta_k)' \\ X_i' &= (1 \ X_{i1} \ \dots \ X_{ik}) \Rightarrow \eta_i = X_i' \beta \\ Y_i &= \beta_1 + \beta_2 X_{i1} + \dots + \beta_k X_{ik} + \varepsilon_i = X_i' \beta + \varepsilon_i \\ E(Y_i / X_i) &= 1.P(Y_i = 1 / X_i) + 0.P(Y_i = 0 / X_i) \\ &= P(Y_i = 1 / X_i) = X_i' \beta \end{aligned} \tag{1}$$

The model given in Eq. (1) is called Linear Probability Model (LPM). The model parameters are estimated by the method of Ordinary Least Square (OLS). However, it is seen that majority of the linear regression model assumptions cannot be satisfied because of the categorical structure of the dependent variable. Alternatively, the use of logit and probit models become widespread, which require less assumptions relatively (Liao, 1984; McCullagh and Nelder, 1989).

The basis of the transition to logit and probit models is the selection of a link function that will form a linear structure with linear combinations of explanatory variables by determining a special function of the conditional expected value  $E(Y_i / X_i)$ . This function cannot be any function, because it must have the ability to limit the probability of  $P(Y_i = 1 / X_i)$  in 0-1 range.

In the light of this information, the model in Eq. (1) is written in the following format and forms a base for the transition to logit and probit models where  $F$  is the specially determined link function:

$$F[E(Y_i / X_i)] = F[P(Y_i = 1 / X_i)] = X_i' \beta \tag{2}$$

In Eq. (2):

- In case that the  $F$  is taken as “logit” link function, which is expressed as  $\log \frac{P(Y_i = 1 / X_i)}{1 - P(Y_i = 1 / X_i)}$ , the conditional probability of an observation belongs to the level “1” coded in binary dependent variable is, as follows:

$$F[E(Y_i / X_i)] = F[P(Y_i = 1 / X_i)] = X_i' \beta$$

$$\log \frac{P(Y_i = 1 / X_i)}{1 - P(Y_i = 1 / X_i)} = X_i' \beta \Rightarrow P(Y_i = 1 / X_i) = \frac{\exp(X_i' \beta)}{1 + \exp(X_i' \beta)} \quad (3)$$

- In case that F is specially selected as  $\Phi^{-1}$  where  $\Phi$  indicates the standard normal cumulative distribution function, the “Probit” model given below is obtained:

$$F[E(Y_i / X_i)] = F[P(Y_i = 1 / X_i)] = X_i' \beta$$

$$\Phi^{-1}[E(Y_i / X_i)] = \Phi^{-1}[P(Y_i = 1 / X_i)] = X_i' \beta \quad (4)$$

$$P(Y_i = 1 / X_i) = \Phi(X_i' \beta)$$

Model parameters can be estimated using Maximum Likelihood Estimation (MLE) method, which does not require restrictive assumptions compared with OLS (Aldrich and Nelson, 1984; Maddala, 1983; McCullagh and Nelder, 1989).

There are three important points to focus on when interpreting the categorical dependent variable models:

- Interpretation of event probabilities
- Interpretation of marginal effect of change in explanatory variable on probability
- Interpretation of “Odds Ratios”, which can only be calculated for logistic regression models

Event probabilities can be calculated for each observation using Eq. (3) and Eq. (4). Odds ratios are found and interpreted by calculating  $\exp(\cdot)$  values of the estimated model parameters in logistic regression.

In case that the dependent variable is binary, marginal effect of a change in explanatory variable on the estimated probabilities is generally calculated using Eq. (5), while in LPM, Probit and Logit models, it is calculated using Eq. (6), (7) and (8), respectively:

$$\frac{\partial P_i}{\partial X_{ik}} = \frac{\partial F(X_i' \beta)}{\partial X_{ik}} = F(X_i' \beta) \frac{\partial X_i' \beta}{\partial X_{ik}} = f(X_i' \beta) \beta_k \quad (5)$$

$$\frac{\partial P_i}{\partial X_{ik}} = \beta_k \quad (6)$$

$$\frac{\partial P_i}{\partial X_{ik}} = \phi(X_i' \beta) \beta_k \quad (7)$$

$$\frac{\partial P_i}{\partial X_{ik}} = \frac{\exp(X_i' \beta)}{(1 + \exp(X_i' \beta))^2} \beta_k \quad (8)$$

In Eq. (5), f indicates the function that is obtained by taking the first-order partial derivative of F link function with respect to explanatory variables; in Eq.

(7),  $\phi$  indicates the standard normal density function, i (i=1,...,n) indicates the observations and k (k=1,...,K) indicates the explanatory variable.

So far, necessary information regarding binary dependent variable models to better explain and understand the sequential modelling terminology were given. In the following sections, architectural structure of a sequential model, data type used, model assumptions, advantages and drawbacks of the model and estimation methods are discussed and sequential logit model interpretations are presented using a real data set.

**General expressions regarding the sequential models and modelling the transition probabilities:**

The main idea in sequential modelling is that the response mechanism begins in the first category. Individual decides whether to stay in the first category or move on to the next one. This two-state decision can be coded as 0/1.

i indicating the index corresponding to observation values (i=1,...,n) and j indicating the dependent variable level (j=1,...,J),  $y_i^{(1)} = 1$  indicates whether the process is ended in the first category or not and in this case it is observed as  $y_i = 1$ . If the process continues,  $y_i^{(1)} = 0$ . The condition for the continuation of the process, transition from the second category to third category, is determined as  $y_i = 2$  with respect to  $y > 2$  and the process ends if  $y_i^{(2)} = 1$ . Generally, transition from category j to j+1 can be shown as follows (Boes and Winkelmann, 2006):

$$y_i^{(j)} \begin{cases} 1 & \text{if the process is completed, there is no transition to the category } j+1 \\ 0 & \text{if the process is going on, there is a transition to the category } j+1 \end{cases}$$

Category j is chosen based on the rejection probability of category (j+1). Therefore, the final transition must be modelled. This process points out the importance of giving the right decision on whether the process will go on or not, when transitions are conditionally modeled and category j is reached.

Conditional transition probabilities need to be determined in order to formulate the conditional probability model with ordered response levels. Event probability  $y_i = j$  can be shown depending on the condition  $y_i > j$ , as follows (Boes and Winkelmann, 2006):

$$P(Y_i = j | Y_i \geq j, x_i) = F(X_i' \beta) \quad (9)$$

In Eq. (9),  $\beta$  indicates the parameter vector,  $y_i$  and  $X_i$  indicate the dependent and explanatory variable values, respectively.

After determining the conditional transition probabilities, unconditional probabilities  $P(y_i = j | x_i)$  (probabilities, which are unconditional on  $y_i > j$ , but still conditional on  $x_i$ ) can be obtained using repeated relations:

$$P_{ij} = P(y_i = j | x_i) = P(y_i = j | y_i \geq j, x_i)P(y_i \geq j | x_i) \quad (10)$$

In fact, the conditional transition probabilities fully characterize the  $y_i$  probability function and can generally be expressed as follows:

$$P_{ij} = P(y_i = j | y_i \geq j, x_i) \prod_{r=0}^{j-1} [1 - P(y_i = r | y_i \geq r, x_i)] = F(\alpha_j + x_i \beta) \prod_{r=0}^{j-1} [1 - F(\alpha_r + x_i \beta)] \quad (11)$$

If Eq. (11) is generalized with respect to  $\beta_j$  parameter vector; the expression is obtained:

$$P_{ij} = P(y_i = j | x_i) = F(\alpha_j + x_i \beta_j) \prod_{r=0}^{j-1} [1 - F(\alpha_r + x_i \beta_j)] \quad (12)$$

Here, while  $\alpha_0 = -\infty$ ,  $F(-\infty) = 0$ . As can be seen, binary model estimations are done in each category of the sequential model. One of the most important features of the sequential model is that there is no restrictions on parameter space when estimating the probability function. In sequential models, probabilities can be expressed as follows with a simple formulation:

$$P_{ij} = P_i \cdot P_{ji} \quad (13)$$

$P_i$  is the probability of  $y_i$  result and  $P_{ji}$  is the  $y_i$  result conditional on observation  $i$ . Finally, we should emphasize that observation values of  $y_1, y_2, \dots$  must be conceptually different and statistically independent from each other (Liao, 1984).

### SOME OF THE STUDIES IN THE LITERATURE RELATED TO THE ARCHITECTURAL STRUCTURE OF THE SEQUENTIAL MODEL

Examples of the architectural structure of the sequential modelling in case that choices are made sequentially in the literature are given below.

The data set used in the study conducted by Maddala (1983) is a good example for sequential response models. The study deals with the educational success of individuals and the possible values of Y are coded, as follows:

- Y = 1: If the individual graduated from high school
- Y = 2: If the individual graduated from high school, but did not graduate from college
- Y = 3: If the individual graduated from college, but did not obtain a professional degree
- Y = 4: If the individual obtained a professional degree

When the data structure is examined in this study, in which a status-based categorical variable is accepted as dependent variable, it is seen that statuses continues in a sequential manner. The sequential structure can be better understood by creating a model schema (Fig. 1).

**The mathematical representation of the estimated probabilities:** As shown in the studies conducted by Amemiya (1981) ve Maddala (1983), the conditional probability that an individual choice corresponds to one of the sequential categories is usually formulated as follows:

- Probability of not graduating from high school:

$$P_1 = F\left(\sum_{k_1}^{K_1} \beta_{k_1} X_{k_1}\right)$$

- Probability of not graduating from college even if it is known that the individual graduated from high school:

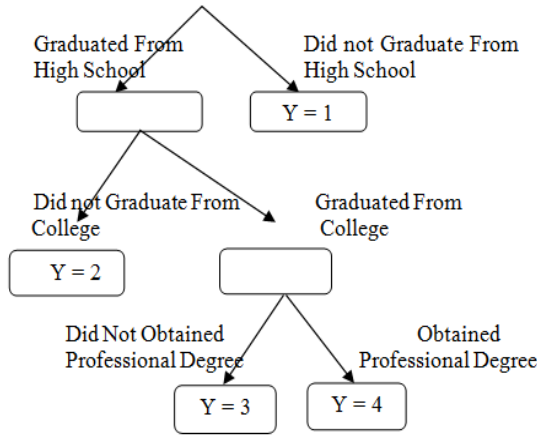


Fig. 1: Branch structure 1 of the dependent variable in sequential modelling

$$P_2 = \left[ 1 - F\left(\sum_{k_1}^{K_1} \beta_{k_1} X_{k_1}\right) \right] F\left(\sum_{k_2}^{K_2} \beta_{k_2} X_{k_2}\right)$$

- Probability of not obtaining a professional degree even if it is known that the individual graduated from high school and college:

$$P_3 = \left[ 1 - F\left(\sum_{k_1}^{K_1} \beta_{k_1} X_{k_1}\right) \right] \left[ 1 - F\left(\sum_{k_2}^{K_2} \beta_{k_2} X_{k_2}\right) \right] F\left(\sum_{k_3}^{K_3} \beta_{k_3} X_{k_3}\right)$$

- Probability of obtaining a professional degree even if it is known that the individual graduated from high school and college:

$$P_4 = \left[ 1 - F\left(\sum_{k_1}^{K_1} \beta_{k_1} X_{k_1}\right) \right] \left[ 1 - F\left(\sum_{k_2}^{K_2} \beta_{k_2} X_{k_2}\right) \right] \left[ 1 - F\left(\sum_{k_3}^{K_3} \beta_{k_3} X_{k_3}\right) \right]$$

In probability equations:

- $k_1, k_2, k_3, k_4$  represent the subscribes of X explanatory variables set, which are included in stages 1, 2, 3, 4, respectively.
- **Stage 1:** parameter  $\beta_{k_1}$  can be estimated by dividing the whole sample into two groups as graduated and not graduated from high school.
- **Stage 2:** parameter  $\beta_{k_2}$  can be estimated by dividing the sub-sample of high school graduates into two groups as graduated and not graduated from college.
- **Stage 3:** parameter  $\beta_{k_3}$  can be estimated by dividing the sub-sample of college graduates into two groups as obtained and not obtained a professional degree.
- As in binary dependent variable models, considering that the sum of probabilities equals to “1”, the estimation of j-1 number of parameter sets is sufficient for a dependent variable with j number of response categories. Accordingly, parameter and probability estimates for each stage of the

sequential model can be obtained using one of the binary logit or probit model algorithms.

Another study by Cragg and Uhler (1970) and discussed by Maddala (1983) is on an automobile demand. As can be seen from the architectural structure of the data below, binary results sometimes cannot be demonstrated on decision tree in an organised manner. This is considered as other type of sequential decision-making method (Fig. 2).

As mentioned before, an important point regarding sequential models is the necessity that choice probability in each stage must be independent from the choice probability in other stages.

Some of the other studies in literature are briefly mentioned below.

In the study conducted by Anderson and Stein (2011), the relationship between the use of marijuana and young female adults was studied using sequential logit model.

In his study, Buis (2011) dealt with the results of unobservable heterogeneity in sequential logit model.

Gürler (2011) analyzed the factors that affect the child poverty in Turkey using combined cross-section data with sequential logit model.

Hossain (2009) focused on alternative specifications in logit models.

In their study in Nagakura and Kobayashi (2009) tried to determine whether sequential logit model or Nested logit model is more appropriate for the data.

In their study on travel request to areas evacuated due to hurricane, Fu and Wilmot (2007) used sequential logit model.

Ophem and Schram (1997) compared sequential logit model with multinomial logit model in their study.

In the applied study conducted by Zhang (1994), the factors that affect socio-economical wealth in Hebei city of China were determined using sequential logit model.

Weiler (1986) tried to determine the factors that might effect getting an attending to college-level educational institutions using sequential logit model.

### Interpretation of the sequential logit model results:

The sequential logit model is obtained by determining the distribution function represented by  $F(\cdot)$  in above sub-section specifically as the logistic distribution function represented by  $\Lambda$ . As explained, The model is given below in its most general form:

$$P_{ij} = P(Y_i = j / x_i) = \Lambda(\alpha_j + x_i' \beta_j) \prod_{r=0}^{j-1} \left[ 1 - \Lambda(\alpha_r + x_i' \beta_r) \right] \quad (14)$$

It is known that the mathematical representation of the logistic distribution function is  $\Lambda(\cdot) = \frac{\exp(\cdot)}{1 + \exp(\cdot)}$ .

When this expression is written instead of  $\Lambda$  in Eq. (14), the sequential logit model given in Eq. (15) is obtained:

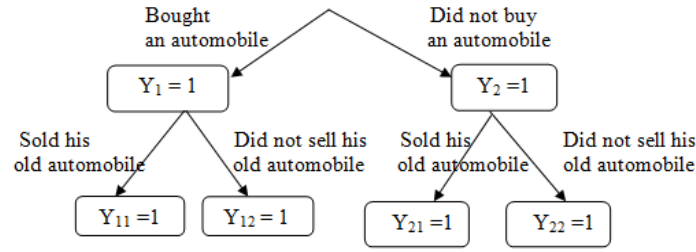


Fig. 2: Branch structure 2 of the dependent variable in sequential modelling

$$P_{ij} = P(Y_i = j / x_j) = \frac{\exp(\alpha_j + x'_j \beta_j)}{1 + \exp(\alpha_j + x'_j \beta_j)} \prod_{r=0}^{j-1} \left[ 1 - \frac{\exp(\alpha_r + x'_r \beta_r)}{1 + \exp(\alpha_r + x'_r \beta_r)} \right] \quad (15)$$

Since sequential logit models are made up of binary logit model sets, interpretation of sequential logit models are based on the interpretation of binary logit models (Liao, 1984).

When interpreting sequential logit model results, interpretation of event probabilities is focused on firstly and more noticeable interpretations can be obtained by interpreting odds ratios. In addition, more detailed interpretations can be obtained through interpretation of marginal effects, which answers the question of how partial change in explanatory variables effects event probabilities. Estimated results are based on estimates calculated from binary logit models. Its only difference from binary result models is that in sequential models, estimated probabilities include the multiplication of probabilities obtained from the related stage. The interpretation of the conditional probabilities and odds ratios are explained in detail in the application section.

Considering that sequential models are created by sorting binary models sequentially, interpreting the effect of any independent variable on event probability or its marginal effect on odds ratios when the effects of other variables are constant is a natural result of interpreting the marginal effects in binary models (Liao, 1984).

In the binary model, the marginal effect of a change of one unit in the value of  $X_k$  explanatory variable is obtained by taking the first-order partial derivative of  $P(Y = 1)$  probability with respect to  $X_k$  variable:

$$\begin{aligned} \frac{\partial P(Y = 1)}{\partial x_k} &= \frac{\exp(\alpha_j + x'_j \beta_j)}{[1 + \exp(\alpha_j + x'_j \beta_j)]^2} \beta_k = \frac{\exp(\alpha_j + x'_j \beta_j)}{[1 + \exp(\alpha_j + x'_j \beta_j)]} \frac{1}{[1 + \exp(\alpha_j + x'_j \beta_j)]} \beta_k \\ &= P(Y = 1)[1 - P(Y = 1)]\beta_k \end{aligned} \quad (16)$$

In the equation,  $\partial$  indicates the partial derivative or the marginal effect.

Based on the general probability representations of the sequential logit model given in Eq. (14) and Eq. (15), the Marginal Effect (ME) of any explanatory variable on the event probability is usually explained as follows, starting from the first category:

$$ME_{ik} = f(\alpha_1 + x'_1 \beta_1) \beta_{1k} \quad (17)$$

Here,  $f(z) = dF(z)/dz$ . The obtained value indicates the approximate change in the probability in case of an one unit increase in  $k$ . element of  $x$ . For each of the remaining categories ( $j = 2, \dots, J$ ), the marginal effects can be calculated in the form of nested marginal effects (Liao, 1984):

$$ME_{ijk} = f(\alpha_j + x'_j \beta_j) \beta_{jk} \prod_{r=1}^{j-1} [1 - F(\alpha_r + x'_r \beta_r)] - F(\alpha_j + x'_j \beta_j) \sum_{r=1}^{j-1} ME_{irk} \quad (18)$$

In case that the explanatory variables are continuous, the formulas given in Eq. (16), (17) and (18) can be used successfully, however in case that dummy explanatory variables are used, obtained marginal effect values may be greater than the actual values due to the fact that the function cannot be differentiated. As a solution to this problem, probabilities for each level of the studied variable could be calculated by giving their average values of the non-studied variables. The difference between these probabilities gives the real marginal effect.

In the following section, architectural structure of the sequential logit model is shown through a real data set on individuals' indebtedness statuses and detailed interpretations are made.

### APPLICATION

The data used in the application section of the study was compiled from "Household Budget Survey" published by Turkish Statistical Institute (TSI) in 2013. Factors that affect the indebtedness statuses of individuals were examined using the Sequential Logit Model. The data used in the study, consisting of 21763 individuals, was modeled using Stata 10 software package. With the help of this study, researchers, practitioners in particular, who work in this field will be able to estimate the most appropriate model and make detailed interpretations in case that they encounter with a sequential dependent variable structure. In the following sub-sections, the branch structure of sequential model, codes of dependent variable levels, the definitions of explanatory variables, estimation results and interpretations are given.

**Branch structure of the sequential logit model:** In our study, the dependent variable Y is defined as "Individuals' Indebtedness Statuses" and coded as follows:

- Individual is not in debt
- Individual is in debt to persons
- Individual is in debt to banks or cooperative institutions
- Individual is in debt to retail stores

Here,

Y = 1 : No debt

Y = 2 : Is in debt to a relative-other persons or in debt in another way (Being in debt to persons)

Y = 3 : Is in debt to a bank or cooperative (Being in debt to institutions)

Y = 4 : Is in debt to a retail store (Being in debt to institutions)

Probability definitions and mathematical formulations of dependent variable levels used in the sequential logit model are as follows.

- $P_{1f}$  : Total probability of individual's having no debt.
- $P_{2f}$  : Total probability of individual's being in debt to persons.
- $P_{3f}$  : Total probability of individual's being in debt to a bank or cooperative.
- $P_{4f}$  : Total probability of individual's being in debt to retail stores.

Mathematical presentations of these probabilities are as follows:

$$\begin{aligned}
 P_{1f} &= F(\beta_1'X) \\
 P_{2f} &= [1 - F(\beta_1'X)]F(\beta_2'X) \\
 P_{3f} &= [1 - F(\beta_1'X)][1 - F(\beta_2'X)]F(\beta_3'X) \\
 P_{4f} &= [1 - F(\beta_1'X)][1 - F(\beta_2'X)][1 - F(\beta_3'X)]
 \end{aligned}
 \tag{19}$$

Branch structure of the data is given below in Fig. 3.

F indicates the logistic distribution function,  $\beta_1$ ,  $\beta_2$  and  $\beta_3$  indicate the parameter vectors in the model and X indicates the explanatory variable vector. Open forms of expressions in Eq. (19) are given below:

$$\begin{aligned}
 P_{1f} &= \frac{\exp(\beta_1'X)}{1 + \exp(\beta_1'X)} \\
 P_{2f} &= \left[1 - \frac{\exp(\beta_1'X)}{1 + \exp(\beta_1'X)}\right] \left[\frac{\exp(\beta_2'X)}{1 + \exp(\beta_2'X)}\right] \\
 P_{3f} &= \left[1 - \frac{\exp(\beta_1'X)}{1 + \exp(\beta_1'X)}\right] \left[1 - \frac{\exp(\beta_2'X)}{1 + \exp(\beta_2'X)}\right] \left[\frac{\exp(\beta_3'X)}{1 + \exp(\beta_3'X)}\right] \\
 P_{4f} &= \left[1 - \frac{\exp(\beta_1'X)}{1 + \exp(\beta_1'X)}\right] \left[1 - \frac{\exp(\beta_2'X)}{1 + \exp(\beta_2'X)}\right] \left[1 - \frac{\exp(\beta_3'X)}{1 + \exp(\beta_3'X)}\right]
 \end{aligned}
 \tag{20}$$

While  $\beta_1$  parameter is calculated using the whole sample,  $\beta_2$  parameter is estimated by observing the remaining sample after subtracting individuals with no debt from the data set.  $\beta_3$  parameter is estimated by observing the remaining sample after subtracting individuals with no debt and individuals in debt to persons from the data set. The explanatory variables which are thought to affect individuals' indebtedness probability are; Household Size; Marital Status (Married-Single); Education (Literate; Not Literate); Gender (Male-Female) and Income.

**Estimation results of sequential logit model:** Stata 10 software package was used in order to obtain sequential logit model results. Estimation results obtained from the combined data are shown in Table 2.

Examining the results given in Table 2, it can be seen that the test used to determine whether the estimated model is statistically significant or not is based on Likelihood Ratio (LR). LR statistic that has a chi square distribution with 15° of freedom is found to be 810.99. According to the related probability value, the model is statistically significant at the 5% level of significance ( $p = 0.000 < p = 0.05$ ).

**First stage: Sequential logit model results for individuals with no debt:** Examining the results of the first stage, it is concluded that all explanatory variables expect for "Income" are statistically significant at the 5% level of significance. Accordingly, since the estimated coefficient of household size variable (0.09455) is a positive number, probability of an individual's having no debt increased as household size increased. Similarly, it is observed that married individuals (0.54618) have a higher probability of

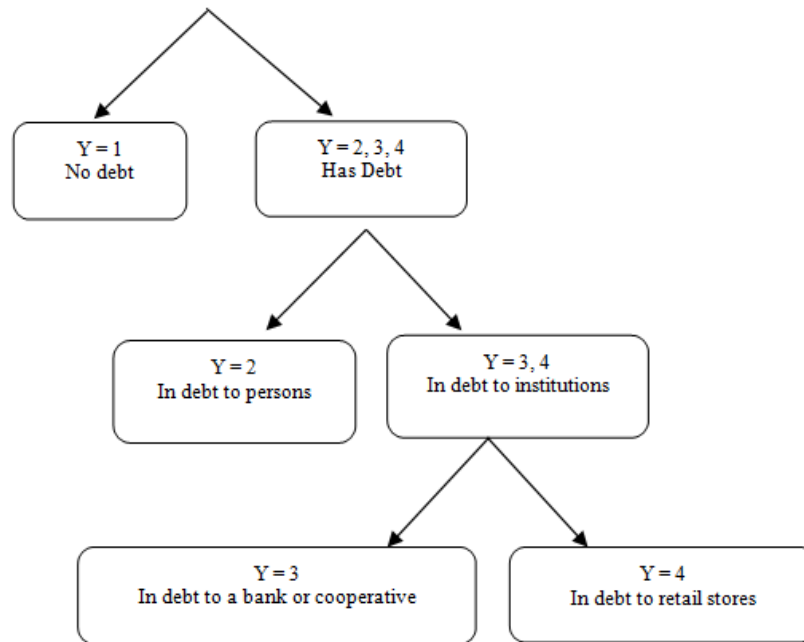


Fig. 3: Branch structure of individuals' indebtedness statuses

Table 2: Results of the sequential logit model steps

Explanatory variables	Model			Model			Model		
	First Step:(P <sub>1</sub> ) Probability of individual's having no debt			Second Step:(P <sub>2</sub> ) Probability of individual's being in debt to persons			Third Step:(P <sub>3</sub> ) Probability of individual's being in debt to a bank or cooperative		
	$\hat{\beta}_1$	SE( $\hat{\beta}_1$ )	p	$\hat{\beta}_2$	SE( $\hat{\beta}_2$ )	p	$\hat{\beta}_3$	SE( $\hat{\beta}_3$ )	p
Constant	-0.20587	0.396	0.603	0.08734	0.498	0.861	1.70729*	0.636	0.007*
Size of Household	0.09455	0.009	0.000*	-0.16325	0.011	0.000*	0.11751*	0.020	0.000*
Marital Status									
Married	0.54618	0.074	0.000*	-0.05705	0.105	0.586	-0.02300	0.142	0.871
Education									
Literate	-0.41076	0.072	0.000*	-0.83108	0.101	0.000*	-0.71100*	0.200	0.000*
Sex									
Male	-0.19494	0.076	0.010*	0.37170	0.103	0.000*	-0.81212*	0.148	0.000*
Income	0.00323	0.020	0.868	0.04420	0.024	0.069	-0.04102	0.031	0.185
Number of obs.	21764								
LR chi2 (15)	810.99 Prob>chi2 : 0.000			Log-likelihood: -28607,484					

\*Coefficient is statistically significant at a significance level of 5%; \*Base Categories: Not married, Illiterate, Female

having no debt compared to single individuals; literate individuals (-0.41076) have a lower probability of having no debt compared to illiterate individuals and similarly men (-0.19494) have a lower probability of having no debt compare to women.

**Second stage: Sequential logit model results for individuals in debt to persons:** In this stage, it is found that all explanatory variables expect for “Marital Status” and “Income” are statistically significant at the 5% level of significance. Accordingly, it is observed that larger household size (-0.16235) and higher level of literacy (-0.83108) decrease the probability of individuals' being in debt to persons. Men (0.37170) have a higher probability of being in debt to persons compared to women.

**Third stage: Sequential logit model results for individuals in debt to a bank or cooperative:**

In the final decision stage, it is seen that “Marital Status” and “Income” variables are not statistically significant at the 5% level of significance. Examining the obtained results, it can be seen that the probability of an individual's being in debt to banks or cooperatives increases as the household size increase (0.11751). Also, men (-0.81212) have a lower probability of being in debt to banks or cooperatives compared to women and literate individuals (-0.71100) have a lower probability of being in debt to banks or cooperatives compared to illiterate individuals.

**Calculation and interpretation of odds ratios:** Odds ratio is a measure of the size of the effect. It is the ratio



Table 3: Odds ratios calculated from significant variables for decision levels

	Model		
	First Step	Second Step	Third Step
Explanatory variables	Odds Ratios		
Size of household	1.099	0.849	1.125
Marital status			
Married	1.727	Non-significant	Non-significant
Education			
Literate	0.663	0.436	2.036
Sex			
Male	0.823	1.450	0.444
Income	Non-significant	Non-significant	Non-significant

of an event’s probability ratio for a group to probability ratio for another group or estimation of this ratio based on a sample. Odds ratios calculated on each step of the model created using sequential logit analysis method are given in Table 3.

**First step: Interpretation of odds ratios for individuals with no debt:** Examining the obtained results, the odds ratio value calculated for household size is 1.099. The fact that this value is close to 1 shows that the household size does not have a significant effect on an individual’s probability of having no debt. On the other hand, a married individual’s probability of having no debt is 1.73 times higher compared to a single individual; an illiterate individual’s probability of having no debt is 1.515 ( $1/0.66 = 1.515$ ) times higher compared to a literate individual and a woman’s probability of having no debt is 1.219 ( $1/0.82 = 1.219$ ) times higher compared to a man.

**Second step: Interpretation of odds ratios for individuals in debt to persons:** Examining the results of the second step, an increase of one unit in household size decreased an individual’s probability of being in debt to persons by 1.178 ( $1/0.849 = 1.178$ ). In addition, an illiterate individual’s probability of being in debt to persons is approximately 2.33 ( $1/0.436 = 2.33$ ) times higher compared to a literate individual; and a man’s probability of being in debt to persons is 1.45 times higher compared to a woman.

**Third step: Interpretation of odds ratios for individuals in debt to a bank or cooperative:** Examining the results of the last step, an increase of one unit in household size increased an individual’s probability of being in debt to a bank or cooperative approximately 1.125 times. According to this conclusion, it can be said that the household size does not have an important influence on the probability of indebtedness. Examining the literacy level of the individuals, a literate individual’s probability of being in debt to a bank or cooperative is approximately 2.03 times higher compared to an illiterate individual; and a woman’s probability of being in debt to a bank or cooperative is 2.27 ( $1/0.44 = 2.27$ ) times higher compared to a man.

**Calculation of marginal effect values on event probability for each step:** In sequential modelling, interpreting the marginal effect of any of the explanatory variable’s on event probabilities is a natural result of interpreting the results of binary models. Accordingly, effects of other explanatory variables being constant, the marginal effects of a change of one unit in  $x_k$  explanatory variable on event probability are obtained by taking the partial derivative of binary response model (Liao, 1984).

Marginal effects in Eq. (21) for each step are obtained by taking the first-order partial derivative of probability expressions with respect to  $x_k$  explanatory variable where  $P_1$ ,  $P_2$  and  $P_3$  indicate probabilities calculated based on explanatory variable values for each step respectively:

$$\begin{aligned} \frac{\partial P_1}{\partial x_{1k}} &= \frac{\exp\left(\sum_{k=1}^K \beta_{1k} x_{1k}\right)}{\left[1 + \exp\left(\sum_{k=1}^K \beta_{1k} x_{1k}\right)\right]^2} \beta_{1k} = P_1(1 - P_1)\beta_{1k} \\ \frac{\partial P_2}{\partial x_{2k}} &= \frac{\exp\left(\sum_{k=1}^K \beta_{2k} x_{2k}\right)}{\left[1 + \exp\left(\sum_{k=1}^K \beta_{2k} x_{2k}\right)\right]^2} \beta_{2k} = P_2(1 - P_2)\beta_{2k} \\ \frac{\partial P_3}{\partial x_{3k}} &= \frac{\exp\left(\sum_{k=1}^K \beta_{3k} x_{3k}\right)}{\left[1 + \exp\left(\sum_{k=1}^K \beta_{3k} x_{3k}\right)\right]^2} \beta_{3k} = P_3(1 - P_3)\beta_{3k} \end{aligned} \tag{21}$$

Parameter estimation values and mean of each explanatory variable necessary for calculating the Marginal Effects (M.E.) of statistically significant explanatory variables on event probabilities are given in Table 4.

Calculation of marginal effect values of “Education” on probability for each step is explained below as an example.

**First step:** After giving other variables their mean values, in order to find the partial effect of individual’s literacy on his probability of having no debt, we need to calculate  $P_1$  probability as probability of being illiterate. We add the effect of being literate status to the model by including estimated beta coefficient of being literate in marginal effect calculation. Accordingly, the related probability and marginal effect value are calculated using the formula given in Eq. (21):

Table 4: Statistics necessary for calculating marginal effects for each step

Explanatory variables	First Step: P <sub>1</sub>			Second Step: P <sub>2</sub>			Third Step: P <sub>3</sub>		
	$\hat{\beta}_1$	$\bar{x}_1$	M.E. <sub>1</sub>	$\hat{\beta}_2$	$\bar{x}_2$	M.E. <sub>2</sub>	$\hat{\beta}_3$	$\bar{x}_3$	M.E. <sub>3</sub>
Constant	-0.206	-	-	0.087	-	-	1.707*	-	-
Size of household	0.094*	3.823	0.023399	-0.163*	4.458	-0.02875	0.118*	3.863	0.029405
Marital status									
Married	0.546*	0.856	0.136481	-0.057	0.907	Non-significant	-0.023	0.939	Non-significant
Education									
Literate	-0.411*	0.049	-0.096088	-0.831*	0.05	-0.193792	0.711*	0.010	0.164411
Sex									
Male	-0.195*	0.873	-0.04379	0.372*	0.901	0.092452	-0.812*	0.949	-0.13354
Income	0.003	20.181	Non-significant	0.044	20.152	Non-significant	-0.04	20.210	Non-significant

$$P_1 = \frac{\exp[-0.206 + 0.094 * 3.823 + 0.546 * 0.856 - 0.411 * (0) - 0.195 * 0.873 + 0.003 * 20.181]}{1 + \exp(-0.206 + 0.094 * 3.823 + 0.546 * 0.856 - 0.411 * (0) - 0.195 * 0.873 + 0.003 * 20.181)} = 0.625052$$

$$\frac{\partial P_1}{\partial x_{1k}} = P_1(1 - P_1)(-0.41) = (0.625052)(1 - 0.625052)(-0.41) = -0.096088$$

**Second step:**

$$P_2 = \frac{\exp[0.087 - 0.163 * 4.458 - 0.057 * 0.907 - 0.831 * (0) + 0.372 * 0.901 + 0.044 * 20.152]}{1 + \exp(0.087 - 0.163 * 4.458 - 0.057 * 0.907 - 0.831 * (0) + 0.372 * 0.901 + 0.044 * 20.152)} = 0.629601$$

$$\frac{\partial P_2}{\partial x_{2k}} = P_2(1 - P_2)(-0.831) = (0.629601)(1 - 0.629601)(-0.831) = -0.193792$$

**Third step:**

$$P_3 = \frac{\exp[1.707 + 0.118 * 3.863 - 0.023 * 0.939 + 0.711 * (0) - 0.812 * 0.949 - 0.04 * 20.210]}{1 + \exp(1.707 + 0.118 * 3.863 - 0.023 * 0.939 + 0.711 * (0) - 0.812 * 0.949 - 0.04 * 20.210)} = 0.636973$$

$$\frac{\partial P_3}{\partial x_{3k}} = P_3(1 - P_3)(0.711) = (0.636973)(1 - 0.636973)(0.711) = 0.164411$$

**Interpretation of marginal effect values on probability for each step:** Detailed interpretation of marginal effect values of statistically significant variables for each step are given below. After giving other variables their mean values.

**Size of household:** An increase of one unit in household size increases individual’s probability of having no debt by approximately 0.024 unit; increases individual’s probability of being in debt to a bank or cooperative by approximately 0.029 unit; and decreases individual’s probability of being in debt to persons by approximately 0.029 unit.

**Married:** It is observed that being married increased an individual’s probability of having no debt by approximately 0.136 unit. Coefficients on other steps are not statistically significant, thus their interpretations are not included.

**Literate:** Being literate decreases individual’s probability of having no debt by approximately 0.096 unit; decreases individual’s probability of being in debt to persons by approximately 0.194 unit; and increases individual’s probability of being in debt to a bank or cooperative by approximately 0.194 unit.

**Male:** Being male decreases individual’s probability of having no debt by approximately 0.045 unit; increases individual’s probability of being in debt to persons by approximately 0.092 unit; and decreases individual’s probability of being in debt to a bank or cooperative by approximately 0.1335 unit.

One of the most important points to consider when calculating the marginal effects is that when the explanatory variable is qualitative, differentiation cannot be performed, thus marginal effects cannot be obtained using this method. An alternative approach is explained in detail in Appendix.

The marginal effects of statistically significant variables on event probabilities and their interpretations for each stage are given in above sub-section. The marginal effects on the total probability will be examined in the following section.

**Calculation of marginal effects on total probability:** First of all, let's remember the total probability definitions once again:

Table 5: Marginal effects on total probability

Explanatory variables	First Step		Second Step		Third Step		Marginal Effect On Final Probability
	P <sub>1</sub>	∂P <sub>1</sub>	P <sub>2</sub>	∂P <sub>2</sub>	P <sub>3</sub>	∂P <sub>3</sub>	
Constant	-	-	-	-	-	-	-
Size of Household	0.532839	0.023399	0.77129	-0.02875	0.528351	0.029405	0.013682
Marital Status							
Married	0.505882	0.136481	0.631965	-0.01326	0.643584	-0.00528	0.047894
Education							
Literate	0.625052	-0.096088	0.629601	-0.193792	0.636973	0.164411	-0.04601
Sex							
Male	0.659517	-0.04379	0.538371	0.092452	0.798631	-0.13354	-0.01709
Income	-	-	-	-	-	-	-

P<sub>1f</sub>: Probability of individual’s having no debt.  
 P<sub>2f</sub>: Probability of individual’s being in debt to persons.  
 P<sub>3f</sub>: Probability of individual’s being in debt to a bank or cooperative.  
 P<sub>4f</sub>: Probability of individual’s being in debt to retail stores.

Mathematical expressions of these probabilities are given in Eq. (20). Accordingly, since the stages are independent in sequential modelling, total probability is the product of probabilities on each stage. Calculation of marginal effects on total probability was explained by Liao (1984) for two stages. Since the data set used in our study consists of three sequential stages, we extended the formulations as follows:

$$\begin{aligned}
 & (P_1 + \partial P_1)(P_2 + \partial P_2)(P_3 + \partial P_3) \\
 &= \underbrace{P_1 P_2 P_3}_{P_1} + \underbrace{P_1 P_2 \partial P_3 + P_1 P_3 \partial P_2 + P_2 P_3 \partial P_1}_{\partial P_1} + \underbrace{P_2 \partial P_1 \partial P_3 + P_3 \partial P_1 \partial P_2 + \partial P_1 \partial P_2 \partial P_3}_{\partial P_1} \\
 &= (P_f + \partial P_f)
 \end{aligned}$$

In order to make the calculations, first we need to find the probabilities related to Eq. (20) for each statistically significant variable and partial derivatives of ∂P<sub>1</sub>, ∂P<sub>2</sub> and ∂P<sub>3</sub>.

For example, the marginal effect of being literate on the total probability must be calculated as follows:

$$\begin{aligned}
 P_1 = 0.625052 \quad \text{and} \quad \partial P_1 &= \frac{\partial P_1}{\partial X_{1k}} = P_1(1 - P_1)(-0.41) = (0.625052)(1 - 0.625052)(-0.41) = -0.096088 \\
 P_2 = 0.629601 \quad \text{and} \quad \partial P_2 &= \frac{\partial P_2}{\partial X_{2k}} = P_2(1 - P_2)(-0.831) = (0.629601)(1 - 0.629601)(-0.831) = -0.193792 \\
 P_3 = 0.636973 \quad \text{and} \quad \partial P_3 &= \frac{\partial P_3}{\partial X_{3k}} = P_3(1 - P_3)(0.711) = (0.636973)(1 - 0.636973)(0.711) = 0.164411
 \end{aligned}$$

The marginal effect of being literate on the total probability is found to be:

$$\begin{aligned}
 \partial P_f &= P_1 P_2 \partial P_3 + P_1 P_3 \partial P_2 + P_2 P_3 \partial P_1 + P_2 \partial P_1 \partial P_3 + P_3 \partial P_1 \partial P_2 + \partial P_1 \partial P_2 \partial P_3 \\
 \partial P_{f(Literate)} &= \left\{ \begin{aligned} & (0.625052)(0.629601)(0.164411) + (0.625052)(0.636973)(-0.193792) \\ & + (0.629601)(0.636973)(-0.096088) + (0.629601)(-0.096088)(0.164411) \\ & + (0.636973)(-0.096088)(-0.193792) + (-0.096088)(-0.193792)(0.164411) \end{aligned} \right\} \\
 \partial P_{f(Literate)} &= -0.04601
 \end{aligned}$$

Marginal effects of other variables on total probability are given in Table 5.

**Interpretation of marginal effects on total probability:** All other variables being constant, the interpretation of the marginal effect of concerned explanatory variables on probability is important. Accordingly.

**Size of household:** The marginal effect of an increase of one unit in household size on the final probability is approximately 0.013682 unit. This value indicates that probability of indebtedness will usually increase by 0.013682 unit in case of an increase of one unit in household size.

**Married:** The marginal effect of being married on the final probability is approximately 0.047894 unit. This value indicates that probability of indebtedness will usually increase by 0.013682 unit if the individual is married.

**Literate:** The marginal effect of being literate on the final probability is found to be -0.04601. This result indicates that there is a decrease in the probability of indebtedness of literate individuals by 0.04601 unit, compared to illiterate individuals. In other words, it can be said that literate individuals have a lower tendency to be indebted.

**Male:** The marginal effect of being male on the final probability is found to be -0.01709 unit. This indicates that there is a decrease in the probability of indebtedness of men by 0.04601 unit, compared to women.

### RESULTS AND DISCUSSION

The main purpose of this study was to provide information about the sequential logit model structure, which is thought to be the best alternative in case that the dependent variable is a nested and sequential categorical variable and explain its interpretations in detail. For this purpose, a real data set reflecting individuals' indebtedness statuses was used. In this way, both the structure of the sequential logit model was introduced and by using a real data set, variables that could affect the individuals' indebtedness statuses were identified. Individual's indebtedness status was taken as the dependent variable and the data set was modelled using the sequential logit model taking into account of the sequential branch of the variable. Household size, marital status, education, sex and income were taken as explanatory variables. Odds ratios related to model were interpreted by calculating the marginal effect values for each stage and the final probability.

The following findings were obtained by interpreting the marginal effects on probability for each stage in the sequential logit model.

In the first stage, it was observed that larger households and married individuals had a higher possibility of having no debt compared to single individuals; literate individuals had a lower possibility of having no debt compared to illiterate individuals and men had a lower possibility of having no debt compare to women.

Examining individuals in debt to persons, it was seen that an increase in household size resulted in a decrease in the probability of being in debt to persons. Additionally, literate individuals had a lower probability of being in debt to persons compared to illiterate individuals and men had a higher probability of being in debt to persons compared to women.

Examining individuals in debt to a bank or cooperative, it was seen that an increase in household size resulted in an increase in the probability of being in debt to a bank or cooperative. Also, men had a lower possibility of being in debt to banks or cooperatives compared to women and literate individuals had a lower possibility of being in debt to banks or cooperatives compared to illiterate individuals.

The results obtained here are similar to research findings in Avery *et al.* (2004) and Sharma and Zeller (1997).

The marginal effect results on the final probability in sequential logit model allowed for the following interpretations.

An increase of one unit in household size usually resulted in an increase of 0.013682 unit in the probability of being in debt, being married led to an increase of 0.047894 unit in the probability of being in debt, being male led to a decrease of 0.01709 unit in the probability of being in debt and being literate led to a decrease of 0.04601 unit in the probability of being in debt.

**Appendix:** As noted earlier, in case that the variable, whose partial effect is researched, is not a continuous variable, the partial derivatives cannot be taken and the marginal values cannot be obtained by using the formula given in Eq. (21). In this case, the real marginal effects are obtained by giving other explanatory variables their mean values and finding the differences between calculated probabilities. Since "Education" is a two-level categorical variable, all explanatory variables except for "Education" will be given the  $\bar{x}$  values shown in Table 4 and probabilities coded as "1" for "literate" level and "0" for "illiterate" level will be calculated. The difference between these probabilities gives the real marginal effect.

Accordingly; other variables are given their mean values that are shown in the second column of Table 4 in order to find the marginal effect of being literate on individual's indebtedness status on the first stage.

$$P_i(Y = \text{Literate}) = P_i(Y = 1) = \frac{\exp \left[ \begin{matrix} -0.206 + 0.094 * (3.823) + 0.546 * (0.856) \\ -0.411 * (1) - 0.195 * (0.873) + 0.003 * (20.181) \end{matrix} \right]}{1 + \exp \left[ \begin{matrix} -0.206 + 0.094 * (3.823) + 0.546 * (0.856) \\ -0.411 * (1) - 0.195 * (0.873) + 0.003 * (20.181) \end{matrix} \right]} = 0.524991$$

$$P_i(Y = \text{Illiterate}) = P_i(Y = 0) = \frac{\exp \left[ \begin{matrix} -0.206 + 0.094 * (3.823) + 0.546 * (0.856) \\ -0.411 * (0) - 0.195 * (0.873) + 0.003 * (20.181) \end{matrix} \right]}{1 + \exp \left[ \begin{matrix} -0.206 + 0.094 * (3.823) + 0.546 * (0.856) \\ -0.411 * (0) - 0.195 * (0.873) + 0.003 * (20.181) \end{matrix} \right]} = 0.625052$$

According to these results, the marginal effect of being literate on individual's probability of having no debt is obtained by finding the difference between two probabilities. That is:

$$ME = P(\text{Literate}) - P(\text{Illiterate}) = (0.524991) - (0.625052) = -0.10006$$

Real marginal effects can be found when similar calculations are done for other stages.

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