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Research Article

Knapsack Model for the Project Allocation Problem (PAP): A Case of Suhum Kraboa Coaltar District

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Abstract: Our study aimed at presenting a model for addressing the long standing problem of allocating resources to political projects in Suhum Kraboa Coaltar District Assembly. The Assembly has noticed recently in its budgetary allocation that large sums of money are allocated to projects whose benefits are stretched out over much larger time horizons than other urgent projects with short term benefits. A combined approach of benefit-cost analysis using analytical hierarchies and knapsack optimization model were used to formulate and modeled our problem in which a number of projects were to be executed by judicious allocation of available resources. The Volume Constraint method for solving knapsack problem was used to solve our problem. Our model resulted in reducing the Assembly's allocated budgeted cost for the execution of the projects by GH¢24,000 and ensured optimal execution of the projects within the budgeted constraints. Our model contributes to the ability to make strategic allocation decision where the problem of more projects being unable to complete is addressed and where optimal resource allocation is demanded by the importance of the project. Our study is also relevant and can be applied in other districts in the country faced by similar problem in resource allocation.

Keywords: Execution, knapsack model, optimal, optimization, project allocation, selection

INTRODUCTION

For years, benefit-cost analysis using Analytical Hierarchies (AH) has been the method of choice in Suhum Kraboa Coaltar District for the allocation of resources to projects as a procedure for maximizing total benefits subject to budget constraints. This method typically requires a common currency for the measurement of benefits and costs. Some heroic effort as a means to overcome the lack of common currency have employed shadow pricing and opportunity cost to broaden the scope of the analysis. However, the measurement of benefits and costs using Analytic Hierarchy always achieve unparalleled scope of analysis. But once estimates of priorities for projects are obtained, it becomes necessary to allocate resources to them by the discretion of the decision maker either efficiently or inefficiently. Instead of making the analyst choose either to measure benefits and costs inclusively using Analytical Hierarchies but then allocate inefficiently, or to allocate resources optimally to poorly measured projects, we presented a study that combined these methodologies with knapsack model for achieving optimal allocation. Our model allocates limited resources to set of projects in realistic situations so as to:

- Maximize total benefit given resource constraints in the form of monetary budget and time.
- Decide whether or not to execute a particular project, by comparing it cost with it benefit and time of completion.
- Consider the set of time frame by which projects would be completed for possible elimination of projects that cannot be completed within the available time frame thereby wasting the scarce resources.
- Execute projects that can be completed within the available time frame by the given constraints.

LITERATURE REVIEW

Klimek and Lebkowski (2011) presented a formal description of resource allocation problem for resource-constrained project scheduling. According to the authors, a schedule could be executed by various resource flow networks which may differ in terms of resistance to disruptions occurring during project execution. The authors defined such criteria of evaluating resource flow network to be more useful than robustness metrics used so far in literature. The authors discussed the importance of robust scheduling

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for execution projects and proposed metrics for resource allocation robustness which took into consideration the stability of the final schedule. Those metrics made it possible to carry out a more precise analysis concerning the properties of the resource flow network in terms of its robustness to disruptions in comparison with a well known flexibility indicator, which they called *flex*.

Resource allocation is the process of assigning resources to tasks throughout the life of a project. Despite the sophisticated software packages devoted to keeping track of tasks, resources and resource assignments, it is often the case that project managers find some resources to be over-allocated and therefore unable to complete the assigned work in the allotted amount of time. Most scheduling Software have provisions for leveling resources, but the techniques for doing so simply add time to the schedule and may cause delays in tasks that are critical to the projects in meeting deadlines. Chilton (2014) presented a model and a software application that ensured that resources are properly balanced at the beginning of the project and eliminated the situation in which resources became over-allocated. According to the author, the model could be used in a multi-project environment and reused throughout the project as tasks, resource assignments and availability and the project scope changes. The application utilized the bounded enumeration technique to formulate an optimal schedule for which both the task sequence and resource availability were taken into account. The model was run on a database server to reduce the running time and made it a viable application for practitioners.

Optimizing construction project scheduling has received a considerable amount of attention over the past 20 years. As a result, a plethora of methods and algorithms have been developed to address specific scenarios or problems. Zhou *et al*. (2013) reviewed the methods and algorithms that have been developed to examine the area of Construction Schedule Optimization (CSO). According to the authors, the developed algorithms for solving the CSO problem can be classified into three methods: mathematical, heuristic and meta-heuristics. The authors also discussed the application of these methods to various scheduling problems and their implications for future research were identified.

Leyman and Vanhoucke (2016) proposed a study on a general scheduler for six NPV optimization models. Three different payment models were discussed in both single- and multi-mode cases. The added values of each part of the proposed methodology were illustrated. The impacts of project parameters on project NPV were analyzed in an extensive computational experiment. The author's model focused on the single- and multi-mode Resource-Constrained Project Scheduling problem with Discounted Cash flows (RCPSPDC and MRCPSPDC) and three payment models. According to the authors the contributions of their paper were twofold. First, they extended a new scheduling technique, which moves activities in order to improve the project net present value. This more general version was applicable to multiple problem formulations and provided an overarching framework in which these models can be implemented. The changes in activity finish times took into account the possible changes in other activities and their finish times, by forming a set of activities which was subsequently moved in time. The scheduling technique was implemented within a genetic algorithm meta-heuristic and employed two penalty functions, one for deadline feasibility and one for non-renewable resource feasibility. Secondly, the authors tested the proposed approach on several datasets from literature and illustrated the added value of each part of the algorithm. The influences of data parameters on the project net present value were highlighted. According to the authors, the detailed results provided in their study could be used as future benchmarks for each of the six models discussed.

Yan *et al*. (2016) presented a study on traditional resource leveling problem that aimed at minimizing the resource usage fluctuations and obtaining sustainable resource supplement, which was accomplished by adjusting non critical activities within their start and finish time. However, there exist limitations in terms of the traditional resource leveling problem based on the fixed project duration. The authors' model assumed that the duration could be changed in a certain range and then analyzed the relationship between the scarce resource usage fluctuations and project cost. The authors proposed an optimization model for the multi-resource leveling problem. Their model took into consideration five kinds of cost: the extra hire cost when the resource demand is greater than the resource available amount, the idle cost of resource when the resource available amount is greater than the resource demand, the indirect cost related to the duration, the liquidated damages when the project duration is extended and the incentive fee when the project duration is reduced. The optimal objective of this model was to minimize the sum of the aforementioned five kinds of cost. Finally, a case study was examined to highlight the characteristic of the proposed model.

Benefit-cost analysis has traditionally required monetary measures of benefits and costs of candidate projects. Bennett (1993) demonstrated that priorities of both tangible and intangible attributes could be used in a benefit-cost setting to address the long standing problem of allocating multiple resources to projects. Although it is widely accepted that proper structuring of analytic hierarchies can achieve aggregation of a number of small projects so that their cumulative benefits are comparable to that of a large project, it frequently happens in budgetary allocation that large sums of money are allocated to a project whose intangible benefits are stretched out over much larger time horizons than other urgent projects with short term benefits. Here, the two hierarchies used to set priorities on benefits and on costs need to consider different time horizons. The Analytic Hierarchy Process and knapsack optimization are used to formulate problems in which a number of projects might be implemented by judicious distribution of resources available. For many applications, one seeks to maximize a function of net benefit, subject only to the constraints imposed by resource limitations. In others, benefit to cost ratios of projects obtained from two separate hierarchies are used. The authors' knapsack or recursive knapsack formulation was a complex combinatorial problem that required approximate heuristic methods for its solution. Several realistic variants of the knapsack problem were illustrated. The computational tractability of practical knapsack problems was discussed. The authors' paper contributed to the ability to make strategic allocation decision where the diversity of benefits and costs demand inclusive measurement and where optimal resource allocation is demanded by the importance of the decisions.

On the basis of the combination of the well-known knapsack problem and a widely used risk management technique in organizations (that is, the risk matrix), an approach was developed by Reniers and Sörensen (2013), to carry out a cost-benefit analysis which efficiently took prevention investment decisions. Using the knapsack problem as a model and combining it with a well-known technique to solve this problem, bundles of prevention measures were prioritized based on their costs and benefits within a predefined prevention budget. Those bundles showing the highest efficiencies and within a given budget, were identified from a wide variety of possible alternatives. Hence, the approach allowed for an optimal allocation of safety resources, did not require any highly specialized information and could easily be applied by any organization using the risk matrix as a risk ranking tool.

Cloud computing is a new computing paradigm which has the provision of computational resources like services accessed over the internet. In this paradigm, computing resources are pooled and allocated according to customer demand. The growing demand for this new type of service has led to increased use of energy on the part of service providers, due to the need to maintain the computing infrastructure, thereby becoming one of the leading providers of cost factors. In this context, solutions have been tried whatever possible to meet the customers' requirements for resources consuming minimum power required. Amarante *et al*. (2013) presented a study using the modeling of the multiple knapsack problem, with a mechanism for allocating resources called Lago Allocator, which addressed the issue of energy saving. Furthermore, a comparative analysis of the proposed solution with the original mechanism to evaluate the performance modification was made.

Duenas *et al*. (2014) presented a model for production resource allocation. The model was applied to a real-life problem within the construction equipment manufacturing industry. A multidimensional knapsack problem formulated was the proposed model based on an evolutionary algorithm using a three dimensional binary coded chromosome. Various tests were carried out to show the appropriateness of the solution. The experimental results obtained were suggested to be satisfactory from the manufacturing companies' perspective.

UNIDO (1972) developed a guideline for evaluating projects with social benefit analysis. Here there is an attempt to quantify both direct market values (benefits) and indirect (non-market) impacts of projects. Social benefit-cost analysis also seeks to integrate aggregate consumption effects with distribution effects, thus escaping the narrow conventional focus on total supply and demand. The estimation of the net aggregate consumption benefit was best carried out in successive stages of approximation. The first approximation was to estimate the net present consumption benefit of a project at market prices. The second approximation involved the adjustment of the market prices of specific resources wherever these prices did not reflect the real contribution of the resources to the aggregate consumption objective. The general principle was to value both outputs and inputs according to the criterion of consumers' willingness to pay.

Kanniappan and Thangavel (1993) studied the problem of selecting various schemes under the integrated rural development program to maximize the number of beneficiaries so as to optimize the annual income generated from each scheme. Typical constraints prescribed by the government in the allocation of the funds to several schemes from the budget were outlaid yearly. Through integer programming model and data from the district rural development agency of Dindigul Anna District, the authors were able to maximize the annual income generated from the schemes.

Pesek *et al*. (2007) proposed a model for local search algorithm that made used of a complex neighborhood relation based on hybridization with a consecutive heuristics for classical Resource Constrained Project Scheduling Problem (RCPSP). The authors performed an experimental analysis to tune the parameters of their algorithm and compared it with a tabu search approach based on a combination of neighborhood relations borrowed from the literature. Finally, the authors showed that their algorithm was competitive with the ones reported in literature.

METHODOLOGY

Mathematical formulation of our proposed problem: In the literature, the project allocation problem is formulated using the benefit-cost analysis. Typically, benefit-cost calculations are made solely in terms of discounted benefits and costs by, the familiar Marglin formula:

$$\sum_{t=1}^{T} \frac{b_t - c_t}{(1+s)^t} - aK \geq 0, \quad (1)$$

where,

- b_t = Gross benefits accruing from project X in the t^{th} period
- c_t = Gross current costs of producing the benefits in the t^{th} period
- s = The Social Time Preference rate (STP)
- K = Capital cost of the project (assumed here to be incurred at once)
- T = Amortization period
- a = 'shadow price' of capital

Usually if (1) is not satisfied, then the project is not worth doing. However, this presumption is not always valid. By placing all costs into a single analytic hierarchy, formula (1) may be rewritten to represent benefits and costs as a ratio:

$$\frac{b^*}{c^*} = \frac{\sum_{t=1}^{T}\left(\frac{b_t}{(1+s)^t}\right)}{\sum_{t=1}^{T}\left(\frac{c_t}{(1+s)^t}\right) + a\,K} \quad (2)$$

where, b^* and c* are taken over all benefits and costs, respectively.

There are however some measurement problems affecting the tradeoff among resources and projects, that is, the optimal allocation of resources to projects. In the first place, authorities sometimes price inputs directly through imposed rates of exchange or indirectly through 'shadow prices,' which may not be inferable in practice.

Secondly, there may be inadequate frequency of exchange to set prices with sufficient stability to utilize them in project evaluation. Thirdly, short time horizons which preclude full economic adjustment can distort allocation. Within a short period, alternative uses for inputs, which should be reflected by "opportunity costs," may be nearly non-existent; the alternative uses might be hypothetical or lie in the future. These effects may impose severe over valuation of human capital when combined with other impediments to labor mobility.

Finally, the time of completion of the project must be given a priority consideration together with the budget constraint. This is as a result of the way prices of inputs increases with time that may affect the budgeted amount and project completion.

Hence, it is argued that the complete decision maker must anticipate all of these difficulties and prepare alternative schemes to markets to find the values of inputs, so that appropriate benefit-cost analysis can be conducted in a large variety of situations. In view of the above, the AHP is said to represent a flexible method to optimally allocate resources to projects and hence cannot always ensure optimal allocation of the constraint resources to the projects.

Ideally, the project allocation problem is described by a set of projects P_j for j = 1, .., n (n ∈ N), a set of project benefits V_j for j = 1, .., n (n∈N), a set of time for each of the project to be completed t_j (j = 1, .., n, n ∈ N and t ∈T), a set of cost associated with the project execution c_j for j = 1, .., n (n ∈ N), the budget constraint for the projects b and the time constraint of the projects execution T which in our case defines the term of office of the district chief executives in the assembly since there is no continuity policy for the district.

The aim is to determine a set of optimal allocation of the constraint resources to the projects such that the:

- Selected projects can be executed within the constraint time frame;
- Selected project can also be executed without overstretching the total budget; and
- Total objective would be maximized;

with the mathematical formulation:

$$\text{Maximize } \sum_{j=1}^{n} P_j\, x_j \quad (3)$$

$$\text{Subject to } \sum_{j=1}^{n} w_{ij}\, x_j \leq C,\ i = 1, ..., m \quad (4)$$

$$\sum_{j=1}^{n} t_{ij}\, x_j \leq T,\ i = 1, ..., m \quad (5)$$

$$x_j \geq 0 \text{ and integer, } j = 1, .., n. \quad (6)$$

$$x_j = \begin{cases} 1 & if\ project\ j\ is\ executed; \\ 0 & otherwise \end{cases} \quad (7)$$

Equation (3) ensures that the objective function aims at maximizing the total objective, Eq. (4) and (5) guarantees that the resource availability for the projects to be executed are not overstretched, Eq. (6) ensure that there is non-negativity in the project execution and Eq. (7) state that a project is either executed or not.

ALGORITHM

Given our problem as a knapsack model, with projects type represented by n mutually exclusive subsets of N, thus n ∈ N. The input to this algorithm is the feasible region of a binary integer program {x ∈ {0, 1}^{n}: Ax ≤ b}, mutually exclusive n sets that represents the number of project types of the binary integer program, the positive integers of the cost (w_i), the profits (p_i), the time frame (t_i) and the knapsack limit (b) as well as the time limit (T). The basic idea of the algorithm is to find non-cover optimum solutions for z^*, W_c, P_c, the project from the various n sets project types that will be executed to obtain the optimum solution to the optimization 0-1 knapsack problem x_j.

1. [INITIALIZATION]
S: = b;
L: $=$ 0, Max_{Val}: $=$ 0, T_W: $=$ 0;
k: = n;
x (k): $=$ {0,....,n}
2. [MAIN STEP]
Max_{n_p} = 1
For each subset item $i_s \in n_p = 0$ to Max_{n_p}
Solve the following
L $= i_s \in n_p: \sum_{i_s=1}^{n_p} i_s T_{i_s}$
$W_{cal} = i_s \in n_p: \sum_{i_s=1}^{n_p} i_s W_{i_s}$
$t_i \in$ T: $t_i \leq$ T
3. [CHECK FOR FEASIBILITY]
While $W_{cal} <$ S, $t_i \leq$ T and $Max_{Val} <$ L
Max_{Val} = L; $T_W = W_{cal}$
For j = 1, k
x (j) = {i_s }
End For
End While
End For
4. [TERMINATION]
Output: x (j), Max_{Val}, and T_W as the solutions.

DATA ANALYSIS AND RESULTS

The study seeks to find the optimum project selection in order to maximize the benefit of the project and as well ensure that all the selected projects are executed and completed within the time frame in SKCD without over stretching the district's budget. The data for the set of fifteen projects below needed to be executed and has gone through tender bidding was obtained: Construction of Assembly Hall, Refurbishment of Offices, Boreholes, Bungalows, Area Council Offices, Community Latrines, Solar Power Projects, Drains, Refuse Collection Points, Market Sheds, 1 CHPS Compound, Social and Resource Centre, 6 Unit Classroom Blocks, NHIA District Offices and Landfill Site. The assembly has a budgeted amount of ₵780, 0000 with fifteen executable projects to be under taken. The projects are to be executed within four year term (ninety six months) of the DCE. The projects are respectively itemized as x_1,.., x_{15} in Table 1.

The net present value and the benefit-cost analytical hierarchical method were applied to determine the priority and value or benefit of the projects. We then employed our proposed knapsack model for the optimal project selection.

RESULTS AND DISCUSSION

To carry out the computation of the proposed model, we applied our proposed algorithm coded in Fortran 90. The feature of the software permits the input data to be fixed into the code. The Software displays the final optimal solution for our proposed problem as shown in Table 2.

The various feasible combinations of projects to be selected to achieve optimal benefit at minimum cost can be seen from Table 2. This gave an optimal benefit of 5,590,000 by executing the selected projects shown in the Table 2 at a cost of GH¢6,410,000 and a total time of 94 months consisting of Construction of: Boreholes, Solar Power Projects, Drains, Refuse Collection Points, Market Sheds, 1 CHPS Compound, 6 Unit Classroom Blocks, NHIA District Offices and Landfill Site out of the fifteen projects (Table 3).

Table 1: Cost, benefits and time of completion of the various projects

Projects	COST (GH¢'000)	BENEFITS ('000)	Project time (Months)
x_1	70	30	18
x_2	50	43	16
x_3	39	29	24
x_4	87	27	24
x_5	77	33	20
x_6	35	41	18
x_7	40	56	3
x_8	35	42	6
x_9	30	70	4
x_{10}	60	46	4
x_{11}	133	96	18
x_{12}	45	40	28
x_{13}	125	78	24
x_{14}	128	82	24
x_{15}	51	60	5

Table 2: Optimal solutions for our proposed problem

	Status of columns of Projects executed and selected per our proposed algorithm														
Project	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}
Selected	0	0	1	0	0	0	1	1	1	1	1	0	1	1	1

Table 3: Optimal solutions per the Assembly's existing procedure

	Status of columns of Projects executed and selected per the assembly's existing method														
Project	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}
Selected	1	1	1	1	1	1	0	0	0	1	1	0	1	1	0

For the same data used, per the Assembly records, the various feasible combinations of projects to be selected to achieve optimal benefit at minimum cost using their method can be seen from Table 3. The best optimal benefit achievable was 5,050,000 at the cost of GH¢8,040,000 and a total time of 172 months; Consisting of Construction of, Assembly Hall, Refurbishment of Offices, Boreholes, Bungalows, Area Council Offices, Community Latrines, Market Sheds, 1 CHPS Compound, 6 Unit Classroom Blocks and NHIA District Offices. This would have led the Assembly to exceed its budget by GH¢240,000 with the implication being that at least a project was going to be uncompleted, a measure of waste of scarce resource and suboptimal allocation of resources.

CONCLUSION

The adoption of our proposed model and algorithm gave an optimal allocation of resource to the projects in the district. It enabled a cut down of the number of uncompleted projects in the district. The selected projects were also able to be executed and completed within the available budget constraint and time frame. Because of the optimal operational time and resource, the projects were able to be executed at a minimal cost without exceeding the budgeted allocation. The new decision procedures for allocation of projects would go a long way in solving the problem of indiscriminate allocation of projects by decision makers, since it would select only the feasible projects that can be executed and completed within the allocated budgetary and time constraints at optimal cost and reduce the high rate of uncompleted projects in the district. The algorithm would provide optimal benefit, reduce cost of project execution and ensure that all selected projects are completed. The algorithm can be successfully applied to project management problem across the districts in other part of the country.

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