Robot Path Planning Based on Simulated Annealing and Artificial Neural Networks

Xianmin Wei
School of Computer Engineering, Weifang University, Weifang 261061, China

Abstract: As for the limitations of algorithms in global path planning of mobile robot at present, this study applies the improved simulated annealing algorithm artificial neural networks to path planning of mobile robot in order to better the weaknesses of great scale of iteration computation and slow convergence, since the best-reserved simulated annealing algorithm was introduced and it was effectively combined with other algorithms, this improved algorithm has accelerated the convergence and shortened the computing time in the path planning and the global optimal solution can be quickly obtained. Because the simulated annealing algorithm was updated and the obstacle collision penalty function represented by neural networks and the path length are treated as the energy function, not only does the planning of path meet the standards of shortest path, but also avoids collisions with obstacles. Experimental results of simulation show this improved algorithm can effectively improve the calculation speed of path planning and ensure the quality of path planning.

Keywords: Energy function, markov chain, network weight, robot path planning, simulated annealing artificial neural network

INTRODUCTION

Mobile robot path planning technology is an important research branch. According to robot’s knowing extent of environmental information, mobile robot path planning can be divided into two types, global path planning with fully known environmental information and local path planning with completely unknown or partially unknown environmental information. Currently, many scientists have made much research of path planning and proposed some methods, such as visual graph method, free-space method and grid method etc., but these algorithms have some limitations.

Visual graph method, viewing robot as one point, combinations and connects the robot, the target point and polygon obstacles points and requires connections do not cross the barrier between robot and obstacles, between target point and the obstacles points and between points and points of obstacles (Ning and Chen, 2012). Optimization algorithms can delete unnecessary connections points to simplify visual graph and shorten search time. Visual graph method can obtain the shortest path, but neglect the size of the assuming robot, making the robot too close even contacting to the obstacle through vertexes and a long search time. Free-space method is applied to robot path planning by using pre-defined shape to structure free space and searching a connected graph which free space is expressed as. Free-space method is more flexible, the starting point targets and changes of destination points will not cause reconstruction of a connected graph, but the complexity of this algorithm is proportional with the number of obstacles and it cannot obtain the shortest path under any circumstances (Yang et al., 2012). Grid method decomposes robot work environment into a series of grid cells with binary information, the work environment is expressed with quadtree or octree and by optimizing algorithm it completes path search. The method records environmental information in grid unit and the environment is quantified into a grid with certain resolution, grid size directly affects the storage size of environmental information and the length of planning time (Tian and Gao, 2009).

Traditional simulated annealing algorithm is a heuristic random search method, which includes the Metropolis sampling algorithm (M algorithm) and Annealing Process (AP) algorithm. Since traditional algorithm simulated annealing process to get the global optimal solution, but it requires a lot of iteration, convergence process is slow (Dorigo and Di, 1999).

Simulated annealing algorithm is derived from the simulation of the thermodynamic annealing process, given at an initial temperature, through the slow decline in temperature parameters, the algorithm given an approximate optimal solution during polynomial time. Annealing is similar as the metallurgical annealing, but it is very different from the metallurgical quenching, the formers is the slow decline in temperature, the latter is the temperature fell rapidly.

Principle of simulated annealing is the same as the principle of metal annealing approximately, we will apply the theory of thermodynamics to statistical, every point within the search space is thought of as the air molecules and molecular energy is the kinetic energy of its own. While every point of search space likes air molecules with the same energy to express the
appropriate level of the proposition. First algorithm starts from an arbitrary point in search space, for every step to select a neighbor and then calculated probability from the existing location to neighbors.

Simulated annealing algorithm can be decomposed into three parts of the solution space, the objective function and the initial solution (Hidenori et al., 2000).

**IMPROVED SIMULATED ANNEALING ARTIFICIAL NEURAL NETWORK THEORY**

Firstly, improved simulated annealing artificial neural network algorithm introduces best retaining simulated annealing algorithm, and combines Powell algorithm to form improved simulated annealing combinatorial optimization algorithm, this not only increases a good solution both protection, but also overcomes the slow convergence of simulated annealing algorithm itself; then an obstacle collision penalty function represented by the neural network and the path length are treated as energy function of the simulated annealing combinatorial optimization algorithm, which makes solutions (mapped out the path) not only satisfy the shortest path, but also avoid obstacles collision.

**Simulated annealing with best retention (ISA):** In the usual process of simulated annealing algorithm, algorithm terminates at a predetermined stopping criteria, the stopping criteria has a variety of options, such as in a number of successive Markov chains there is no change in the solution; the value of control parameter T is smaller than a sufficiently small positive number; error of current solution is less than required errors. However, simulated annealing search process is random and when T is larger, some bad solution can be accepted, then as the T value decreases, probability of accepted bad solutions decreases until zero. On the other hand, some of current solutions must pass through the temporarily deteriorated “ridge” to achieve the optimal solution. Algorithm stop criterion cannot guarantee that the final solution must be derived from the best, even the final solution cannot guarantee that the entire search process has been reached the best one, that is general simulated annealing algorithm has not protective measures for good solutions. To add a store device for algorithm (Nasser and Msc Huthaifa, 2010), to save the best results which search process had encountered, when the annealing process is complete, to compare the final solution with solution from store device and the better is the last result, thus algorithm improved quality of the obtained solutions. Improvements are the followings.

**Construction of update sequence:** At first, ISA constructs the guideline value of optimal solution update sequence SS (k) which is monotonously reduced by the search sequence.

Search sequence \{S (0), S (1), ..., S (k), ...\} constructs a update sequence:

\[
\begin{align*}
SS(0) &= S(0) \\
SS(k) &= \begin{cases} 
S(k), & f[S(k)] < f[SS(k-1)] \\
SS(k-1), & \text{else}
\end{cases}
\end{align*}
\]

Since the constructed update sequence cannot change the original control process and control track sequences, the highlight advantages of original simulated annealing algorithm bypass the local optimal solution is reserved and finally the optimal solution must be the one which experienced in all states of search process. The optimal improved algorithm is better than the original optimal algorithm.

**AP algorithm and the M algorithm termination conditions:** M algorithm termination conditions are described as follows: If starting from a certain k, \( SS(k) = SS(k+1) = ... = SS(k+q) \), \( q_0 \) is threshold value, if \( q \geq q_0 \) then M algorithm ends.

AP algorithm termination conditions are described as follows: At one \( T_i \), after the M algorithm is called, the solution of \( SS(i) = SS(T_i) \), if \( SS(i) = SS(i+1) = ... = SS(i+p) \), \( p_0 \) is the threshold, if \( p \geq p_0 \) then AP algorithm ends (Zahid and Deo, 2009).

**Implementation of new solution generator:** Assuming \( S_o, S_n \) as the old solution and new solution respectively, then \( S_n \) is generated by the following formula:

\[
\begin{align*}
S_n &= S_o + \sigma e, \sigma > 0 \\
S_n &= S_{\max}, S_n \geq S_{\max} \\
S_n &= S_{\min}, S_n \leq S_{\min}
\end{align*}
\]

where,

\( \sigma \): The step value which concerns with initial value and value range

\( e \): Random disturbance, which is generated by the following methods:

- With a random \((0, 1)\)
- \( e \sim N(0,1) \) with normal distribution

For normal distribution of the variable generator, \( \eta_1 \) and \( \eta_2 \) are uniformly distributed random variables in \([0, 1]\), then generated functions are followings:

\[
\begin{align*}
\xi_1 &= [\frac{-2 \ln \eta_1}{\eta_1^2} \cdot \cos(2\pi \eta_2)] N(0,1) \\
\xi_2 &= [\frac{-2 \ln \eta_2}{\eta_2^2} \cdot \cos(2\pi \eta_1)] N(0,1)
\end{align*}
\]

If \( e = \xi_1 \), under the normal \(N(0,1)\):

\[
\sigma = (S_{\max} - S_{\min})/(2 \times 1.96)
\]

- \( e \sim C(0,1) \), Cauchy distribution
Choice of cooling schedule parameters: Cooling schedule is a set of control parameters which ensures convergence of simulated annealing algorithm, to approximate the asymptotic convergence of simulated annealing; the algorithm returns a near optimal solution after limited execution (Fang and He, 2004). A cooling schedule should provide the following parameters:

- Control initial $T_0$ of parameter $T$, based on the principle of compromise, through experiments to optimize the selection of $T_0$ value.
- Selection of attenuation function:
  \[ T_i = \frac{T_0}{(1 + \ln(i))} \]  
  where,  
  $i$ : Number of iteration  
  $T$ : Thermodynamic temperature
- Attenuation on control parameters of decay function decreases with algorithm process and it can reduce the decreasing rate of control parameter values and which benefits stability of experiments performance of simulated annealing algorithm (Sun et al., 2011).
- Control parameter selection of final value of $T_f$.
- Stopping criteria which was proposed by Kirkpartick, that is when in the Markov chain solution has not any changes (including optimization or worse), the algorithm terminates.
- Selection of Markov chain length of $L_k$. $\tilde{L}$ is choosed to limit the value of $L_k$, usually:
  \[ \tilde{L} = \alpha.n \]  
  where,  
  $n$ : The scale of problem  
  $\alpha$ : A constant greater than or equal to 1 and it is decided by experiments

Fast combinatorial optimization algorithm (F-PSA) combined Powell algorithm with simulated annealing: Fast Simulated Annealing combinatorial optimization algorithm (F-PSA) is the optimization algorithm from best reserved simulated annealing search strategy into local Powell optimization algorithm. Initially, according to the Powell algorithm to optimize, a local minimum point was quickly searched, using the best reserved simulated annealing search strategy. After more benefits were obtained, it immediately transferred to the direct method, the quickly search the bottom, followed by interaction. Not only can get global optimal solution, but also reduce the number of iterations.

Specific process of fast simulated annealing combinatorial optimization algorithm can be summarized as follows:

- From the initial point, implementation of Powell algorithm obtains a local extreme point.
- Determine the initial temperature $T_0$ of best reserved simulated annealing algorithm.
- From the obtained local extreme points, using the best reserved simulated annealing algorithm, to make global search by random strategy (Hou and Zhu, 2011). A pre-designated number of searches of $n$ ($n$ is larger), in $n$ times if the point objective function value is less than the local minimum point (more benefits), then starting from a more advantages in Powell algorithm to optimize; in the $n$ times search another advantage could not be found, iteration can stop, extreme solution obtained is the global optimum.

Improved simulated annealing artificial neural network: Combination with Fast Simulated Annealing optimization algorithm (F-PSA) as a training algorithm of neural network, which is an improvement of BP network. Best reserved simulated annealing algorithm is an integrated gradient descent algorithm and heuristic search method of random process, which is used to solve the external solutions of neural network to jump out of local optimum to obtain the global optimal solution for the corresponding algorithm to improve forecast accuracy and which can further improve network convergence, combined with Powell algorithm.

In specific process of algorithm implementation, F-PSA treats all weight sets of the network as a vector of solution. Objective function in F-PSA is constructed as the following formula, so that the minimum value is corresponding to the optimal solution:

\[ E = \frac{1}{2p} \sum_p \sum_k (t_{pk} - o_{pk})^2 \]  

where,  
$p$ : The number of training samples  
$k$ : The number of output layer neurons  
$t_{pk}$ : The desired output of $k$-th neurons on $p$-th samples  
$o_{pk}$: The network output of $k$-th neurons on $p$-th samples

Procedure of improved simulated annealing artificial neural network can be summarized as follows:

**Step 1:** Initialization, initial network weights of $S_0$ are produced randomly and set the initial temperature $T_0>0$, the number of iterations $i = 0$ and test accuracy of $\varepsilon$. $f_{out} = f(S_0)$, $f^* = f(S_0)$, $S_p = S_0$.

**Step 2:** To treat network weights of $S_p$ as the initial starting point $S(0)$, which is optimized by the Powell algorithm and quickly search for a local minimum point to get a new set of network weights $S_p$, so that $S_l = S_p$, $f_{out} = f(S_l)$, $f^* = f(S_l)$.

**Step 3:** Network weights of $S_l$ were looked as iteration value of $x$, to set the current solution $S(i) = x$, $T = T_i$, to operate with best reserved for
According to accepted guidelines, to get a new set of network weights \( S_{i+1} \). This paper determined laws of annealing decline of \( T, T_i = T_0 / (1 + \ln (i)) \) and \( i = i + 1 \).

**Step 4:** If after simulated annealing operation the network weights of \( S_{i+1} \) meets the precision requirements or the number of iterations, the algorithm ends; otherwise, if \( f(S_{i+1}) < f_{\text{out}} \), then let \( S_p = S_{i+1} \) and turn to step two. If \( f(S_{i+1}) < f_{\text{out}} \), then \( S_i = S_{i+1} \) and go to step 3. Corresponding algorithm flow is shown in Fig. 1.

**MATHEMATICAL MODEL OF MOBILE ROBOT PATH PLANNING**

Mobile robot path planning is divided into two types, global path planning with fully known environmental information and local path planning with completely unknown or partially unknown environmental information. This algorithm is mainly used in mobile robot global path planning problem. Improved simulated annealing artificial neural network is properly applied in mobile robot path planning; we must first set up neural networks with an obstacle collision penalty function and energy function of fast simulated annealing algorithm for combinatorial optimization.

Because the path for an object is represented by a set of via points, its collision with obstacles can be considered as a sum of the collision between its via points and obstacles. To determine the degree of collision between a point and an obstacle, the collision penalty function is defined as a three layer neural network for each obstacle as shown. Each of the three units in the bottom layer represents respectively the x, y, z coordinate of a point. Each unit in the middle layer corresponds to one inequality constraint of the obstacle. The connections between the bottom layer and the middle layer are assigned to the coefficients of x, y, z in the inequality constraints and the threshold of a middle layer unit is assigned to the constant term in its inequality constraint. The connections between the top layer and the middle layer are all assigned to 1 and the threshold of the top layer unit is assigned to 0.5 less than the number of constraints.

**Neural network with obstacle collision penalty function:** To quantify collision nature between obstacles and paths, collision penalty function of paths is defined as the sum of the collision penalty function of path points and collision penalty function of one point is represented with connect network to the various obstacles. Also that the mobile robot moving in a limited two-dimensional space was assumed, obstacles in work space can be described with the convex polygon, which is a set of linear inequalities, as a particle of robot can be neglected, then all points in obstacles must satisfy all inequality constraints. The connected network structure of N obstacles collision penalty function is shown in Fig. 2.

Network consists of input layer, two middle layer and output layer, two neurons on input layer represent path point coordinates of x and y; each node on first middle layer corresponds to inequality constraints of a barrier, as each range of obstacles was constrained with four inequality, so there are a total of 4*N intermediate layer nodes in the network; in the second middle layer there are N nodes, that is N obstacles; output layer has one node, representing the collision penalty function of path points of \((x_i, y_i)\). Connection weights coefficient between input layer and first intermediate layer is one coefficient in front of x and y in inequality constraints for each obstacle, threshold of each node in first intermediate layer is the constant item in corresponding the inequality constraints for each obstacle; weights are 1 from the first intermediate layer to the second intermediate layer and from the second intermediate layer to output layer, threshold of second middle layer nodes for each obstacle is the negative number after inequality number minus 0.5, because the scope of each obstacle was constrained with four inequalities, all thresholds of each node in the second middle layer are -3.5. Network operations are as follows:

\[
I_{wn} = o_{w_{x_i}}x_i + o_{w_{y_i}}y_i + \theta_{wn}
\]

\[
O_{wn} = f(I_{wn})
\]

\[
T_i = \sum_{m=1}^{M} O_{wn} + \theta_i
\]
Fig. 2: Network structure of collision penalty function of N obstacles

\[ C_i^k = f(T_k)k = 1, 2, ..., K \]  

(12)

\[ C_i = \sum_{k=1}^{K} C_i^k \]  

(13)

where,

- \( I_{km} \): The input of k-th obstacle, relative to m-th node in the first intermediate layer
- \( \theta_{km} \): Threshold of k-th obstacle to m-th node in first middle layer
- \( O_{km} \): Output of k-th obstacle to m-th node in first middle layer
- \( T_k \): Input of nodes in second intermediate layer
- \( \theta_k \): Node thresholds in second middle layer
- \( C_i \): Node output in second middle layer
- \( C_i^k \): The output of output layer

Two activation functions of the middle layer are Sigmoid function, \( f(x) = 1/(1+e^{-x}) \), which reflects the collision degree in the path point and the obstacle, the greater the output number, the closer path points approaching at the center of obstacle, on the contrary, the more far away from the path points to obstructions.

Energy function of fast simulated annealing combinatorial optimization algorithm: Collision-free path planning problem can be equivalent to specific optimization problem with two constraints. The one is avoiding collision between path points and obstacles; the other is to require planning paths as short as possible. Through quantifying these two constraints, path planning problem has come into an extreme problem or an optimization problem, energy function of optimization composed of two parts of path length and penalty function of obstacles. Collision penalty function of one path is defined as the sum of collision penalty function of path points and collision penalty function of a path point can be calculated through the appropriate neural network, energy function of the entire path can be expressed as:

\[ E = \lambda_1 E_1 + \lambda_c E_c \]  

(14)

\[ E_1 = \sum_{i=1}^{N} \sum_{j=1}^{K} ((x_{i+1} - x_j)^2 + (y_{i+1} - y_j)^2) \]  

(15)

\[ E_c = \sum_{i=1}^{N} C_i \]  

(16)

where,

- \( E_1 \): The square sum of all segments length on path, reflecting the entire length of the path
- \( E_c \): The collision penalty function of entire path
- \( N \): The number of path points
- \( K \): The number of obstacles
- \( \lambda_1, \lambda_c \): The corresponding weights

Based on fast simulated annealing combinatorial optimization algorithm, path planning problem has come into: \( \min E, X \in \mathbb{R}^2 \). That is optimization problem to find the minimum value of energy function \( E \) in the solution space, since the whole energy \( E \) is a function of path points, using fast simulated annealing algorithm for solving combinatorial optimization, the global optimal solution of energy function can be obtained and a collision-free optimal way.

Specific implementation process of improved simulated annealing artificial neural network:

Step 1: Initialization, any feasible path from starting point to target point was randomly produced, SA looks each possible solution path as a solution vector and the initial feasible path is chosen as uniform distribution points on a straight line from start to finish. To set M path points in the generated path, indicated with the serial number from 1 to M, as the robot’s workspace was divided by grid, path-point number in each path was corresponding with its
Cartesian coordinates. And set the initial temperature $T_0 > 0$, the number of iterations $i = 0$, test accuracy is $\varepsilon$:

$$f_{opt} = f(S_0), f_x = f(S_x), S_p = S_0$$

Step 2: Feasible path of $Sp$ is the initial starting point $S(0)$, which is optimized by the Powell algorithm and quickly search a local minimum point and get a new feasible path $Sp'$, so:

$$S_i = S_p, f_{opt} = f(S_i), f_x = f(S_x).$$

Step 3: Feasible path of $Si$ is iteration value of $x$, set the current solution $S(i) = x, T = Ti$, to operate with simulated annealing. According to accepted guidelines, to get a new feasible path $S_{i+1}$. This book uses the following formula $T = T_0(1 + \ln(i))$ of $T$ for annealing, $i = i + 1$.

Step 4: After simulated annealing operation, if a feasible path of $Si+1$ meets the accuracy requirements or iterations number, then turn to step 5; Otherwise, if $f(S_{i+1}) < f_{opt}$, then $S_p = S_{i+1}$, turn to step 2. If $f(S_{i+1}) < f_{opt}$, then $S_i = S_{i+1}$, turn to step 3.

Step 5: To change path points and to truncate redundant serial number between two identical serial number in possible path, together with one of the two in a same number to simplify the path, the algorithm ends, the specific algorithm flow chart refers to Fig. 1.

**SIMULATION RESULTS**

Under VC++ 6.0 environment, the corresponding algorithm program was compiled with C++ and carry on simulation experiments for algorithms, the selected parameter is, the initial temperature of $T = 30\,^{\circ}\mathrm{C}, T_{min} = 0.1, \lambda_1 = \lambda_2 = 0.5$, the simulation results shown in Fig. 3, in the figure boxes indicate obstacle, S represents the starting point, O represents the target point. Figure 3a and 3b are planned paths with the same environment information but from different start points and ends, from the simulation results it can be seen, the algorithm paper proposed is correct and effective, not only do satisfy the shortest path and avoiding collisions with obstacles, but also improved convergence rate significantly.

**CONCLUSION**

This paper presents mobile robot path planning algorithm combined improved simulated annealing algorithm with artificial neural networks, on the one hand, due to the introduction of best reversed simulated annealing algorithm and its combination with Powell algorithm, to speed up the convergence rate, shorten the computing time of path planning, the global optimal solution can quickly be got; on the other hand, energy function of improved simulated annealing combinatorial optimization algorithm was made with obstacle collision penalty function expressed with neural network and the path length, so the solution makes not only satisfy the shortest path and avoid collision with obstacles. The simulation results prove the effectiveness and practicality of the method and which can effectively improve the calculation speed and ensure the quality of path planning.

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