Covariance Intersection Kalman Fuser for Two-sensor System with Time-delayed Measurements

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Abstract: For a two-sensor linear discrete time-invariant stochastic system with time-delayed measurements, by the measurement transformation method, an equivalent system without measurement delays is obtained and then using the Covariance Intersection (CI) fusion method, the covariance intersection steady-state Kalman fuser is presented. It can handle the estimation fusion problem between local estimation errors for the system with unknown cross-covariances and avoid a large computed burden and computational complexity of cross-covariances. It is proved that its accuracy is higher than that of each local estimator and is lower than that of optimal Kalman fuser weighted by matrices with known cross-covariances. A Monte-Carlo simulation example shows the above accuracy relation and indicates that its actual accuracy is close to that of the Kalman fuser weighted by matrices, hence it has good performances.

Keywords: Consistency, covariance intersection fusion, information fusion kalman fuser, time-delayed measurement, unknown cross-covariance

INTRODUCTION

Multisensor information has been applied to many fields, such as Guidance, defense, robotics, tracking, signal processing (Hall and Llinas, 1997; Li et al., 2003; Bar-Shalom and Li, 2001). There are two kinds of optimal information fusion methods: the centralized and distributed information fusion, where the distributed weighted fusion method applies widely because of its less computational burden. Even if, to compute the optimal distributed weighted fusion Kalman estimators (Sun and Deng, 2004; Bar-Shalom and Campo, 1986), it is required to compute the cross-covariances among local Kalman estimator errors, which needs a large computational burden, because these cross-covariances are generally unknown (Deng et al., 2008) or their computation is very complex (Sun and Deng, 2009; Sun et al., 2009; Liggins et al., 2009). In order to overcome this limitation, a Covariance Intersection (CI) fusion method was presented and developed in Julier and Uhlmann (1997), Chen et al. (2002), Julier and Uhlmann (1996), Julier and Uhlmann (2009) and Arambel et al. (2001), which can solve the fused filtering problems with unknown cross-covariances and has the consistency and robustness (Julier and Uhlmann, 1997). It has wide applications (Julier and Uhlmann, 2007; Guo et al., 2010; Bolzani et al., 2007). Besides that, the systems with time-delayed measurement exist widely in control and estimation fields, which results that it is more difficult to compute the cross-covariances. Now there are augmented state method (Kailath et al., 2000) and non-augmented state method (Sun and Deng, 2009; Zhang and Xie, 2007) to convert the system with time delays into that without time delays.

In this study, for the two-sensor system with measurement delays, a measurement transformation approach is presented, by which the fused estimation problem is converted into that for systems without measurement delays (Sun and Deng, 2009; Sun et al., 2009; Liggins et al., 2009). And, to overcome the difficulty and complexity of computing cross-covariances, using the CI method (Julier and Uhlmann, 1996, 1997, 2009; Chen et al., 2002; Guo et al., 2010; Bolzani et al., 2007), a CI fusion steady-state Kalman fuser is presented, which avoids to find the cross-covariances and can reduce the computational burden. It can handle the system with unknown cross-covariances and it has higher accuracy comparing with local filters and is lower than that of optimal fuser weighted by matrices. The CI fuser is consistent and the good performance.

PROBLEM FORMULATION

Consider the two-sensor linear discrete time-invariant stochastic system:
\[ x(t + 1) = Fx(t) + Gw(t) \quad (1) \]
\[ z_i(t) = Hx(t - \tau_i) + \xi_i(t), \quad i = 1, 2 \quad (2) \]

where, \( t \) is the discrete time, \( \tau_i \) is the measurement delay, \( x(t) \in \mathbb{R}^n \) is the state, \( z_i(t) \in \mathbb{R}^{m_i} \) is the measurement, \( w(t) \in \mathbb{R}^r \) and \( \xi_i(t) \in \mathbb{R}^{m_i} \) are uncorrelated white noises with zero mean and variances \( Q_w \) and \( Q_{\xi_i} \), respectively.

The objectives are to find the local steady-state optimal Kalman estimators \( \hat{x}_i(t + |t + \tau_i) \). From (2), we have the new measurement equation without measurement delays.

\[ y_i(t) = Hx(t) + v_i(t), \quad i = 1, 2 \quad (5) \]

where, the new measurement noise \( v_i(t) \) is white noise with zero mean and variance \( Q_{v_i} = Q_{\xi_i} \).

Denoting the linear space spanned by the stochastic variables \( (z_i(t+N), z_i(t+N-1), \ldots) \) as \( L_z(z_i(t+N), z_i(t+N-1), \ldots) \) and the linear space spanned by the stochastic variables \( (y_i(t + N - \tau_i), y_i(t + N - \tau_i - 1), \ldots) \) as \( L_y(y_i(t + N - \tau_i), y_i(t + N - \tau_i - 1), \ldots) \), we have the relation between the two linear measurement spaces as:

\[ L_z(z_i(t+N), z_i(t+N-1), \ldots) = L_y(y_i(t + N - \tau_i), y_i(t + N - \tau_i - 1), \ldots) \quad (6) \]

So we have the relation as:

\[ \tilde{x}_i^N(t + |t + N) = x_i(t + |t + N - \tau_i) \quad (7) \]

where \( \tilde{x}_i(t + |t + N - \tau_i) \) is the local minimum variance filters of \( x(t) \) based on \( (y_i(t + N - \tau_i), y_i(t + N - 1 - \tau_i), \ldots) \) and \( \tilde{x}_i^N(t + |t + N) \) is the local minimum variance filters based on \( L(z_i(t+N), z_i(t+N-1), \ldots) \). So the relation between the estimation errors \( \tilde{x}_i^N(t + |t + N) \) and \( \hat{x}_i(t + |t + N) \) is:

\[ \tilde{x}_i^N(t + |t + N) = \tilde{x}_i(t + |t + N - \tau_i), \quad i = 1, 2 \quad (8) \]

where, \( \tilde{x}_i^N(t + |t + N) = x(t) - \tilde{x}_i^N(t + |t + N) \), \( \hat{x}_i(t + |t + N) = x(t) - \hat{x}_i(t + |t + N) \).

Defining the steady-state estimator error variances and cross-covariances are \( P_i^N(N) = E[\tilde{x}_i^N(t + |t + N)\tilde{x}_i^N(t + |t + N)] \) and \( p_{ij}^N(N) = E[\tilde{x}_i^N(t + |t + N)\tilde{x}_j^N(t + |t + N)] \), respectively. So we have obtained

\[ P_i^N(N) = P_i(N - \tau_i, N - \tau_j), \quad i = 1, 2 \quad (9) \]

where, \( P_i(N - \tau_i, N - \tau_j) \) is the local steady-state Kalman estimator based on the measurement \( (y_i(t + N), y_i(t + N - 1), \ldots) \) and the cross-covariance \( p_{ij}^N(N) \) is the local steady-state Kalman estimator based on the measurement \( (y_i(t + N - \tau_i), y_i(t + N - 1), \ldots) \).

**Lemma 1:** (Sun and Deng, 2009) For the two-sensor system (1) and (5), the local steady-state Kalman predictor \( \tilde{x}_i(t + 1|t) \) is given by:

\[ \tilde{x}_i(t + 1|t) = \Psi_{\mu i}x_i(t + |t - 1) + K_{\mu i}y_i(t), \quad i = 1, 2 \quad (10) \]

where, \( \Psi_{\mu i} = \Phi - K_{\mu i}H_i \quad (11) \)

\[ K_{\mu i} = \Phi\Sigma_iH_i^T(H_i\Sigma_iH_i^T + Q_{\mu i})^{-1} \quad (12) \]

The prediction error variance matrix \( \Sigma_i \) satisfies the steady-state Riccati equation:

\[ \Sigma_i = \Phi(\Sigma_i - \Sigma_iH_i^T(H_i\Sigma_iH_i^T + Q_{\mu i})^{-1}H_i\Sigma_i + \Gamma Q_{\mu i}\Gamma^T) \quad (13) \]

and the cross-covariance matrix \( \Sigma_{ij} = E[\tilde{x}_i(t + |t + 1)\tilde{x}_j(t + |t + 1)] \) between local prediction errors satisfies the Lyapunov equation:

\[ \Sigma_{ij} = \Psi_{\mu i}\Sigma_i\Psi_{\mu j}^T + \Gamma Q_{\mu i}\Gamma^T, \quad i, j = 1, 2, i \neq j \quad (14) \]

The -\( k_j \) steps steady-state Kalman predictor \( \tilde{x}_i(t + |t + k_j) \) is given by:

\[ \tilde{x}_i(t + |t + k_j) = \Phi^{-k_j}x_i(t) + k_j + 1|t + k_j), \quad k_j \leq -2 \quad (15) \]

The -\( k_j \) steps Kalman prediction error variance \( P_i(k_j, k_j) = E[\tilde{x}_i(t + |t + k_j)\tilde{x}_i(t + |t + k_j)] \) and cross-covariance \( p_{ij}(k_j, k_j) = E[\tilde{x}_i(t + |t + k_j)\tilde{x}_j(t + |t + k_j)] \) are given by:

\[ P_i(k_j, k_j) = \Phi^{-k_j-k_j}\Sigma_i\Phi^{-k_j-k_j} + \sum_{j=1}^{k_j} \Phi^{-k_j-j}\Gamma Q_{\mu i}\Gamma^T, \quad k_j \leq -2 \quad (16) \]
\[
P_i(k_i,k_j) = \Phi^{-k_i} \Sigma_i \Phi^{k_j-1} + \sum_{j=0}^{\max(k_i,k_j)-2} \Phi^j \Gamma Q \Gamma^j \Phi^{k_j-1}, k_i,k_j \leq -2
\]

The steady-state Kalman filter \( \hat{x}_i(t|t) \) is as follows:

\[
\hat{x}_i(t|t) = \Psi \Phi \chi_i(t-1|t-1) + K_i e_i(t), i = 1,2
\]

where,

\[
\Psi_i = (I - K_i H_i) \Phi
\]

\[
K_i = \Sigma_i H_i (H_i \Sigma_i H_i^T + Q_i)^{-1}
\]

The corresponding steady-state Kalman filtering error variance \( P_i = E[\hat{x}_i(t|t)\hat{x}_i(t|t)^T] \) and cross-covariance \( P_{ij} = E[\hat{x}_i(t|t)\hat{x}_j(t|t)] \) are given by:

\[
P_i = (I - K_i H_i) \Sigma_i
\]

\[
P_{ij} = \Psi^T \Sigma_i H_i^T (H_i \Sigma_i H_i^T + Q_i)^{-1}
\]

The steady-state Kalman smoother \( \hat{x}_i(t|t+k_i) \), \( k_i > 0 \) is given by:

\[
\hat{x}_i(t|t+k_i) = x_i(t|t-1) + \sum_{r=0}^{k_i} K_i(r) e_i(t+r) i = 1,2
\]

where,

\[
K_i(r) = \Sigma^T \Psi^T H_i^T (H_i \Sigma_i H_i^T + Q_i)^{-1}
\]

\[
e_i(t) = y_i(t) - H_i \hat{x}_i(t|t-1)
\]

The Kalman smoothing error variance \( P_i(k_i,k_i) = E[\hat{x}_i(t|t+k_i)\hat{x}_i(t|t+k_i)^T] \) and cross-covariance \( P_{ij}(k_i,k_j) = E[\hat{x}_i(t|t+k_i)\hat{x}_j(t|t+k_j)] \) are given by:

\[
P_i(k_i,k_i) = \Sigma_i - \sum_{r=0}^{k_i} K_i(r) H_i \Sigma_i H_i^T + Q_i K_i(r), k_i > 0
\]

\[
P_i(k_i,k_j) = \sum_{r=0}^{\max(k_i,k_j)-1} K_i(r) H_i \Sigma_i H_i^T K_i(r), k_i,k_j > 0
\]

**Lemma 2:** (Sun and Deng, 2009) For the two-sensor system (1) and (5), the steady-state Kalman filtering error variance \( P_0(k_i,k_j) = E[\hat{x}_i(t|t+k_i)\hat{x}_j(t|t+k_j)] \) and cross-covariance \( P_{0i}(k_i,k_j) = E[\hat{x}_i(t|t+k_i) \hat{x}_j(t|t+k_j)] \) are given by:

\[
P_0(k_i,k_j) = P_0
\]

\[
P_{0i}(k_i,k_j) = P_0
\]

\[
P_i(k_i,k_j) = \Sigma_i - \sum_{r=0}^{k_i} K_i(r) H_i \Sigma_i H_i^T + Q_i K_i(r), k_i > 0
\]

\[
P_i(k_i,k_j) = \sum_{r=0}^{\max(k_i,k_j)-1} K_i(r) H_i \Sigma_i H_i^T K_i(r), k_i,k_j > 0
\]

**Case 2:** when \( k_i < 0, k_j < 0 \)

\[
P_i(k_i,k_j) = \Sigma_i, (k_i = -1)
\]

\[
P_i(k_i,k_j) = \Sigma_i, (k_i = -1, k_j = -1)
\]

**Case 3:** when \( k_i > 0, k_j > 0 \)

\[
P_i(k_i,k_j) = \Sigma_i - \sum_{r=0}^{k_i} K_i(r) H_i \Sigma_i H_i^T + Q_i K_i(r), (k_i > 0)
\]

\[
P_i(k_i,k_j) = \Sigma_i - \sum_{r=0}^{k_i} K_i(r) H_i \Sigma_i H_i^T + Q_i K_i(r), (k_i > 0, k_j > 0)
\]

**Case 4:** when \( k_i < 0, k_j < 0 \)

\[
P_i(k_i,k_j) = \Sigma_i - \sum_{r=0}^{k_i} K_i(r) H_i \Sigma_i H_i^T + Q_i K_i(r), (k_i < 0, k_j < 0)
\]
where the weighted matrix is given by:

\[
P_m(k, k) = \Psi^{T} \Sigma \Phi^{T} \Psi - \sum_{j=1}^{d-2} \Psi^{T} \Gamma \Omega \Gamma^{T} \Phi^{T} \Psi + \sum_{i=1}^{d-2} \sum_{j=1}^{d-2} K_i(s) H \Psi^{T} \Gamma \Omega \Gamma^{T}
\]

Lemma 3: (Sun and Deng, 2004) For the two-sensor system (1) and (5), the optimal Kalman fuser weighted by matrices is given by:

\[
\hat{x}_m^w(t | t + N) = \Omega \hat{x}_1^w(t | t + N) + \Omega \hat{x}_2^w(t | t + N)
\]

where, \( \hat{x}_1^w(t | t + N) = \Omega \hat{x}_1^w(t | t + N) + \Omega \hat{x}_2^w(t | t + N) \) and \( \hat{x}_2^w(t | t + N) = \Omega \hat{x}_2^w(t | t + N) + \Omega \hat{x}_1^w(t | t + N) \)

where the weighted matrix is given by:

\[
\Omega = \Omega \Omega^T = \langle e^T P^{-1}(k_i, k_i) e \rangle + \langle e^T P^{-1}(k_i, k_i) e \rangle^T
\]

The optimal Kalman fuser \( \hat{x}_m^w(t | t + N) \) weighted by scalars is given by:

\[
\hat{x}_m^w(t | t + N) = a_1 \hat{x}_1^w(t | t + N) + a_2 \hat{x}_2^w(t | t + N)
\]

where the optimal weighted coefficients \( a = [a_1, a_2] \) is given as:

\[
a = \frac{\phi^T P^{-1} \phi}{e_\phi^T (P^w)^{-1} e_\phi}
\]

where,

\[
e_\phi^T = \begin{bmatrix} 1 & 1 \end{bmatrix}
\]

\[
P^w = \begin{bmatrix} \phi^T P^{-1} \phi & \phi^T P^{-1} \phi \end{bmatrix}
\]

The optimal Kalman fuser \( \hat{x}_m^w(t | t + N) \) weighted by diagonal matrices is given by:

\[
\hat{x}_m^w(t | t + N) = A_1 \hat{x}_1^w(t | t + N) + A_2 \hat{x}_2^w(t | t + N)
\]

where the optimal weighted diagonal matrices \( A_j \) is given as:

\[
A_j = \begin{bmatrix} a_{j1} & \cdots & a_{jn} \end{bmatrix}
\]

The component expression of \( \hat{x}_m^w \) and \( \hat{x}_d^w \) are given by:

\[
\hat{x}_m^w = \begin{bmatrix} x_{m1}^w \\ \vdots \\ x_{mn}^w \end{bmatrix}
\]

So the equation (50) is equivalent with:

\[
\hat{x}_m^w(t | t + N) = \sum_{l=1}^{2} a_{jl} x_l^w(t | t + N), l = 1, \ldots, n
\]

where the optimal weighted coefficient vector is:

\[
a_j = [a_{j1} \ldots a_{jn}]
\]

and defining:

\[
P^w = \begin{bmatrix} P_{11}^w & P_{12}^w \\ P_{21}^w & P_{22}^w \end{bmatrix}
\]

where \( P_{ll}^w \) is the (l, l)th diagonal component of \( P_j(k_i, k_j) \) and the optimal component fused error variance \( P_{ll}^w \) and the optimal fused error variance \( P_d \) are given by:

\[
P_d = [e^T (P^w)^{-1} e_j], l = 1, \ldots, n
\]

The steady-state CI Kalman Fuser

From the proposed interpretation, it is clear that the computation of getting the cross-covariances between local filtering is very complex and the computed burden is very large. So a CI fusion method was presented (Julier and Uhlmann, 1997; Chen et al., 2002).

Consider a two-sensor system (1) (2) and (5), when \( P_1 \) and \( P_2 \) are known, but the cross-covariance \( P_{12} \) is unknown, the CI Kalman fuser without \( P_{12} \) is given by:

\[
\hat{x}_m^w(t | t + N) = \sum_{l=1}^{2} a_{jl} x_l^w(t | t + N) + (1 - \omega) \hat{x}_d^w(t | t + N)
\]

where the CI fusion error variance matrix \( P_{CI} \) is given by:

\[
P_{CI} = [a (P^w)^{-1} + (1 - \omega) P_{CI}^{-1}]
\]
where \( P_i \) \( \oplus P_j \) \( i, j = 1, 2 \), \( P_i \) \( \oplus P_j \) \( i, j = 1, 2 \) and the weighted coefficient \( \omega \in [0, 1] \) and minimizes the performance index:

\[
J = \min_{\omega} \text{tr} P_{CI} = \min_{\omega \in [0,1]} \text{tr}[(\omega P^{-1}_1 + (1-\omega)P^{-1}_2)]^{-1}
\]

(59)

where the notation \( \text{tr} \) denotes the trace of matrix. The optimal weighting coefficient \( \omega \) can be solved by the gold section method or Fibonacci method (Julier and Uhlmann, 1996).

### THE CONSISTENCY AND ACCURACY OF CI KALMAN FUSER

**Theorem 1:** The actual CI fusion error variance matrix \( \overline{P}_{CI} \) is given by:

\[
\overline{P}_{CI} = P_{CL} + \omega(1-\omega)P^{-1}_1 P_i P^{-1}_2 + \omega\text{tr}(\omega P^{-1}_1 + (1-\omega)P^{-1}_2)^{-1}
\]

(60)

**Proof:** From (58), we can obtain \( P_{CI}[\omega P^{-1}_1 + P^{-1}_2] = I \), which yields

\[
x(t) = P_{CL}[\omega P^{-1}_1 x(t) + (1-\omega)P^{-1}_2 x(t)]
\]

(61)

Subtracting (57) from (61), we get the actual CI fuser error:

\[
\tilde{x}_{CI}(t|t + N) = P_{CL}[\omega P^{-1}_1 \tilde{x}_i(t|t + N) + (1-\omega)P^{-1}_2 \tilde{x}_i(t|t + N)]
\]

(62)

The actual CI fusion error variance \( \overline{P}_{CI} = E[\tilde{x}_{CI}(t|t + N)\tilde{x}_{CI}^T(t|t + N)] \), where \( E \) denotes the mathematical expectation, \( T \) denotes the transpose. Substituting (62) into \( \overline{P}_{CI} \) yields that (60) holds. The proof is completed.

**Theorem 2:** For local and fused Kalman filters, we have the accuracy relations:

\[
P_a \leq P_i \leq P_a \leq P_{CI} \leq P_a, i = 1, 2
\]

(63)

\[
\text{tr} \overline{P}_{CI} \leq \text{tr} P_{CI}
\]

(64)

\[
\text{tr} P_a \leq \text{tr} P_i \leq \text{tr} P_a \leq \text{tr} P_{CI}, i = 1, 2
\]

(65)

\[
\text{tr} P_a \leq \text{tr} \overline{P}_{CI} \leq \text{tr} P_{CI}, i = 1, 2
\]

(66)

**Proof:** from (41), \( \tilde{x}_m^z(t|t + N) \) is the Unbiased Linear Minimum Variance (ULMV) estimate of \( x(t) \) based on the linear space \( L_m = L_m(\tilde{x}_1^z(t|t + N), \tilde{x}_2^z(t|t + N)) \) spanned by \( (\tilde{x}_1^z(t|t + N), \tilde{x}_2^z(t|t + N)) \). From (45), (50), (57), we have \( \tilde{x}_m^z(t|t + N) \), \( \epsilon L_m, \theta = s, d, CI, i, \)

which yields (63). It has been proven \( \overline{P}_{CI} \leq \overline{P}_{CI} \) and \( \text{tr} P_m \leq \text{tr} P_i \leq \text{tr} P_a \), \( i = 1, 2 \) in the reference (Sun and Deng, 2004; Julier and Uhlmann, 1997; Chen et al., 2002), so we have (64) and (65) hold. In the performance index (59), taking \( \omega = 0 \), we have \( J = \text{tr} P_2 \) and taking \( \omega = 1 \), we have \( J = \text{tr} P_1 \). Hence the optimal weighting coefficient \( \omega \in [0, 1] \) yields \( \text{tr} P_{CI} \leq \text{tr} P_2 \leq \text{tr} P_1 \), i.e. \( \text{tr} P_{CI} \leq \text{tr} P_i \), \( i = 1, 2 \). Taking trace operation for (63) and (64), we have (66) holds. The proof is completed.

### SIMULATION EXAMPLE

Consider the tracking system with two-sensor and with time-delayed measurements:

\[
x(t + 1) = \begin{bmatrix} 1 & T_s \\ 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0.5 \xi^z_1(t) \\ T_s \end{bmatrix}
\]

\[
z_i(t) = H_ix(t - \tau_i) + \xi_i(t), i = 1, 2
\]

\[
H_i = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, H_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \tau_1 = 1, \tau_2 = 2
\]

where, \( x(t)\in \mathbb{R}^2 \) is the state, \( z_i(t) \) is the measurement with delays for sensor \( i \), \( w(t) \) and \( \xi(t) \) are white noise with zero mean and variances \( Q_w \) and \( Q_\xi \), respectively. According to the corresponding transformation, we obtain the measurement \( y_i(t) \) without time delays:

\[
y_i(t) = z_i(t + \tau_i) = H_i x(t) + v_i(t)
\]

where \( v_i(t) = \xi_i(t + \tau_i) \) is the transformed measurement white noise with zero mean and variance \( Q_\xi = Q_\zeta \). Thus the problem of finding the local steady-state optimal Kalman smoother \( \tilde{x}_i(t|t + N), i = 1, 2 \) based on the measurement \( z_i(t) \) is converted into the problem of finding the local steady-state optimal Kalman filter \( \tilde{x}_i(t|t) \) and Kalman predictor \( \tilde{x}_2(t|t - 1) \) based on the new measurement \( y_i(t) \), respectively. Further, applying the Kalman filtering method yields the steady-state optimal fusion Kalman smoothers \( \tilde{x}_m^z(t + N, I, xdsttt + I, xszttt + I \) weighted by matrices,
diagonal matrices and scalars and CI fused Kalman smoother $\tilde{x}_{CI}^E(t|t+1)$. In simulation, we take

$$T_o = 0.4, Q_0 = 1, Q_{ci} = 0.36, Q_{ci} = \begin{bmatrix} 4 & 0 \\ 0 & 0.16 \end{bmatrix}$$

In order to give a powerful geometric interpretation with respect to accuracy relations of local and fusers, the covariance ellipse for a covariance matrix $P$ is defined as the locus of points $\{x: x^T P^{-1} x = c\}$ where $c$ is a constant. In the sequel, $c = 1$ will be assumed without loss of generality. It was proved (Julier and Uhlmann, 1996) that $P_a \leq P_b$ is equivalent to that the ellipse for $P_a$ is enclosed in the ellipse for $P_b$. The simulation results are shown in Fig. 1.

From Fig. 1, it is clearly shown that the covariance ellipse for $P_m$ lies within the intersection of ellipses for $P_1$ and $P_2$, the ellipse for CI fused theoretical covariance $P_{CI}$ encloses the intersection region of ellipses for $P_1$ and $P_2$ (Chen et al., 2002) and passes through the four points of intersection of ellipses for $P_1$ and $P_2$. The ellipse for CI fused actual covariance $P_{CI}$ lies within the ellipse for theoretical covariance $P_{CI}$.

In order to verify the above theoretical results for accuracy relation, 200 Monte-Carlo runs are performed for $t = 1, \ldots, 300$ the results are shown in Fig. 2, where the straight lines denote $\text{tr} P_i$, $i = 0, 1, 2$, CI and $\text{tr} P_{CI}$, the curves denote the corresponding mean square errors $\text{MSE}_i(t)$. We see the CI fuser has good performance, whose accuracy is approximates to the accuracy of the optimal smoother, because $\text{MSE}_{CI}(t)$ is approximates to $\text{MSE}_m(t)$.

**CONCLUSION**

For the two-sensor system with time-delayed measurements and with unknown cross-covariance matrices, a CI fusion steady-state Kalman fuser has been presented, which has the advantage that the computation of cross-covariances can be avoided. It is proved that its accuracy is higher than that of each local estimator and is lower than that of optimal fuser weighted by matrices. A two-sensor system Monte-Carlo simulation example shows that its accuracy is approximates to the accuracy of the optimal fuser and is higher than those of the fusers weighted by diagonal matrices and scalars.

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